MINISTRY OF TRANSPORT AND TELECOMMUNICATIONS OF UKRAINE

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CHAIR OF INFORMATION SECURITY AND DATA COMMUNICATIONS

NUMERICAL TECHNIQUES OF HILL-CLIMBING EXTREMUM SEEKING WITHIN THE FIXED INTERVAL

METHODICAL INSTRUCTIONS FOR PRACTICAL SEMINARS ON THE SUBJECT “METHODS OF OPTIMIZATION”

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1 AIM OF THE WORK

1. Studying of the numerical techniques of hill-climbing extremum seeking within fixed interval through the realization of these stages:

a) the determination of interval limits

b) the reduction of the interval seeking

2. Studying of the method of interval bisection and of the method of golden section.

3. Solving of problems using the investigated procedures of the reduction of the interval seeking.

2 BASIC THESES

2.1 Delimitation of an interval

On this stage we choose the reference point and then, using the elimination rule we scheme a rather wide interval which contains a point of extremum. Usually a seeking of limiting points of this interval is carried out with the help of heuristics methods of seeking, though in different cases one can use the methods of extrapolation. In accordance with one of the heuristics methods, offered by Swenn (k+1), the trial point can be determined with the help of the formula:

$$x_{k+1} = x_k + 2^k\Delta, \qquad k = 0, 1, 2, \ldots ,$$

where x_k – is arbitary point, Δ – is the size of step selected by some method. The symbol Δ defined by comparison of the quantity of $f(x_0)$, $f(x_0+|\Delta|)$ with $f(x_0-|\Delta|)$. If

$$f(x_0-|\Delta|) \geq f(x_0) \geq f(x_0+|\Delta|),$$

then, according to the hypotheses of unimodality, the minimum point must be righter than the point x_0 and the size of Δ is taken positive (fig.1).

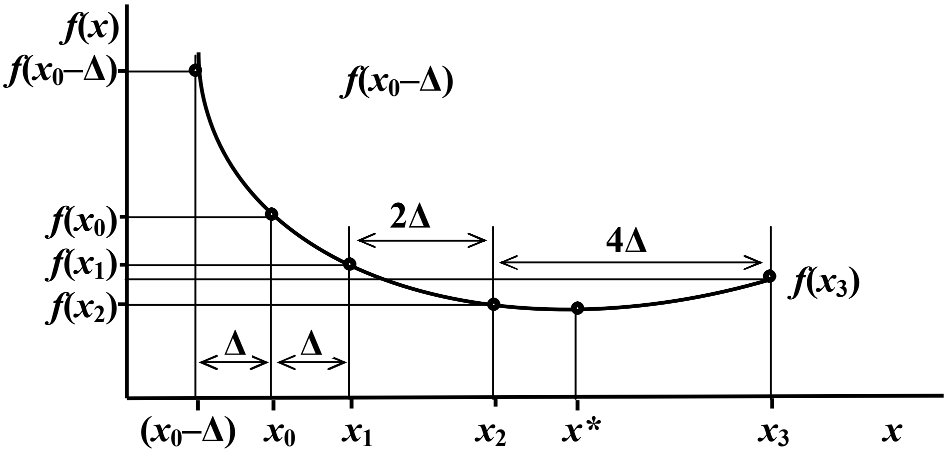

Figure1 – Graphical illustrations to the procedure of the limiting points seeking.

If to change the symbols of inequality on opposite symbols, then we choose the negative meaning of Δ. If

$$f(x_0-|\Delta|) \geq f(x_0) \leq f(x_0+|\Delta|), \tag{2.1}$$

is satisfieded, the minimum point is between $(x_0-|\Delta|)$, $(x_0+|\Delta|)$ and the search of limiting points is completed.

The case when

$$f(x_0-|\Delta|) \leq f(x_0) \geq f(x_0+|\Delta|),$$

goes against is contrary to hypotheses of unimodality in the seeking of minimum point. The execution of this condition shows that the function is not unimodal.

