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Department of Higher Mathematics

REFERENCE BOOK
ON HIGHER MATHEMANICS
Part I

For Students Doing a Course of Higher Mathematics in English

Odessa – 2010

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Справочник содержит разделы «Арифметика», «Алгебра», «Комплексные числа», «Линейная алгебра», «Аналитическая геометрия», «Предел функции одной переменной», «Дифференциальные исчисления функции одной и многих переменных». Перед каждым разделам приведен англо-русский словарь необходимых терминов. В конце справочника приведен русско-английский словарь, включающий термины всех рассмотренных тем.

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CONTENTS

Arithmetic

Vocabulary

add – прибавлять
addition – сложение
be undefined – не определено
composite number – составное число
counting numbers – натуральные числа
change a mixed number to improper fraction – перевести смешанное число в неправильную дробь
change an improper fraction to a mixed number - перевести неправильную дробь в смешанное число
decimal – десятичная дробь
decrease – уменьшать
denominator – знаменатель
difference – разность
divide (by) – делить
dividend – делимое
divisibility – делимость
division – деление
divisor – делитель
evaluate – вычислять
example – пример
factor – множитель
fraction – дробь
greater than or equal to – больше или равно
Greatest Common Factor – наибольший общий делитель
how much – сколько
improper fraction– неправильная дробь
increase – увеличивать
is divisible by – делится на
known – известный
Least Common Multiple – наименьшее общее кратное
less than or equal to – меньше или равно
long division– деление в столбик
minuend – уменьшаемое
mixed number– смешанное число
multiplication – умножение
multiply (by) – умножать
numerator – числитель
once – один раз
prime factorization– разложение на простые множители
prime number – простое число
product – произведение
proper fraction– правильная дробь

quotient – частное (от деления)
remainder – остаток
repeating decimal – периодическая дробь
subtract – вычитать
subtraction – вычитание
subtrahend – вычитаемое
sum – сумма
summand – слагаемое
terminating decimal – конечная десятичная дробь
the least common denominator– наименьший общий знаменатель
three times – трижды
to (find) – для того, чтобы (найти)
twice – дважды
unknown – неизвестный

Arithmetic Operations and Properties

The formula	Read
$a + b = c$	• *a* plus *b* equals *c* • *b* added to *a* is *c* • the sum of *a* and *b* is *c*
$a - b = c$	• *a* minus *b* equals (is) *c* • the difference of *a* and *b* is *c*
$a \times b = c$, $a \cdot b = c$;	• *a* multiplied by *b* equals *c* • *a* by *b* is *c* • *a* times *b* is *c* • the product of *a* and *b* is *c*
$a : b = a / b = \frac{a}{b} = c$	• *a* divided by *b* is *c* • a quotient of *a* and *b* is *c*
a^n	*a* to the *n*th power
Properties of addition and multiplication	
Name	Formula
Commutative property	• $a + b = b + a$ • $a \cdot b = b \cdot a$
Associative property	• $(a + b) + c = a + (b + c)$ • $a(bc) = (ab)c$
Distributive property	• $a(b \pm c) = ab \pm ac$
Identity property	• There is a unique number, namely *0*, such that for any number *a*: $a + 0 = 0 + a = a$ • There is a unique whole number, namely 1, such that for every number *a*: $a \cdot 1 = 1 \cdot a = a$
Property of zero	If $a \neq 0$, then • $0 : a = 0$; • $0 \cdot a = a \cdot 0 = 0$

