MINISTRY OF TRANSPORT AND TELECOMMUNICATIONS OF UKRAINE

ODESA NATIONAL O.S. POPOV ACADEMY OF TELECOMMUNICATIONS

CHAIR OF INFORMATION SECURITY AND DATA COMMUNICATIONS

THE ITERATION ALGORITHM OF THE HARMONIC EQUALIZER PARAMETERS OPTIMIZATION BY COORDINATEWISE DESCENT METHOD

METHODICAL INSTRUCTIONS
FOR PRACTICAL SEMINARS AND LABORATORY LESSON
ON THE SUBJECTS
«METHODS OF OPTIMIZATION» AND
«THE BASIS OF RADIOELECTRONIC EQUIPMENT DESIGN AUTOMATION»

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1 Learning objective

Learning of the iteration algorithm of harmonic equalizer parameters optimization by coordinatewise descent method.

2 References

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3 Home assignment

1 Study theoretical points using given methodical instructions or recommended literature.

2 Be ready to answer the following test questions.

4 Test questions

1 Mathematically formulate the task of optimization of linear distortions parameters in telecommunication channel.

2 Block diagram of optimization of polynomial equalizer parameters.

3 Block diagram and transfer function of harmonic equalizer.

4 Root-mean-square error of a correction as a function of equalizer regulable coefficients.

5 The reason of numerical methods application for searching of equalizer coefficients optimal values.

6 The essence of iterative optimization methods.

7 Peculiarities of iteration algorithm of harmonic equalizer (HE) parameters optimization by coordinatewise descent method.

5 Laboratory task

- display and write down in Table 2 the values of the channel attenuation-frequency characteristic (AFC) samples before the correction (A, dB);
- according to the written values construct a graph of channel AFC before correction;
- display and write down in Table 3 the values of the samples of channel group traveling time (GTT) variation before the correction (T, ms);
- construct a graph of GTT variation in channel before the correction;

− run the optimization of harmonic equalizer parameters (adjustment), introducing on computer demand the following initial data:

- **number of HE taps (regulators)** **(*N*) –** **2**
- **accuracy of HE regulators setting** **(*E*1) –** **0.001**
- **accuracy of correction** **(*E*2) –** **0.01**
- **initial values of HE coefficients (regulators *G*0 and *G*1) are to be selected from Table 1 according to the number of working place in the laboratory:**

Table 1 Initial values of equalizer regulators *G*0 and *G*1

№	0	1	2	3	4	5	6	7	8	9
***G*0**	**0**	**0**	**0,5**	**1**	**1,5**	**2,5**	**3**	**0**	**1**	**2**
***G*1**	**0**	**3**	**3**	**3**	**3**	**3**	**3**	**4**	**4**	**4**

Table 2 Attenuation-frequency characteristic

Attenuation-frequency characteristic, *A*, dB										
f, Hz		62,5	125	250	375	500	625	750	875	1000
Channel	*before correction*									
Equalizer	*after adjustment*									
Channel + equalizer	*chain after equalizer adjustment*									

Table 3 Group traveling time

Group traveling time, *T*, ms										
f, Hz		62,5	125	250	375	500	625	750	875	1000
Channe	*before correction*									
Equalize	*after adjustment*									
Channel + equalizer	*chain after equalizer adjustment*									

Press «Enter» button for the execution of sequent step of adjustment.

Write down the values of equalizer regulators *G*0 and *G*1 and the values of correction error D^2 at every step of adjustment into Table 4.

- look over the AFC and GTT channel characteristics at every step of equalizer adjustment;
- **write down** into Tables 2 and 3 AFC and GTT characteristics of <u>channel before correction</u> and <u>the chain (channel + equalizer) after adjustment completion</u> (at the last iteration);
- **construct the graphs of AFC and GTT in threes**: <u>channel before correction, the chain after the first correcting step</u> and <u>the chain after completion of correcting adjustment</u>;

- **redraw** from the display into the report the level lines of function D^2 and **draw** with a polygonal line a search path with the iterations number pointing;
- **construct a graph** of values of correction error D^2 depending on iteration number (№ of a step) according to the data from Table 4;
- **compare** obtained channel characteristics after the correction with the channel characteristics before the correction by the variation value:

$$\Delta A = A_{f=125\text{ Hz}} - A_{f=500\text{ Hz}}; \quad \Delta T = T_{f=125\text{ Hz}} - T_{f=500\text{ Hz}};$$

- **draw conclusions** on advantages and disadvantages of coordinatewise descent method taking into consideration the results of executed work.

Table 4 Correction error values at the iterations numbers

Iteration number	Regulator ***G*0** value	Regulator ***G*1** value	Correction error $\boldsymbol{D^2}$ value
0			
1			
2			
3			
4			
5			
6			
7			

Note:

1) Numerical data into the Tables 2, 3 and 4 are to be inroduced with three significant figures.