According to (2.1) during the transition to $(k+1)$ of the trial point, the search of limits of the interval may be completed if the inequality

$$f(x_{k-1}) \geq f(x_k) \leq f(x_{k+1}) \text{ is satiafied.} \tag{2.2}$$

In that case the minimum point is between x_{k-1} and x_{k+1} and the search of limiting points is completed.

We must notice that the efficiency of the search directly depends on the size of the step Δ. If Δ is big, we can receive only the approximate evaluations of the coordinate of the limiting points and the interval, which we have been built is extremely large. On the other hand if Δ is small, rather big volume of calculations may be required for the determination of limiting points.

2.2 Stage of the interval reduction

After the limits of the interval, which contains the extremum point are fixed, one can use more complicate procedure of the reduction of interval seeking in order to get the specified evaluation of extremum coordinate. The size of subinterval excluded on every step,dependes on the location of the trial points x_1 and x_2 within the seeking interval. As the position of the point of extremum is priori unknown it is every likely to suppose, that the location of the trial points must guarantee the reduction of the interval in one and the same relation. Besides, in order to increas the efficiency of the algorithm we must require the maximum of the mentioned ratio. This strategy may be called the maximum strategy of the seeking.

2.2.1 Method of the interval elimination

In fact, all the one-measured methods of seeking which are used in practice, are based on the hypotheses, that the examined function in the admissible field may have at least the quality of unimodality. The efficiency of this quality is determined by the fact, that the comparison of meanings *f(x)* in two different points of the seeking interval permit to define in which of the intervals determined by two points, the point of extremum is absent.

Theorem 1

Let the function $f(x)$ be unimodal in the closed interval $a \le x \le b$, and let the minimum is approached in the point x^*. We shall examine the points x_1 and x_2, which are situated on the interval, in such way that $a<x_1<x_2<b$. Comparing the meaning of the function in the points x_1 and x_2, we can draw the conditions.

1. If $f(x_1)>f(x_2)$, the minimum point $f(x)$ is not in the interval (a, x_1), then $x^* €$ (x_1, b) (fig. 1, a).
2. If $f(x_1)<f(x_2)$, the minimum point $f(x)$ is not in the interval (x_2, b), then $x^* €$ (a, x_2) (see fig. 1, b)

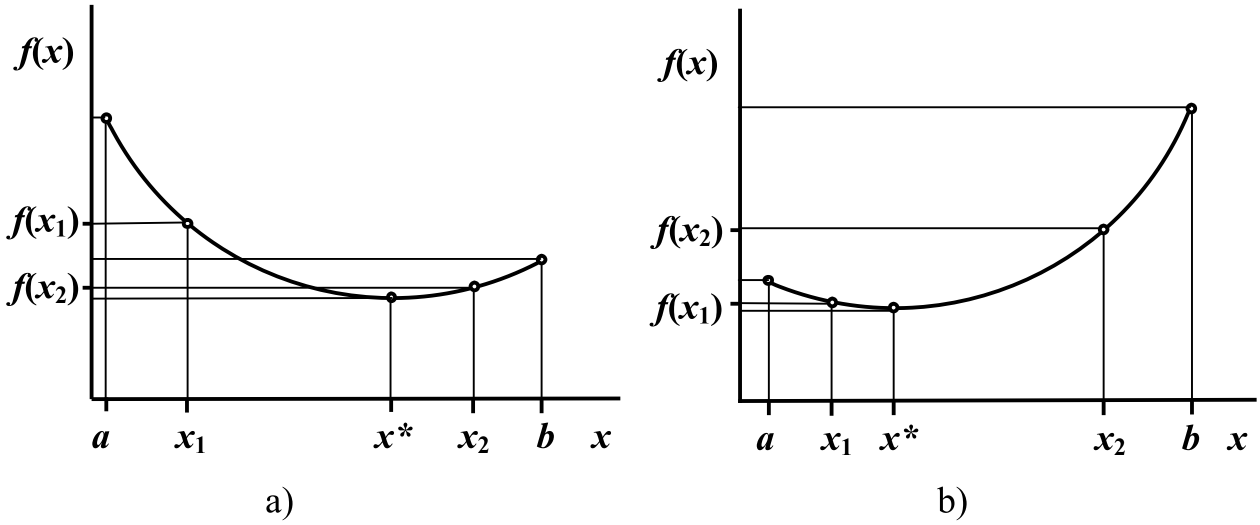

Figure 2 – Graphical illustration to the theorem 1.