Definitions, theorems	
Divisor	*a* is a divisor of *b* if *a* is a factor of *b*
Multiple of *a*	*b* is a multiple of *a* if *b* is divisible by *a*
Test for divisibility by 2	A number is divisible by 2 if and only if its ones digit is 0, 2, 4, 6, or 8
Test for divisibility by 3	A number is divisible by 3 if and only if the sum of its digits is divisible by 3.
Test for divisibility by 5	A number is divisible by 5 if and only if its ones digit is 0 or 5.
245	Two hundred forty five.
2436	Two thousand four hundred thirty six, or twenty four thirty six.
The Greatest Common Factor (GCF(*a, b*))	The greatest common factor of two nonzero whole numbers *a* and *b* is the largest whole number that is a factor of both these numbers.
The Least Common Multiple LCM(*a, b*)	The least common multiple of two nonzero whole numbers *a* and *b* is the smallest nonzero whole number that is the multiple of each of these numbers.
Common fraction	• $\frac{a}{b}$, or a/b, represents *a* of *b* equivalent parts • *a* is called the numerator • *b* is called the denominator.
Proper fraction	A fraction whose numerator is less than a denominator.
Improper fraction	A fraction whose numerator is greater than or equal to a denominator.
Mixed numbers	A number containing the whole an fractional parts.
2/3	Two of thirds or two over three (the proper fraction).
$4\frac{3}{5}$	Four and three of the fifths, or four and three over five (the mixed number).
Addition and subtraction of fractions with common denominators	$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$
Addition and subtraction of fractions with different denominators	1)find the least common denominator of these fractions ; 2) subtract them as before.

Multiplication of fractions	$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$
Multiplication of a mixed number by a whole number	$4\frac{2}{15}\cdot 7=4\cdot 7\frac{2\cdot 7}{15}=28\frac{14}{15}$
Multiplication of a mixed number by a mixed number	$2\frac{2}{15}\cdot 1\frac{3}{8}=\frac{32}{15}\cdot\frac{11}{8}=\frac{32\cdot 11}{15\cdot 8}=\frac{44}{15}=2\frac{14}{15}$
Division of fractions	$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$
Division of a mixed number by a mixed number	$1\frac{19}{21}:1\frac{5}{6}=\frac{40}{21}:\frac{11}{6}=\frac{40\cdot 6^2}{{}_7 21\cdot 11}=\frac{80}{77}=1\frac{3}{77}$
Decimal	Decimal is a common fraction denominator of which is 10^n, where $n=1,2,\ldots$. For example: $\frac{123}{10000}=\frac{123}{10^4}=0.0123$
0.0123	read "zero point, zero, one, two, three".
1.875	read "one point, eight, seven, five".
Terminating decimals	$\frac{a}{b}$ has a terminating decimal representation if and only if b contains only 2s and (or) 5s in its prime factorization: $\frac{3}{50}=\frac{3}{2\cdot 5\cdot 5}=\frac{3\cdot 5\cdot 2\cdot 2}{(2\cdot 5)\cdot(5\cdot 2)\cdot(5\cdot 2)}=\frac{60}{1000}=\frac{6}{100}=0.06$ zero point, zero, six.
Repeating decimals	$\frac{a}{b}$ has a repeating decimal representation that does not terminate if and only if b has a prime factor other than 2 or 5 in its prime factorization: $\frac{34}{99}=0.343434\ldots=0.(34)=0.\overline{34}$.
A ratio of two numbers a and b	$a:b=\frac{a}{b}$ is the ordered pair of numbers, with $b\neq 0$.
A proportion	$\frac{a}{b}=\frac{c}{d}$ is a statement that two given ratios are equal.
1 percent (1%) of a number a	is one hundredth of this number : $a\cdot\frac{1}{100}=\frac{a}{100}$

ALGEBRA

Vocabulary

absolute value- абсолютная величина
acute – острый
adjacent – прилежащий, смежный
base - основание
closed interval – закрытый интервал
combine like terms – привести подобные члены
constant term – свободный член
common logarithm - десятичный логарифм
distance - расстояние
element - элемент
empty set – пустое множество
even – четный
exponent – показатель степени
integers (whole numbers) – целые числа
intersection – пересечение
interval - интервал
irrational numbers – иррациональные числа
leading coefficient – коэффициент при старшей степени
monomial – одночлен
natural logarithm – натуральный логарифм
negative - отрицательный
odd – нечетный
open interval – открытый интервал
polynomial – полином, многочлен
positive – положительный
radical – корень, радикал
radicand – подкоренное выражение
rational numbers – рациональные числа
real number – действительные числа
subset - подмножество
the nth power of – энная степень
the nth root of – корень энной степени
union - объединение

Fundamental Concepts

Name	Formula
Let A and B be sets then:	
x is an element of the set A (x belongs to A)	$x \in A$
an empty set	$A = \acute{\emptyset}$
a union of A and B	$A \cup B$
an intersection of A and B	$A \cap B$
A is a subset of B	$A \subset B$
to follow	$\Rightarrow$
if and only if	$\Leftrightarrow$
any, for any	$\forall$
to exist	$\exists$
such that	**:**