2) For a zero iteration number the initial values of equalizer regulators G0 and G1 are to be written according to the given variant.

6 Report contents

1 **Block diagram** of harmonic equalizer, used in the laboratory lesson.

2 **The principles** of iteration algorithm of adjustment by coordinatewise descent method.

3 **Three graphs of AFC on one figure**: channel before correction, chain after the first step of correction and chain after completion of correcting adjustment (according to the data of Table 2).

4 **Three graphs of GTT on one figure**: channel before the correction, chain after the first step of the correcting and chain after the completion of correcting adjustment (according to the data of Table 3).

5 **Graph of level lines of** function D^2 and search path with the iterations number pointing.

6 **Graph of the values of correction error $\boldsymbol{D^2}$** depending on the iteration number (according to the data of Table 4).

7 **Calculated data** of variation ΔA and ΔT before and after the correction.

8 **Conclusions** on the laboratory lesson results:

a) on the characteristics behaviour,

b) on advantages and disadvantages of coordinatewise descent method.

7 Theory

Optimization of parameters (correction and rectification) of linear distortions in telecommunication channel can be realized by concatenated connection into the transmission chain of linear correcting devices (equalizers), providing in the given frequency range $\omega_l \le \omega \le \omega_u$ the execution of the condition

$$H(j\omega)G(j\omega) = F(j\omega), \tag{1}$$

where $H(j\omega)$, $G(j\omega)$ and $F(j\omega)$ - transfer functions of distorting telecommunication channel, equalizer and «ideally» corrected transmission chain correspondingly.

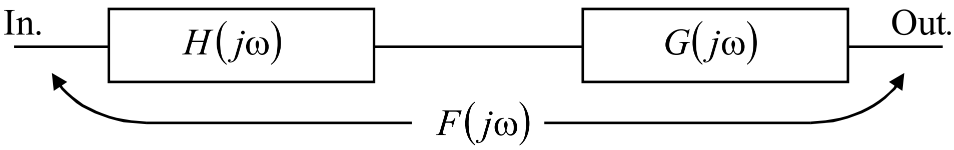

Figure 1 – Connection of equalizer into transmission chain

If the channel transfer function $H(j\omega)$ is known beforehand, equalizer can be calculated and synthesized as a device with fixed parameters, but in many practical tasks function $H(j\omega)$ is not defined beforehand (for example, on switched telecommunication network), and sometimes can be variable during a transmission (for example, in multipath radiochannels). Then the correction task resolves into the synthesis of retunable linear quadrilateral with transfer function $G_{impl}(j\omega)$.

Presently polynomial equalizers are widely used with the transfer functions as a generalized polynomial

$$G_{impl}(j\omega) = \sum_{k=0}^{N} g_k \Psi_k(j\omega) \tag{2}$$

where $\Psi_k(j\omega)$, $(k = 0, 1, ..., N)$ – previously realized (so called basis) functions, and g_k, $(k = 0, 1, ..., N)$ – varied parameters (real numbers), by changing of which the synthesis of necessary equalizer transfer function is done. A functional diagram, shown at fig. 2, corresponds to a formula (2).

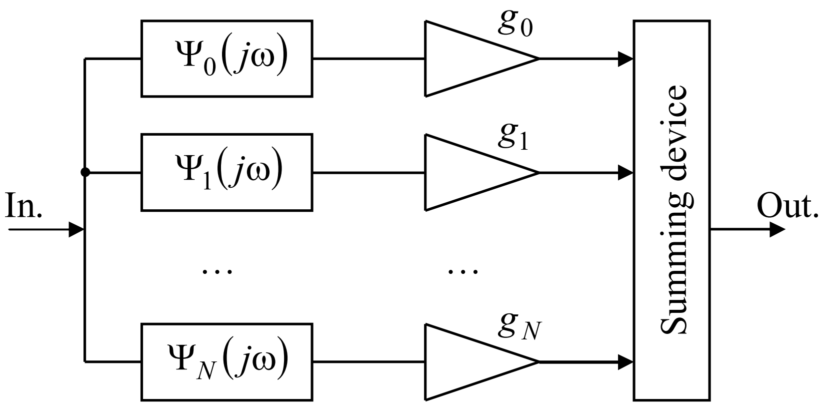

Figure 2 – Functional diagram of polynomyal equalizer

Criteria for the estimation of quality of parameters optimization. An index, which can be considered as the best characterizing quality of transmission, is the error probability (error rate). Under the influence of linear distortion its value increases, as because the influence of intersymbol interference sample of a signal can vary towards a neighbouring permitted value, and the slight interference will be enough for assigning of the incorrect value to the received signal at sample moment. However, both calculation and measuring of error probability in the conditions when not only white noise is reproduced as an interference, entail the high expenses of time.