Note. If $f(x_1) = f(x_2)$, we can exclude both extreme intervals (a, x_1) and (x_2, b), the minimum point must be the interval (x_1, x_2).

2.2.2 Method of interval bisection

Considered method permits to exclude exactly half of an interval on every iteration.

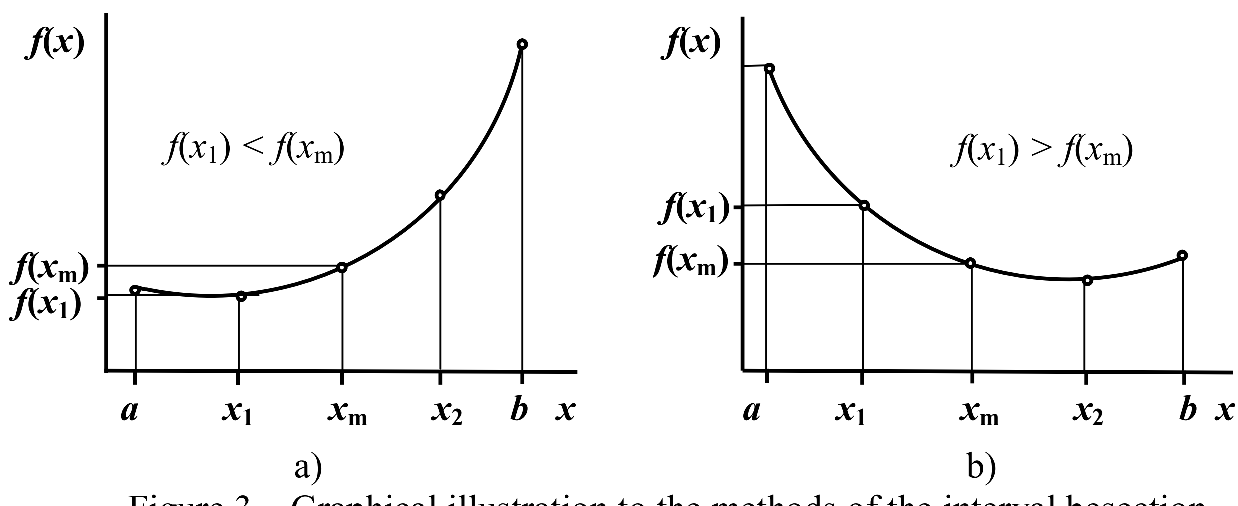

Figure 3 – Graphical illustration to the methods of the interval besection

Sometimes this method is called three-pointed seeking on the equal intervals, as its realization is based on the choice of three trial points placed in the interval of seeking. Later we shall describe the principle steps of the procedure of this seeking, which permit us to find the minimum point of function $f(x)$ in the interval (a, b).

Step 1. If $x_m = (a + b) / 2$ and $L = b - a,$ we must calculate the value of $f(x_m)$.

Step 2. If $x_1 = a + L/4$ and $x_2 = b - L/4$, we must notice, that the points x_1, x_m, and x_2 divide interval (a, b) on four equal parts. One calculates values $f(x_1)$ and $f(x_2)$.

Step 3. We compare $f(x_1)$ and $f(x_m)$.

(1) If $f(x_1) < f(x_m)$ (fig. 3, a), one must exclude the interval (x_m, b), and set $b = x_m$. The medium point of the new seeking interval becomes the point x_1. Consequently we must set $x_m = x_1$ and pass to the step 5.