The Real Number System

Natural numbers (positive integers)	$N = \{1,2,3,..., n,...\}$
Integers (whole numbers)	$\boldsymbol{Z} = \{...,-3,-2,-1,0,1,2,3....\}$
Rational numbers – $\{$all terminating or repeating decimals$\}$	$\boldsymbol{Q} = \left\{\frac{m}{n}\right\}$, where $m, n \in Z, n \neq 0$
Irrational numbers	$\left\{\begin{array}{l}\text{all nonterminating} \\ \text{nonrepeating decimals}\end{array}\right\}$
Real numbers	$\boldsymbol{R} = \{\text{all rational and irrational numbers}\}$
Absolute value of the real number a	$\lvert a \rvert = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

The interval notations:

$(a,b) = \{x : a < x < b\}$ - an open interval	all real numbers between a and b, not including a and not including b.
$[a,b] = \{x : a \leq x \leq b\}$ a closed interval	all real numbers between a and b, including a and including b.
$(a,b] = \{x : a < x \leq b\}$	all real numbers between a and b, not including a and including b
$[a,b) = \{x : a \leq x < b\}$	all real numbers between a and b, including a and not including b.
$(-\infty, b) = \{x : x < b\}$	all real numbers less than b.
$(-\infty, b] = \{x : x \leq b\}$	all real numbers less than or equal to b.
$(a, \infty) \quad = \{x : x > a\}$	all real numbers greater than a.
$[a, \infty) \quad = \{x : x \geq a\}$	all real numbers greater than or equal to a.
$(-\infty, \infty) = R$	all real numbers

Exponents and Radicals	
The *n*th power of *b*	$b^n = \underbrace{b \cdot b \cdot b \cdot \ldots \cdot b}_{n \text{ factors of } b}$, where *b* is the **base**, *n* is the **exponent**,
The *n*th root of *a* (radical)	$\sqrt[n]{a} = b: \quad b^n = a$ *a* is called the **radicand,** *n* is called the index of a root.
$\sqrt[2]{a} = \sqrt{a}$	a square root of *a*
$\sqrt[3]{a}$	the cube root of *a*;
Remember:	• $\sqrt[n]{a^n} = \begin{cases} a, & \text{if } n = 2k+1 \, (n \text{ is odd}), \\ \lvert a \rvert, & \text{if } n = 2k \, (n \text{ is even}). \end{cases}$ • $b^0 = 1,\ b \neq 0$ • $b^{-n} = \frac{1}{b^n}$ and $\frac{1}{b^{-n}} = b^n$. $b \neq 0$ and $n \neq 0$, • $\sqrt[n]{b} = b^{1/n}$, *n* is a positive integer and *b* is a real number

Properties of Exponents	
Product	• $b^m b^n = b^{m+n}$ • $(ab)^n = a^n b^n$
Quotient	• $\frac{b^m}{b^n} = b^{m-n}$ • $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Exponent	• $\left(b^m\right)^n = b^{mn}$

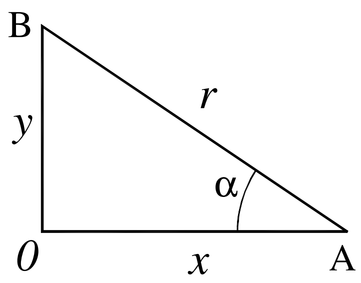
Properties of Radicals	
Product	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
Quotient	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \quad (b \neq 0)$
Radical	$\sqrt[m]{\sqrt[n]{b}} = \sqrt[mn]{b}$
Exponent	$\left(\sqrt[n]{b}\right)^n = \sqrt[n]{b^n} = b$
Fundamental identity	$\sqrt[km]{b^{kn}} = \sqrt[m]{b^n}$