In many cases for the estimation of transmission quality more simple criteria can be used, indirectly related to error probability. The frequently used index is a root-mean-square (RMS) error. We'll explain its sense.

The required transfer function of equalizer $G(j\omega)$ is always realized with some error. Denoting as $G_{impl}(j\omega)$ and $F_{impl}(j\omega)$ transfer functions of the technically implemented equalizer and the chain «channel plus equalizer» correspondingly (see fig. 1), obtain the expression for correction error

$$e(j\omega) = F(j\omega) - F_{impl}(j\omega) = F(j\omega) - H(j\omega)G_{impl}(j\omega). \qquad (3)$$

While the correction of channels frequency characteristics with the polynomial equlizer expression for the error of required channel transmission function synthesis with an account of (2) can be written as

$$e(j\omega) = F(j\omega) - H(j\omega)\cdot G_{impl}(j\omega) = F(j\omega) - H(j\omega)\cdot \sum_{k=0}^{N-1} g_k \cdot \Psi_k(j\omega),$$

$e(j\omega)$ - complex-valued function of frequency ω. However, observation of $e(j\omega)$ during the equalizer adjustment is difficult.

In practice instead of (3) the most often used value is

$$E[e(j\omega)] = \int_{\omega_l}^{\omega_u} |e(\omega)|^2 \, d\omega, \qquad (4)$$

where ω_l и ω_u – lower and upper frequencies of the telecommunication channel bandwidth. Expression (4), called as root-mean-square error (RMS error) of channel correction, is numerically equal to the square under the curve $|e(\omega)|^2$ (fig. 3).

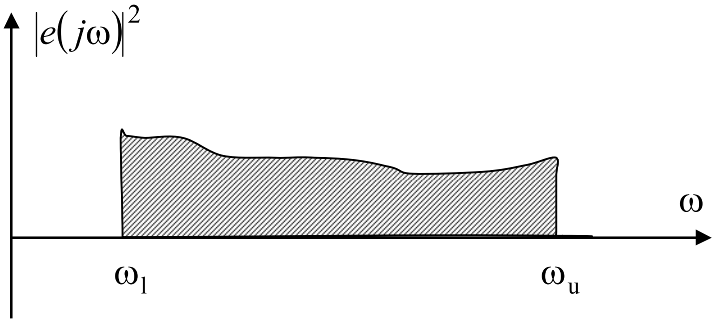

Figure 3 – To the concept of root-mean-square error

Introduce its new designation:

$$D^2(g_0, g_1, \ldots, g_{N-1}) = E[e(j\omega), g_0, g_1, \ldots, g_{N-1}].$$

Process of the finding of error D^2 minimum at varying the parameters g_k, $(k = 0, 1, 2, ..., N-1)$ in the case of apparatus minimization – is called parameters optimization or polynomial equalizer adjustment.

If the optimization of equalizer parameters is carried out by root-mean-square criterion D^2, the error can be monitored in time domain also. Indeed, let for simplicity in (4) $\omega_l = 0$, $\omega_u = \Omega$, where Ω – the highest frequency of low-frequency band, then:

$$D^2 = \int_{-\Omega}^{\Omega} \left| F(j\omega) - \sum_{k=0}^{N-1} g_k \Phi_k(j\omega) \right|^2 d\omega = 2\pi \int_{-\infty}^{\infty} \left[f(t) - \sum_{k=0}^{N-1} g_k \varphi_k(t) \right]^2 dt \quad (5)$$

where

$$f(t) = \frac{1}{2\pi} \int_{-\Omega}^{\Omega} F(j\omega) \cdot e^{j\omega t} d\omega, \quad \varphi_k(t) = \frac{1}{2\pi} \int_{-\Omega}^{\Omega} \Phi_k(j\omega) \cdot e^{j\omega t} d\omega$$

- impulse responces of the circuits with the transfer functions $F(j\omega)$ and $\Phi_k(j\omega)$, correspondingly. At that

$$\Phi_k(j\omega) = H(j\omega) \cdot \Psi_k(j\omega).$$

If the highest frequency in the spectrum of time functions $f(t)$ and $\varphi_k(t)$, $(k = 0, 1, 2, ..., N-1)$ does not exceed Ω, the error D^2 (according to the sampling theorem) can be found by sample values of these functions:

$$D^2 = \frac{2\pi}{T} \sum_{m=-\infty}^{\infty} \left[f(mT) - \sum_{k=0}^{N-1} g_k \varphi_k(mT) \right]^2, \text{ where } T = \frac{\pi}{\Omega}. \quad (6)$$

Block diagram of entire complex, consisting directly of polynomial equalizer and device for adjustment, is shown on fig. 4.