(2) If $f(x_1) \geq f(x_m)$ (fig. 3, b), and pass to the step 4.

Step 4. We must compare $f(x_2)$ and $f(x_m)$,

(1) If $f(x_2) < f(x_m)$, we must exclude the interval (a, x_m), $a = x_m$. As the medium point of the new interval becomes x, we must set $x_m = x_2$ and pass to the step 5.

(2) If $f(x_2) \geq f(x_m)$, we must exclude the intervals (a, x_1) and (x_2, b). We must set $a = x_1$ and $b = x_2$. Notice that x_m continue to be the medium point of the new interval. And then we must pass to the step 5.

Step 5. One calculates $L = b - a.$ If the size of $|L|$ is small, we must finish the seeking.
If not – we must return to the step 2.

Remarks

1. On every iteration of the algorithm exactly half of seeking interval is excluded.

2. The medium point of the consequently got intervals always coincides with one of the trial points x_1, x_2 or x_m, found during the previous iteration. Then on every iteration we need not more than two calculations of the function;

3. If we made n calculations of the function values, the length of the given interval is $(1/2)^{n/2}$ of the value of the initial interval.

4. In our work we show that from all methods of seeking on the equal intervals (two-pointed seeking, three-pointed seeking, four-pointed) three-pointed seeking or the method of the interval bisection differe with the most effectiveness.

2.2.3 Seeking by the method of the gold section

From above mentioned discussions of the methods of interval elimination and of the minimaximal strategies of the seeking, we may make following conclusions:

1. If the quantity of the trial points is equal to two, they must be located on the same distance from the middle of the interval.

2. In accordance with the general minimaximal strategy the trial points must be located in the interval in the symmetrical scheme so that the length ratio of the excluded subinterval to the size of the seeking interval remains constant.

3. In each iteration of the seeking procedure only one value of function must be calculated.

According to these conclusions we examine the symmetrical location of two trial points on the initial interval of the unit length, which is represented on the fig. 3. The choice of the unit interval is determined by the reasons of convenience.

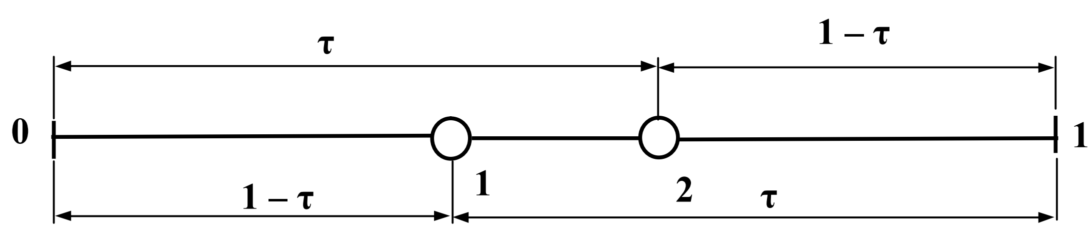

Figure 3 – The search with the help of the method of golden section

Trial points are on the τ distance away from the limiting points. Under such symmetrical locations of the points the length of the after alimination interval, is always equal τ, irrespectively to what of the value of function in the trial points is the less. Suppose that the right subinterval is excluded. We show on the fig. 4 that the subinterval of length τ contains one trial point located at a distance (1 – τ) from the left limiting point.

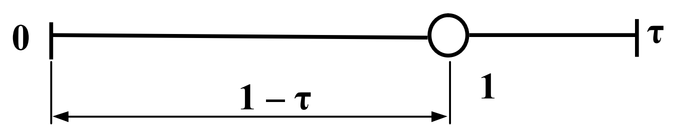

Figure 4 – Intervals received by golden section method

To save the symmetry of trial model, the length (1 – τ) must be τth-part of the length of the interval (which is equal to τ). Under such choice of τ the following trial point is at a distance equals to τth-part of all the length of the interval from the right limiting point of the interval (fig.5).