Polynomials	
The general form of a polynomial	$P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$
leading coefficient	a_n
constant term	a_0
Particular cases of polynomials	• $P_0(x) = a_0$ • $P_1(x) = a_1 x + a_0$ • $P_2(x) = a_2 x^2 + a_1 x + a_0$ ....
Sum (difference) of polynomials	combine like terms
Product of polynomials	multiply every monomial of the first polynomial by every monomial of the second one and combine like terms.
Special Product Formulas	
Difference of the squares	$x^2 - y^2 = (x - y)\cdot(x + y)$
Sum (difference) of the cubes	$x^3 \pm y^3 = (x \pm y)\cdot\left(x^2 \mp xy + y^2\right)$
Square of the sum (difference)	$(x \pm y)^2 = x^2 \pm 2xy + y^2$
Cube of the sum (difference)	$(x \pm y)^3 = x^3 \pm 3x^2 y + 3xy^2 \pm y^3 =$ $= x^3 \pm y^3 \pm 3xy(x \pm y)$
Rational fraction (algebraic fraction)	$\dfrac{P_n(x)}{Q_m(x)}$
$\dfrac{P_n(x)}{Q_m(x)}$ – proper rational fraction	$n < m$
$\dfrac{P_n(x)}{Q_m(x)}$ – improper rational fraction	$n \geq m$
Logarithms	
Logarithm of x to the base a (logarithm is an exponent of a base a to receive an argument	$\log_a x \left(\log_a x = n \Leftrightarrow a^n = x\right)$
Properties of Logarithms	
Fundamental identities	• $\log_b b = 1$ • $\log_b 1 = 0$

An inverse property	$\log_b (b^p) = p$ $b^{\log_b p} = p$ (for $p > 0$)
Product property	$\log_b MN = \log_b M + \log_b N$
Quotient property	$\log_b \frac{M}{N} = \log_b M - \log_b N$
Power property	$\log_b (M^p) = p \log_b M$
One-to-one property	$\log_b M = \log_b N \Rightarrow M = N$
Logarithm of each side property	$M = N \Rightarrow \log_b M = \log_b N$
Change-of-Base Formula $a, x,$ *and* b are positive real numbers with $a \neq 1$ and $b \neq 1$,	$\log_b x = \frac{\log_a x}{\log_a b}$
common logarithm ($\log_{10} x = \lg x$)	$\lg x$
natural logarithm $\ln x = (\log_e x)$	$\ln x$

Trigonometric Functions of an Acute Angle

Let α be an acute angle of a right triangle. The values of four trigonometric functions of α are



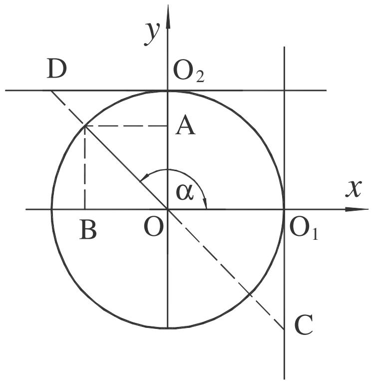
$$\sin\alpha = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{y}{r}$$

$$\cos\alpha = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} = \frac{x}{r}$$

$$\tan\alpha = \frac{\text{length of opposite side}}{\text{length of adjacent side}} = \frac{y}{x}$$

$$\cot\alpha = \frac{\text{length of adjacent side}}{\text{length of opposite side}} = \frac{x}{y}$$

α	$30^0 = \pi/6$	$45^0 = \pi/4$	$60^0 = \pi/3$
$\sin\alpha$	1/2	$1/\sqrt{2} = \sqrt{2}/2$	$\sqrt{3}/2$
$\cos\alpha$	$\sqrt{3}/2$	$1/\sqrt{2} = \sqrt{2}/2$	1/2
$\tan\alpha$	$1/\sqrt{3} = \sqrt{3}/3$	1	$\sqrt{3}$
$\cot\alpha$	$\sqrt{3}$	1	$1/\sqrt{3} = \sqrt{3}/3$



Trigonometric Circle

In general

$\sin\alpha = \frac{y}{r}$, $\cos\alpha = \frac{x}{r}$,

$\tan\alpha = \frac{y}{x}$, $\sin^2 x + \cos^2 x = 1$,

where $r = OP = \sqrt{x^2 + y^2}$.