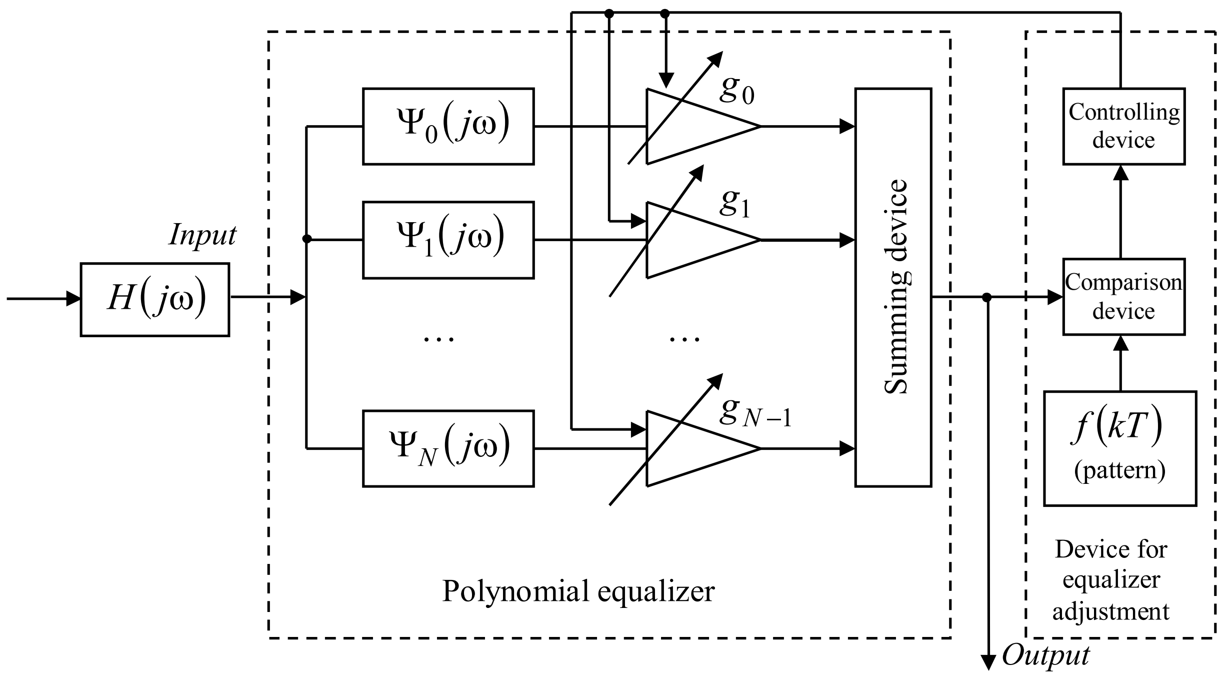

Figure 4 – Block diagram of optimization of polynomial equalizer parameters

The sensor of pattern function (for example, $f(kT)$) sends signals to one of the entrances of comparison device. To its other entrance comes the signal from equalizer output. Comparing device calculates error D^2, and controlling device changes the positions of equalizer adjustment units g_k, $(k = 0, 1, 2, ..., N - 1)$ until the correction error will not attain its minimum value.

Polynomial equalizer with orthogonal basis functions is called harmonic equalizer (HE). It obtained a wide circulation in telecommunication technique due to realization simplicity. The basis functions of HE are following

$$\Psi_k(j\omega) = e^{-j\omega T},\ T = \frac{\pi}{\Omega},\ k = -m, -m+1, \dots (n-1), n.$$

Harmonic equalizer (HE) is implemented on delay line with taps and summing device. Fig. 5 shows the block diagram of HE, employed in the laboratory lesson, with the number of regulators N equal to 2.

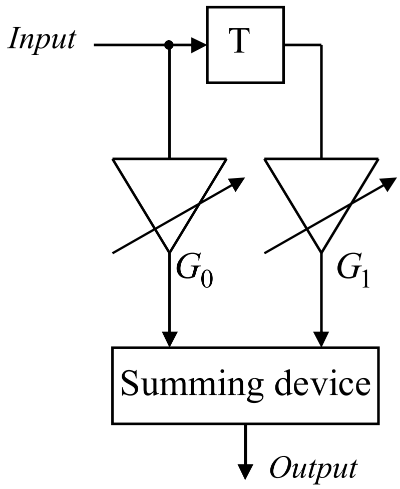

Figure 5 – Block diagram of HE with two variable parameters

Transfer function of such HE has the form

$$G_{impl}(j\omega) = \sum G_k e^{-j\omega kT} = G_0 + G_1 \cdot e^{-j\omega T}. \qquad (7)$$

T – time delay, introduced by the delay element and equal to the sampling interval.

The basic parameters of harmonic equalizer are:

- correcting precision;
- time of adjustment;
- complication of the automatic adjustment system.