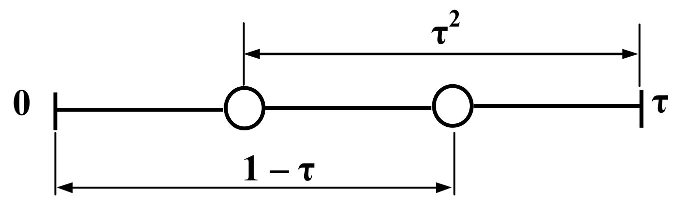

Figure 5 – Symmetry of the interval gold section

Therefore choosing τ in accordance to the condition $1 - \tau = \tau^2$ the symmetry of the model, showed on the fig.3, is left during the passing to the reduced interval, showed on the fig.5. Solving this square equation we get

$$\tau = (-1 \pm \sqrt{5})/2,$$

where the positive resolution τ =0,61803... .

We shall examine the correlation of got segments, on the next step of the seeking

$$\frac{\phi - (1 - \phi)}{1 - \phi} = \frac{1 - \phi}{\phi},$$

where $\tau^2 - \tau(1 - \tau) = (1 - \tau)^2$.

We will divide both parts of the equation on $(1 - \tau)$.

After these transformations we finally receive the earlier written condition of a symmetry of the searched model $\tau^2 = 1 - \tau$.

The scheme of the search, where the trial points divide the interval in this relation is known as the method of golden section. We must notice, that after two primary calculations of the function values, each following calculations allow to exclude the subinterval, which value is $(1 - \tau)$th-part of the length of the searched interval.

Thus, if the primary interval has the unit length, the size of the interval, received as the result N calculations of the function values is equal to τ^{N-1}. We can show that this search with the help of method of golden section is the most effective way of realization of minimaximum strategy of the search.

In general if the right and the left limiting points of the indefinite interval (named as XR and XL) are well-known, then the coordinates of all following trial points, got in accordance to the method of golden section, we can calculate by the formula

$w = \mathrm{XR} - \tau^n$ or $w = \mathrm{XL} - \tau^n$

depending on what right or left subinterval has been excluded during the preceding iteration. In the above mentioned formula we denoted by "T" the *n*-degree of τ, where n – is a quantity of function values.

The search with the help of the method of golden section may be ended or started from the given quantity of calculations of function value, under the relative precision of the searched function value. The most preferable is the usage of both criterion at the same time.

2.2.4 Comparison of the methods of intervals elimination

It is given below the comparison of the relative efficiency of the examined methods of the interval eliminating. The length of the primary interval will be marked as L_1, and the length of the interval got as a result of calculations of function value as L_N. As an exponent of efficiency of the different methods of interval elimination we shall introduce into our examination a characteristic of the relative reduction of a primary interval $K_L(N) = L_N / L_1$.

We must notice that using the method of interval bisection and the method of golden section, the length of the interval that we have got is $L_1(0{,}5)^{N/2}$ and $L_1(0{,}618)^{N-1}$ accordingly. Thus, the relative reduction of the interval after N – calculations of the function values is equal to:

$K_L(N) = L_1(0{,}5)^{N/2}$ – for the method of interval bisection;

$K_L(N) = L_1(0{,}618)^{N-1}$ – for the method of golden section.

For comparison let concider also the method of an equal search, in accordance with it the determination of a function is realized in N points equally removed from each other, for that the interval L_1 is divided into $(N + 1)$ equal intervals with the length $L_1/(N + 1)$. Let x^* – be a point, in which there is minimum function $f(x)$.

Then the point of the true minimum $f(x)$ is included in the interval

$$\{[x^* - L_1/(N+1)],\quad [x^* + L_1/(N+1)]\},$$

and then $L_N = 2L_1/(N+1)$.
Consequently for the method of equal search

$$K_L(N) = 2/(N+1).$$

We have represented in the table below the values of $K_L(N)$, according to chosen N for methods of search. The table demonstrates that the search with the help of method of golden section guarantees the most relative reduction of the primary interval during the same quantity of calculations of the function value.