Let the radius of circle be equal to 1, then
the vertical diameter is called the **sines-line** $\Rightarrow \sin\alpha = y = OA$
the horizontal diameter – the **cosines-line** $\Rightarrow \cos\alpha = x = OB$
the upper tangent – **line of cotangent** $\Rightarrow \tan\alpha = O_1C$
the right tangent – line of tangent $\Rightarrow \cot\alpha = O_2D$

Trigonometric Formulas	
The Fundamental trigonometric Identities	• $\sin^2 x + \cos^2 x = 1$ • $\tan x = \frac{\sin x}{\cos x}$ • $\cot x = \frac{\cos x}{\sin x}$ • $1 + \tan^2 x = \frac{1}{\cos^2 x}$ • $1 + \cot^2 x = \frac{1}{\sin^2 x}$
Sum and difference Identities	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Double-Angle Identities	$\cos 2x = \cos^2 x - \sin^2 x$ $\sin 2x = 2\sin x \cos x$ $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$
Half-Angle Identities	$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$ $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$ $\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$
Co-function Identities	$\sin(90^0 - x) = \cos x$ $\cos(90^0 - x) = \sin x$ $\tan(90^0 - x) = \cot x$ $\cot(90^0 - x) = \tan x$
Lowering of the Order	$\sin x \cos x = \frac{\sin 2x}{2}$ $\cos^2 x = \frac{1+\cos 2x}{2}$ $\sin^2 x = \frac{1-\cos 2x}{2}$
Product-to-Sum Identities	$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))$ $\sin\alpha\sin\beta = -\frac{1}{2}(\cos(\alpha+\beta)-\cos(\alpha-\beta))$ $\cos\alpha\sin\beta = \frac{1}{2}(\sin(\alpha+\beta)-\sin(\alpha-\beta))$ $\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$
Sum-to-Product Identities	$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$ $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$ $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$ $\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$ $a\sin x + b\cos x = k\sin(x+\alpha)$, where $k = \sqrt{a^2+b^2}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$, $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$.

Equations		
Linear equation $ax+b=0$		$x=-\frac{b}{a}$
Quadratic equations:		
1.	$ax^2+bx+c=0$	$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\begin{cases} x_1+x_2=-\frac{b}{a}, \\ x_1x_2=\frac{c}{a}. \end{cases}$
2.	$ax^2+2kx+c=0$	$x_{1,2}=\frac{-k\pm\sqrt{k^2-ac}}{a}$
3.	Vieta's theorem: if x_1 and x_2 are roots of an equation $x^2+px+q=0$, then	$\begin{cases} x_1+x_2=-p, \\ x_1x_2=q. \end{cases}$
Exponential equation $a^{f(x)}=b$, $a>0, a\neq 1$		$f(x)=\log_a b$
Logarithmic equation $\log_a f(x)=b$, $a>0, a\neq 1$		$\begin{cases} f(x)=a^b \\ f(x)>0 \end{cases}$
Trigonometric equations		
$\sin f(x)=a, \lvert a\rvert\leq 1$		$f(x)=(-1)^n \arcsin a+n\pi, n\in Z$
$\cos f(x)=a, \lvert a\rvert\leq 1$		$f(x)=\pm\arccos a+2n\pi, n\in Z$
$\tan f(x)=a$		$f(x)=\arctan a+n\pi, n\in Z$
$\cot f(x)=a$		$f(x)=arc\cot a+n\pi, n\in Z$

- $\arcsin a=x$: $\sin x=a$, where $\lvert a\rvert\leq 1, x\in[-\pi/2,\pi/2]$
- $\arccos a=x$: $\cos x=a$, where $\lvert a\rvert\leq 1, x\in[0,\pi]$
- $\arctan a=x$: $\tan x=a$, where $x\in(-\pi/2,\pi/2)$
- $arc\cot a=x$: $\cot x=a$, $x\in(0,\pi)$

COMPLEX NUMBERS

Vocabulary

complex number – комплексное число
conjugate complex number – сопряженное комплексное число
exponential form – показательная форма
geometrical representation – геометрическая интерпритация
imaginary axis – мнимая ось
imaginary part – мнимая часть
imaginary unit – мнимая единица
integral powers and roots of complex numbers – целые степени и корни комплексных чисел
principal argument – главный аргумент
real axis – действительная ось
real part – действительная часть
quadrant – четверть, квадрант

• By a **complex number** we mean an ordered pair of real numbers $z=(x, y)$ where x – **real part,** y – **imaginary part**

• **The equality relation and the arithmetical operations**:

1) $(x_1, y_1) = (x_2, y_2) \Leftrightarrow x_1 = x_2,\ y_1 = y_2$;

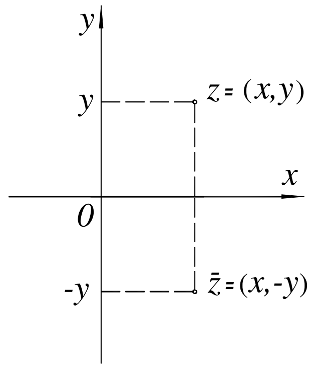
2) $(x_1, y_1) \pm (x_2, y_2) = (x_1 \pm x_2, y_1 \pm y_2)$;

3) $(x_1, y_1) \cdot (x_2, y_2)(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)$.

• **Imaginary unit:** $i = (0,1)$;

• $$i^n = \begin{cases} 1, & \text{if } n = 4k \\ i, & \text{if } n = 4k+1 \\ -1, & \text{if } n = 4k+2 \\ -i, & \text{if } n = 4k+3 \end{cases}, \text{ where } k \in N$$

• **Standard** or **rectangular form:** $z = x + yi$



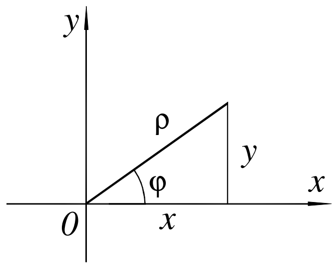
• **Modulus**, or **absolute value:** $|z| = \rho = \sqrt{x^2 + y^2}$.

• **The conjugate** to $z = x + yi$ is $x - yi$ and is denoted by $\bar{z}$:

$\bar{z} = x - yi$

• **Division of complex numbers:**

if $z_2 \neq 0$ then $z = \dfrac{z_1}{z_2} = \dfrac{z_1 \overline{z_2}}{z_2 \overline{z_2}}$.



If the point $z = (x, y) = x + yi$ is represented by polar coordinates ρ and φ ,we can write

$$\begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{cases} \text{ then}$$

• $z = \rho(\cos\varphi + i\sin\varphi)$ – is the **trigonometric form** of a complex number .

• The *x*-axis is called the **real axis.**

• The *y*-axis is the **imaginary axis**.

• The unique real number φ which satisfies the condition $-\pi < \varphi \leq \pi$ is called the **principal argument** of z and is denoted by $\arg z$: $\varphi = \arg z$ and can be calculated by formulas:

$$\varphi = \arg z = \begin{cases} \arctan\dfrac{y}{x}, & \text{if } z \in \text{I or IYquadrant,} \\ \pi + \arctan\dfrac{y}{x}, & \text{if } z \in \text{II or III quadrant.} \end{cases}$$

• $z = \rho e^{i\varphi}$ ($z = \rho\exp(i\varphi)$) is an **exponential form** of a complex number .

• Sometimes there is used the next form of complex number: $z = \rho\angle\varphi$.

Let $z_1 = \rho_1 \angle \varphi_1$ **and** $z_2 = \rho_2 \angle \varphi_2$ **be two complex numbers**, then

Operation	Trigonometric form	Exponential form
Product $z_1 z_2 =$	$= \rho_1 \rho_2 \left(\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2)\right)$	$= \rho_1 \rho_2 e^{i(\varphi_1 + \varphi_2)}$
Ratio $\frac{z_1}{z_2} =$	$= \frac{\rho_1}{\rho_2} \left(\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2)\right)$	$= \frac{\rho_1}{\rho_2} e^{i(\varphi_1 - \varphi_2)}$
Integral power $z^n =$	$= \rho^n (\cos n\varphi + i \sin n\varphi)$	$= \rho^n e^{in\varphi}$
Integral root $\sqrt[n]{z} = w_k$ $k = 0,1,2,\dots,n-1.$	$w_k = \sqrt[n]{\rho}\left(\cos\frac{\varphi + 2k\pi}{n} + i \sin\frac{\varphi + 2k\pi}{n}\right),$ $k = 0,1,2,\dots,n-1.$	$w_k = \sqrt[n]{\rho}\exp\left(i\frac{\varphi + 2k\pi}{n}\right),$ $k = 0,1,2,\dots,n-1.$