Correcting precision depends on the number of equalizer regulable parameters $g_k(G_k)$ and on the value of frequency characteristics variation of telecommunication channel.

Time of equalizer adjustment – is a time during which initial value of error reduces to the required level, which enables the steady operation of the data transmission system.

HE adjustment rate depends on applied optimization method, lying in a basis of automatic adjustment system construction.

Adjustment of harmonic equalizer in accordance with minimum of RMS error

Adjustment of equalizer consists in finding of such values of regulators $(g_0,\ g_1,\ ...,\ g_{N-1})$ in the HE delay line taps, at which root-mean-square error of correction $D^2(g_0,\ g_1,\ ...,\ g_{N-1})$ will be minimal.

Function $D^2(g_0,\ g_1,\ ...,\ g_{N-1})$, minimum of which has to be found while varying the coefficients g_i, is denoted as goal function.

In many practical tasks a goal function is not specified directly, so neither function itself, nor its derivatives can be described analytically. However, it is possible to execute experiments and measurings to define the values of goal function and approximate values of its derivatives.

Several kinds of methods are known, allowing to find a minimum of nonlinear function $D^2(g_0,\ g_1,\ ...,\ g_{N-1})$, which is not specified analytically. These methods are named «numerical» and allow by the series of steps (iterations) to approach the minimum of this function with the sufficient degree of accuracy.

Numerical methods of optimization of functions of several variables generally can be devided into:

- *zero-order methods or direct methods;*
- *gradient methods.*

Zero-order methods are based on the use of only measured or calculated values of goal function. Advantage of these methods is evident, because in a number of practical tasks there is only information about values of the function. Gradient methods are based on calculation of goal function derivatives.

Procedure of stepwise approaches to the minimum demands the unimodality feature of investigated goal function. This means, that given function in the limited region of acceptable values of variable parameters must have one extremum point – minimum. It is proved, that correction root-mean-square error $D^2(g_0,\ g_1,\ ...,\ g_{N-1})$ satisfies the condition of unimodality.

Function $D^2(g_0,\ g_1,\ ...,\ g_{N-1})$ can be represented in an explicit form as function of second power from N variables.

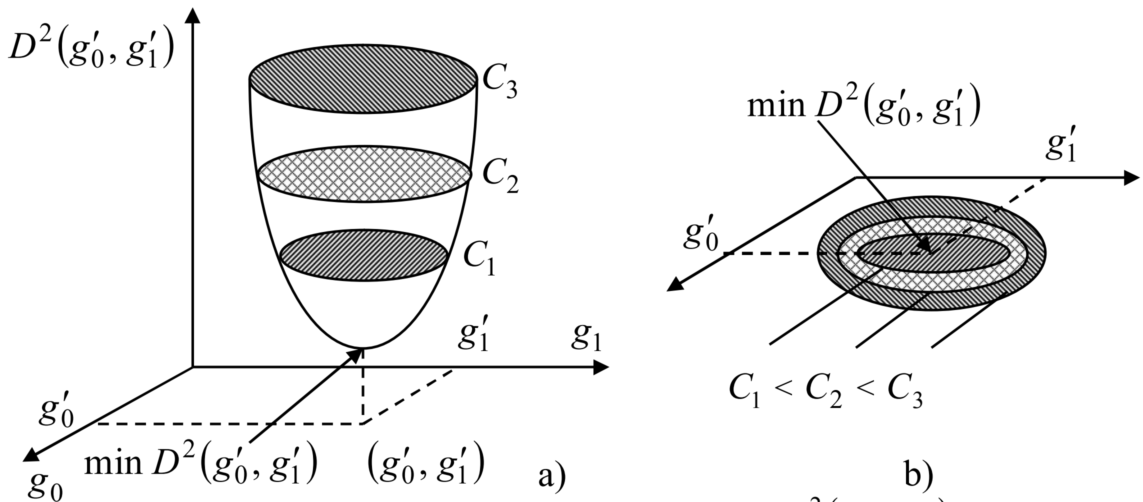

Figure 6 – Root-mean-square error $D^2(g_0,\ g_1)$.

a) – two-dimensional parabolic surface; б) – level lines of surface

On fig. 6,a function of two variables $D^2(g_0,\ g_1)$ – two-dimensional parabola is resulted, and on fig. 6,b – level lines of this function. As level lines of a function are named the projections of sections of this function by planes, parallel to the domain of definition.

Coordinatewise descent method

The most simple method of multidimensional optimization of goal function of N variables is the coordinatewise descent method, or method of step-by-step search by every variable. Its essence while finding of, for example, minimum is in stepwise changing of each of independent variables g_i until the investigated function $D^2(g_i)$ will not cease to decrease, and following turning to other variable until all variables will not be looked over. Thereafter return to that variable, from which a search began, opening in this way the second iteration of process and so on.