Table 2.1 – The size of the relative reduction of an interval

Method of search	The quantity of calculation to calculate the size of function				
	$N = 2$	$N = 5$	$N = 10$	$N = 15$	$N = 20$
Method of interval bisection	0,5	0,177	0,031	0,006	0,0009
Method of golden section	0,618	0,146	0,013	0,001	0,0001
Method of equal search	0,667	0,333	0,182	0,125	0,095

On the other hand we can also compare a quantity of calculations of function value, which is necessary to receive for the set value of the relative reduction of an interval with the set degree of precision. If the value $K_L(N) = E$ is set, the meaning is calculated by formula:
for the method of interval bisection

$$N = 2\ \ln(E)/\ln(0{,}5),$$

for method of golden section

$$N = l + [\ln(E)/\ln(0{,}618)],$$

for method of equal search

$$N = (2/E) - 1.$$

We have placed in the table 2.2 the statistics for determination of the coordinate of the minimum point with the set precision.

Table 2.2 – The quantity of calculation necessary to calculate the function value

Method of search	set precision			
	E = 0,1	E = 0,05	E = 0,01	E = 0,001
Method of interval besection	7	9	14	20
Method of golden section	5	8	11	16
Method of equal search	19	36	199	1999

We must remark that the method of golden section is more effective than other methods, comparing with other two methods because it requires the lost number of function values, to get one and the same precision.

3 MAIN QUESTIONS

3.1 Explain the method of intervals elimination.

3.2 Define the main point of the determination of interval limits.

3.3 Illustrate the minimaximum strategy of search.

3.4 Explain the method of interval bisection.

3.5 Describe the search of the indefinite interval with the help of the method of golden section.

3.6 Compare the relative efficiancy of the different methods of search of the indefinite interval.

4 HOME WORK

4.1 Learn the theoretical content of these "methodical instructions".
4.2 Made an algorithm of the problems for every method step by step.
4.3 Answer the test questions.

5 WORK IN THE LECTURE HALL

5.1 Study the examples of the solutions
5.2 Fulfill the task 1,2,3 in accordance to the given variant.

6 PROBLEMS TO SOLVE (EXAMPLES)

Example 1. The determination of limits of the interval

We shall examine the problem of the minimization of the function $f(x) = (100 - x)^2$ if $x_0 = 30$ and step value $|\Delta| = 5$ are set. The sign Δ is defined on the base of values. comparison.

$f(x_0) = f(30) = 4900$,
$f(x_0 + |\Delta|) = f(35) = 4225$,
$f(x_0 - |\Delta|) = f(25) = 5625$.
as $f(x_0 - |\Delta|) \geq f(x_0) \geq f(x_0 + |\Delta|)$,
the value of Δ must be positive, and the coordinate of minimum point x^* must be more than 30.

We have $x_1 = x_0 + \Delta = 35$. Than $x_2 = x_1 + 2\Delta = 35 + 2\times 5 = 45$,
$f(x_2) = f(45) = (100 - 45)^2 = 3025 < f(x_1)$,
where $x^* > 35$.
$x_3 = x_2 + 2^2\Delta = 65$, $f(x_3) = f(65) = (100 - 65)^2 = 1225 < f(x_2)$,
where $x^* > 45$.
$x_4 = x_3 + 2^3\Delta = 105$, $f(x_4) = f(105) = (100 - 105)^2 = 25 < f(x_3)$,
where $x^* > 65$.
$x_5 = x_4 + 2^4\Delta = 185$, $f(x_5) = f(185) = (100 - 185)^2 = 7225 > f(x_4)$.

In case, the inequality is done passing to the 5-th trial point x_5 in accordance with (2.2), the search of limiting points may be completed.
$f(x_3) \geq f(x_4) \leq f(x_5)$,
$1225 \geq 25 \leq 7225$.

Thus, five steps of calculations in the trial points allow to reveal the limits of the interval $65 \leq x^* \leq 185$, where the point x^* is placed.

Example 2. Method of the interval bisection

We minimize $f(x) = (100 - x)^2$ in the interval $60 \leq x \leq 150$.
Here $a = 60$, $b = 150$, length of the interval $L = 150 - 60 = 90$.
$x_m = (60 + 150)/2 = 105$.
$f(x_m) = f(105) = (100 - 105)^2 = 25$

Iteration 1

$x_1 = a + (L/4) = 60 + (90/4) = 82{,}5$,
$x_2 = b - (L/4) = 150 - (90/4) = 127{,}5$,
$f(x_1) = f(82{,}5) = (100 - 82{,}5)^2 = 306{,}25 > f(x_m) = 25$,
$f(x_2) = f(127{,}5) = (100 - 127{,}5)^2 = 756{,}25 > f(x_m) = 25$.

So, the intervals (60, 82,5) and (127,5, 150) are eliminated. Interval of uncertainty is equal (82,5, 127,5).

The length of the interval of search is reduced from 90 to 45.

Iteration 2

$a = 82{,}5$, $b = 127{,}5$, length of the interval $L = 127{,}5 - 82{,}5 = 45$
$x_m = (82{,}5 + 127{,}5)/2 = 105$,
$f(x_m) = f(105) = (100 - 105)^2 = 25$
$x_1 = a + (L/4) = 82{,}5 + (45/4) = 93{,}75$,
$x_2 = b - (L/4) = 127{,}5 - (45/4) = 116{,}25$,
$f(x_1) = f(93{,}75) = (100 - 93{,}75)^2 = 39{,}06 > f(x_m) = 25$,
$f(x_2) = f(116{,}25) = (100 - 116{,}25)^2 = 264{,}06 > f(x_m) = 25$.

Thus, the intervals (82,5, 93,75) and (116,25, 127,5) are eliminated. The interval of uncertainty is equal (93,75, 116,25).

The length of the interval of search is reduced from 45 to 22,5.

Iteration 3

a = 93,75, b = 116,25, length of the interval L = 116,25 – 93,75 = 22,5,

x_m = (93,75+ 116,25)/2=105,

$f(x_m) = f(105) = (100 - 105)^2 = 25$

$x_1 = a + (L/4)$ = 93,75+(22,5/4) = 99,375,

$x_2 = b - (L/4)$ = 116,25– (22,5/4) =110,625,

$f(x_1) = f(99{,}375) = (100 - 99{,}375)^2 = 0{,}3906 < f(x_m) = 25$,

As $f(x_1) < f(x_m)$, in this case $f(x_2)$ is not calculated.

Thus, we eliminated the interval (105, 116,25). The new interval of uncertainty is equal (93,75, 105), its middle point x_m is 99,375 (point x_1 on the iteration 3).

We must notice the fact, that during 3 iterations (6 calculations of the function values) the initial interval of search of the length 90 was reduced to the value $90\times(1/2)^3 = 11{,}25$.

Example 3. The method of golden section

We shall examine the problem, from the example 2, where we need to minimize $f(x) = (100 - x)^2$ the interval $60 < x < 150$.

For we can pass to the interval of singular length, we must change the variable x, arranging $w = (x - 60)/90$. So, the problem takes such form:

We must minimize $f(w) = (40 - 90w)^2$

Under the limit $0 \leq w \leq 1$.

Iteration 1. The interval I_1 = (0, 1); L_1 = 1. we shall realize two primary calculations of the function values:

$w_1 = \tau = 0{,}618$, $f(w_1) = (40 - 90w_1)^2 = (40 - 90\times0{,}618)^2 = 244{,}0$,

$w_2 = 1 - \tau = \tau^2 = 0{,}382$, $f(w_2) = (40 - 90w_2)^2 = (40 - 90\times0{,}382)^2 = 31{,}6$.