Coordinatewise descent method belongs to the direct methods of finding of a function $D^2(g_0,\ g_1,\ ...,\ g_{N-1})$ minimum, and allows either automatic, or manual adjustment of equalizer.

Formula of iteration algorithm of coordinatewise descent method looks like:

$g_i^{(k+1)} = g_i^{(k)} + \alpha_i^{(k)} V_i^{(k)}$, where k – iteration number;

i – number of independent variable (of HE regulator) $(i = 0,\ 1,\ ...,\ N-1)$;

$\alpha_i^{(k)}$- parameter, characterizing a length of a step;

$V^{(k)}$- vector, determining the varied (active) variable on s-th step

$$V^{(k)} = [e_0, e_1, ..., e_k], \qquad e - \textit{unitary vectors:}$$

$$V_0^{(k)} = [1,\ 0,\ 0,\ ...,\ 0]$$

$$V_1^{(k)} = [0,\ 1,\ 0,\ ...,\ 0]$$

$$V_2^{(k)} = [0,\ 0,\ 1,\ ...,\ 0]$$

$$\dots\dots\dots\dots\dots\dots$$

$$V_{N-1}^{(k)} = [1,\ 0,\ 0,\ ...,\ 1]$$

Block diagram of coordinatewise descent method algorithm is given on fig. 7.

Efficiency of coordinatewise descent method depends on the type of level lines. As far as every link of the search path is parallel to one of coordinate axis, and general motion path within the domain of definition is «stair-like», coordinatewise descent method is most effective, when the level lines are of the form of circles or ellipses, the main axes of which are parallel to coordinate axes. In this case it is enough to execute single exhaustion of all independent variables to attain a minimum. If the main axes of ellipses are turned in relation to coordinate axes, it is necessary to change directions of search many times, before minimum will be attained.

It is proved that if a channel does not have amplitude-frequency distortions, the level lines of root-mean-square error are circles, and application of coordinatewise descent method in this case is most effective.