As $f(w_2) < f(w_1)$ and $w_2 < w_1$, the interval $w \geq w_1$ is eliminated.

Iteration 2. The interval I_2 = (0, 0,618); $L_2 = 0{,}618 = \tau$. The next calculation is realized in the point:

$w_3 = \tau - \tau^2 = \tau(1 - \tau) = \tau^3 = 0{,}236$.

$f(w_3) = (40 - 90w_3)^2 = (40 - 90\times0{,}236)^2 = 352$.

As $f(w_3) > f(w_2)$ and $w_3 < w_2$, the interval $w \leq w_3$ is eliminated.

Iteration 3. The interval I_3 = (0,236, 0,618), $L_3 = 0{,}382 = \tau^2$. The next calculation of function we make in the point, which is situated at a distance τ × (length of the obtained interval from the left limiting point of interval), or on the distance $(1 - \tau)$ × (length of the obtained interval from the right limiting point of interval). Thus:

$w_4 = 0{,}618 - (1 - \tau) L_3 = 0{,}618 - \tau^2 L_3 = 0{,}618 - \tau^2(\tau^2) = 0{,}618 - \tau^4 = 0{,}472$,

$f(w_4) = (40 - 90w_4)^2 = (40 - 90\cdot0{,}472)^2 = 6{,}15$.

As $f(w_4) < f(w_2)$ and $w_4 > w_2$, the interval $w \leq w_2$ is eliminated.

As a result we have obtained the interval of uncertainty: $0{,}382 \leq w \leq 0{,}618$ for the variable w.

Passing to the variable x we get searched minimum limits of the interval of uncertainty $94{,}4 \leq x \leq 115{,}6$ for the variable x, so that is the solution of the problem.

We shall compare an efficiency of interval searching methods in the tasks 2 and

3. If during the search with the help of the method of golden section we have made 6 calculations of function values, so the length of the resulting interval for variable w is equal to

$\tau^{N-1} = \tau^5 = 0,09,$

which conform to the interval of the length 8,1 for the variable x. To compare we must remind that in the same situation the method of interval bisection has led to the result of getting interval 11,25.

7 SOLITARY WORK

Task 1. The determination of the interval limits

We must determine the limits of function interval $f(x) = (N\times 10 - 2x)^2$ if the initial point is equal to $x_0 = \dfrac{N\times 10}{M}$ and the value of the step $\Delta = \dfrac{N\times 10}{3\times M}$.

N – is the last digit of a student's card number, if it is equal to 0, take $N = 15$.

If the next to last digit of the student's card number is odd, take $M = 5$.

If the next to last digit of the student's card is even or is equal to zero, take, $M = 4$. In the calculations round off to three figures.

Task 2. We shall determine the interval limits where the minimum point of function $f(x)$ is placed with the method of interval bisection.

We must fulfil the triple procedure of the reduction of the search interval with the aim of obtaining of minimum interval limits, within which the minimum point of function $f(x)$ is situated.

The function $f(x) = (N\times 10 - 2x)^2$ in the interval $\dfrac{N\times 10}{M} \le x \le K\times N\times 10$.

N – is the last digit of the student's card, if it is equal to zero, we take $N = 15$.

If the last digit of the student's card is odd, take variant $K = 2$, and $M = 5$.

If the last digit of the student's card is even or zero, take $K = 3$, a $M = 4$.

Task 3. Determination of the minimum interval limits, where the minimum point of function $f(x)$ is situated by the method of golden section.

Initial data and conditions are the same as in the task 2.

8 TABLE OF ACCOUNT CONTENTS

1 Subject and aim of the work.

2 Algorithm of problem solutions step by step on each assignment.

3 Results of calculations for all the seeking steps on each assignment.

4 Table with the data of intervals received during the execution of every seeking step in every assignment.

5 Conclusions based on the results received in every task.

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CONTENT

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