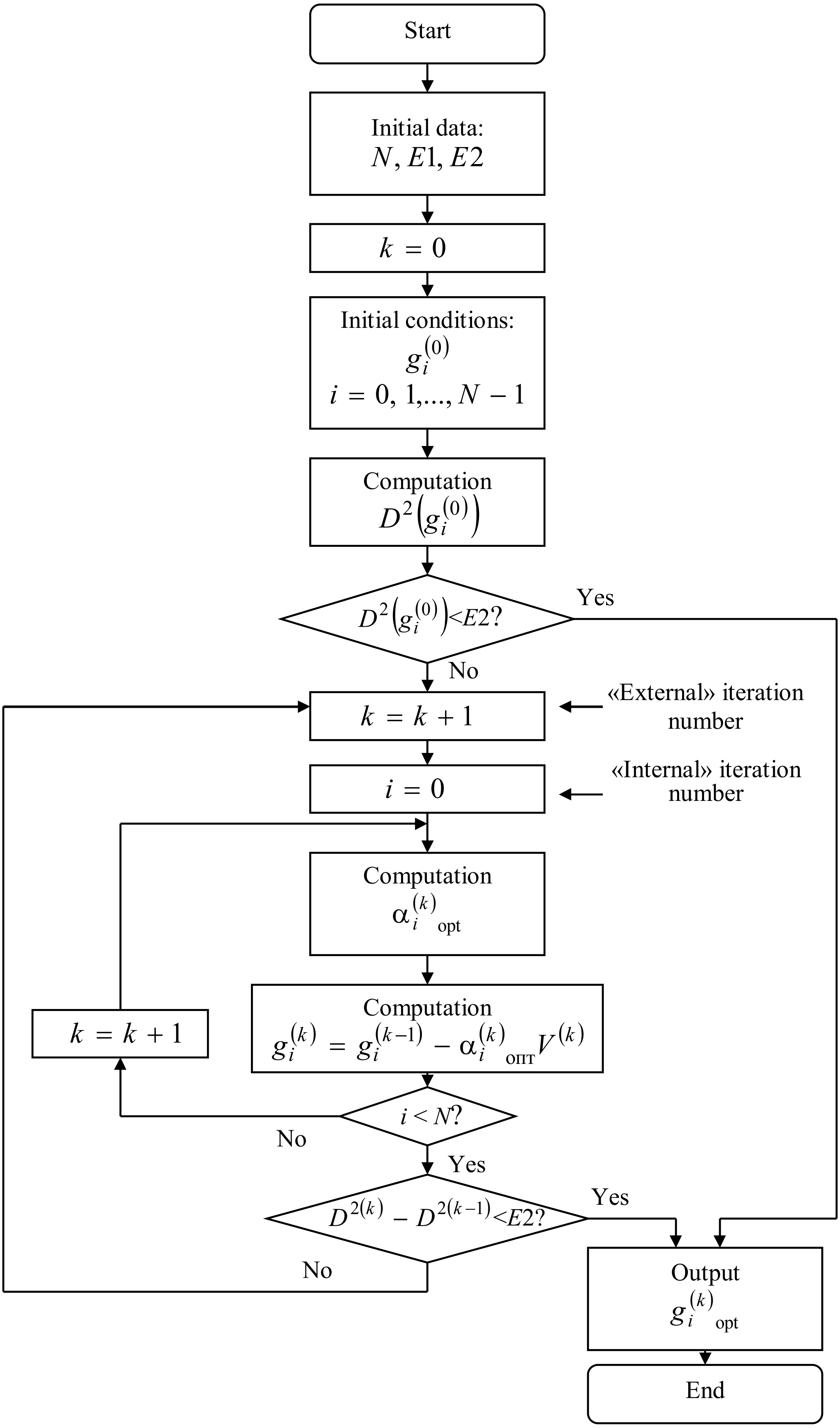

Figure 7 – Coordinatewise descent method algorithm

Example. It is necessary to find the optimum values of coefficients of harmonic equalizer g_0 and g_1, delivering a *minimum* of function (root-mean-square error):

$$D^2(g_0,\, g_1) = 2g_0^2 + 3g_1^2 - 20g_0 - 12g_1 + 62$$

by coordinatewise descent method.

Step 1. The assignment of initial values:

$$g_0^{(0)} = 0; \quad g_1^{(0)} = 0; \quad D^{2(0)}(g_0^{(0)}, g_1^{(0)}) = 62$$

Step 2 (1-st external iteration).

1-st internal iteration:

$$g_0^{(1)} = g_0^{(0)} + \alpha_0^{(0)} = 0 + \alpha_0^{(0)}$$

$$g_1^{(1)} = g_1^{(0)} = 0;$$

- Let's put values $g_0^{(1)}$ and $g_1^{(1)}$ into expression for $D^2(g_0,\, g_1)$. We'll find $\alpha_{0\ \text{opt}}^{(0)}$, for the purpose of what will study the obtained expression (function $D^{2(1)}(g_0^{(1)},\ g_1^{(1)})$) on the extremum with respect to variable α_0 analytically:

$$D^2(\alpha_0) = 2\alpha_0^2 - 20\alpha_0 + 62$$

According to the necessary conditions of function extremum – equality to zero of its first derivative we get

$$\frac{d\left(D^2(\alpha_0)\right)}{d\alpha_0} = 4\alpha_0 - 20 = 0; \qquad \alpha*_{0\ \text{opt}} = 5.$$

- Find the coefficients of equalizer on 1-st iteration

$$g_0^{(1)} = g_0^{(0)} + \alpha_{0\ \text{opt}}^{(0)} = 0 + 5 = 5$$

$$g_1^{(1)} = g_1^{(0)} = 0;$$

- Define the value of goal function on 1-st iteration

$$D^{2(1)}\left(g_0^{(1)}, g_1^{(1)}, \alpha_{0\ \text{opt}}^{(0)}\right) = 2 \cdot 5^2 - 20 \cdot 5 + 62 = 12;$$

- Check the execution of a condition

$$D^{2(1)}\left(g_0^{(1)},\, g_1^{(1)}\right) < D^{2(0)}\left(g_0^{(0)},\, g_1^{(0)}\right)$$

2-nd internal iteration:

$$g_0^{(1)} = 5;$$

$$g_1^{(1)} = g_1^{(0)} + \alpha_1^{(0)} = 0 + \alpha_1^{(0)}.$$

- Let's put values $g_0^{(1)}$ and $g_1^{(1)}$ into expression for function $D^2(g_0,\, g_1)$ and will find $\alpha_{1\ \text{opt}}^{(0)}$, for what will study the obtained expression (function $D^2(g_0^{(1)}, g_1^{(1)})$) on extremum with respect to variable α_1 analytically.

$$D^2(\alpha_1) = 3\alpha_1{}^2 - 12\alpha_1 + 12$$

According to the necessary conditions of function extremum – equality to zero of its first derivative with respect to variable α_1 we get

$$\frac{d\left(D^2(\alpha_1)\right)}{d\alpha_1} = 6\alpha_1 - 12 = 0; \qquad \alpha^*_{1\,opt} = 2$$

- Find the equalizer coefficients on 2-nd internal iteration

$$g_0^{(1)} = 5$$

$$g_1^{(1)} = g_1^{(0)} + \alpha_{1\ opt}^{(0)} = 0 + 2 = 2;$$

- Define the goal function value on 2-nd iteration

$$D^{2(1)}\left(g_0^{(1)}, g_1^{(1)}, \alpha_{1\ opt}^{(0)}\right) = 3 \cdot 2^2 - 12 \cdot 2 + 12 = 0;$$

$$D^{2(1)}\left(g_0^{(1)}, g_1^{(1)}\right) < D^{2(0)}\left(g_0^{(0)}, g_1^{(0)}\right)$$

- Minimum is attained, $D^2 = 0$ at $g_0 = 5$ and $g_1 = 2$.

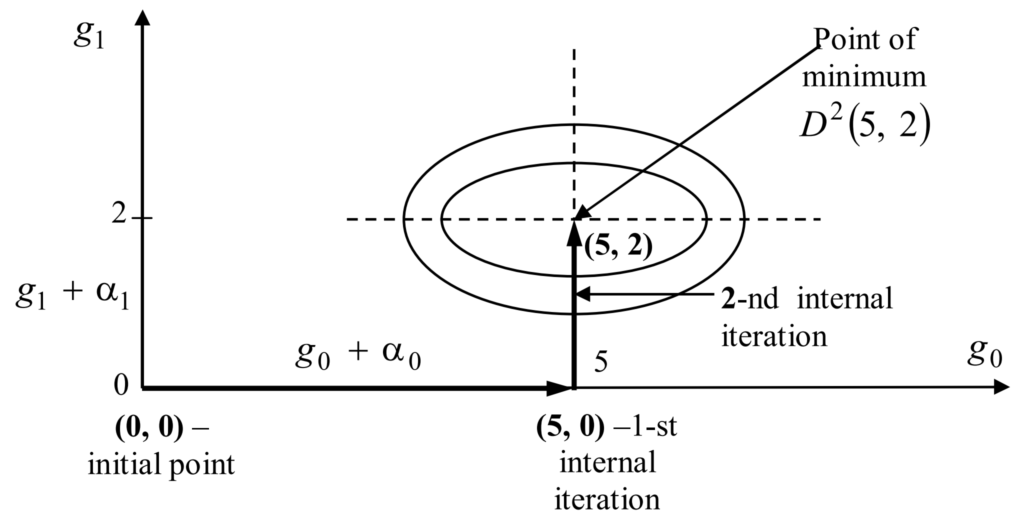

Figure 8 – Search path of function minimum by coordinatewise descent method

$$D^2(g_0, g_1) = 2g_0{}^2 + 3g_1{}^2 - 20g_0 - 12g_1 + 62$$

Notes:

1 Considered example refers to the case, when amplitude-frequency distortions are absent and level lines of function D^2 are ellipses, the main axes of which are parallel to the axes of coordinates. Therefore, the search of a minimum of function occurs for one external iteration (2 internal).

2 For simplification of calculations in given example values α_0 and α_1 were defined by analytical research of a function on extremum, while in practice these calculation are executed, as a rule, by approximative methods.

Disadvantages of coordinatewise descent method:

1 The necessity of solving the task of definition of optimal coefficients α_0 and α_1 values on every step, that results in increasing of calculations amount.

2 The dependence of algorithm efficiency on amplitude-frequency characteristics of a channel (i.e. on type of level lines).

The first disadvantage can be decreased by choosing α_0 and α_1 values constant on every step. Such way simplifies the calculations, but multiplies the number of iterations.

The second disadvantage cannot be excluded in the context of coordinatewise descent method. Therefore, other methods are used, which are based on motion, following the directions, which are not parallel to the coordinate axes, i.e. gradient methods of optimization.

Glossary

adsustment precision	– точность корректирования	– точність коректування
attenuation-frequency characteristic (AFC)	– частотная характеристика затухания (ЧХЗ)	– частотна характеристика згасання (ЧХЗ)
block diagram	– структурная схема	– структурна схема
concatenated connection	– каскадное включение	– каскадне включення
coordinatewise descent method	– метод покоординатного спуска	– метод покоординатного спуску
domain of definition	– область определения	– область визначення
goal function	– целевая функция	– цільова функція
group traveling time (GTT)	– групповое время прохождения (ГВП)	– груповий час проходження (ГЧП)
GTT variation	– неравномерность ГВП	– нерівномірність ГЧП
harmonic equalizer	– гармонический корректор	– гармонійний коректор
iteration algorithm	– итерационный алгоритм	– ітераційний алгоритм
level lines of function	– линии уровня функции	– лінії рівня функції
pattern function	– функция-образец	– функція-зразок
projection of section	– проекция сечения	– проекція перерізу
root-mean-square error	– среднеквадратичная погрешность	– середньоквадратична похибка
search path	– траектория поиска	– траєкторія пошуку
significant figure	– значащая цифра	– значуща цифра
single exhaustion	– однократный перебор	– одноразове перебирання
transfer function	– передаточная функция	– передатна функція
unitary vectors	– векторы-орты	– вектори-орти

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