Remember :

- $e^{-i\varphi} = \cos\varphi - i\sin\varphi$
- $\cos\varphi = \dfrac{e^{i\varphi} + e^{-i\varphi}}{2}$
- $\sin\varphi = \dfrac{e^{i\varphi} - e^{-i\varphi}}{2i}.$

FUNCTIONS

Vocabulary

common logarithm. $(\log_{10} x)$ – десятичный логарифм
composite function – сложная функция
cube parabola – кубическая парабола
domain – область определения
even function – четная функция
exponential function – показательная функция
function - функция
graph of a function – график функция
hyperbola - гипербола
inverse function – обратная функция
linear function – линейная функция
maximum - максимум
minimum – минимум
natural exponential function (e^x) – экспонента
natural logarithm $(\ln x)$ – натуральный логарифм
odd function – нечетная функция
one-to-one function – взаимно-однозначная функция
parabola – парабола
periodic function – переодическая функция
power function – степенная функция
quadratic function–квадратичная функция
range – область значений
root or a zero of a function – корень или ноль функции
slope $(a = \tan\alpha)$ - угловой коэффициент
straight line –прямая
y-intercept point – пересечение с осью *ОУ*

Names	Definitions, theorems
Function: $y = f(x)$	Correspondence of sets D and E : $\forall x \in D\ \exists$ exactly one $y \in E$
Domain of a function: $D(y)$	$D(y) = \{x : f(x)$ takes finite, real values$\}$
Roots (zeros) of $f(x)$	$\{x : f(x) = 0\}$ - Set of x for which $f(x) = 0$
Graph of a function $y = f(x)$	$\{(x; f(x))\}$ – A set of points of a plane with coordinates x and $f(x)$
A function *f(x)* is increasing if	$f(x_1) < f(x_2)$ whenever $x_1 < x_2$

A function *f(x)* is decreasing if	$f(x_1) > f(x_2)$ whenever $x_1 < x_2$
A function *f(x)* is constant $(f(x) = C)$ if	$f(x_1) = f(x_2)$ for all x_1 and x_2
A function $f(x)$ is an even function	$f(-x) = f(x)$ for $\forall x \in D(x)$ and $-x \in D(y)$
A function $f(x)$ is called an odd function if	$f(-x) = -f(x)$ for $\forall x \in D(x)$ and $-x \in D(y)$
A function $f(x)$ is a periodic function with period P if	$f(x+P) = f(x)$ for $\forall x \in D(x)$
Composite function	$f(g(x))$ – function of function
The graphs of the given function $f(x)$ and its inverse $f^{-1}(x)$ are symmetric one to other with respect to the line $y = x$.	
Linear function	$y = ax + b$
Quadratic function	$y = ax^2 + bx + c$
The graph of a quadratic function	parabola
Standard form of a quadratic function	$y = a(x - x_0)^2 + y_0$
Hyperbola	$y = \dfrac{a}{x^n}, n \in N$
An exponential function	$y = a^x, a > 0, a \neq 1$
Logarithmic function	$y = \log_a x,\ a > 0, a \neq 1$

Table for Finding the Domain of a Composite Function

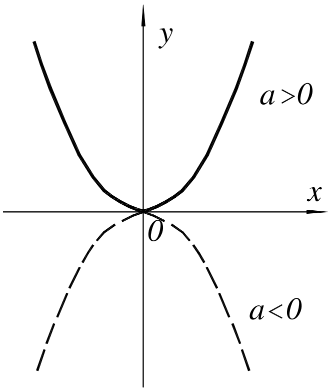
$y=$	$\dfrac{f(x)}{g(x)}$	$\sqrt[2k]{f(x)}$	$\log_a f(x)$	$\tan f(x)$	$\cot f(x)$	$\arcsin f(x)$, $\arccos f(x)$
$D(y)$:	$g(x) \neq 0$	$f(x) \geq 0$	$f(x) > 0$, $a > 0, a \neq 1$	$f(x) \neq \dfrac{\pi}{2} + \pi n$, $n \in Z$	$f(x) \neq \pi n$, $n \in Z$	$\lvert f(x) \rvert \leq 1$

A. Graphs in the Cartesian System of Co-ordinates

I. Power Functions

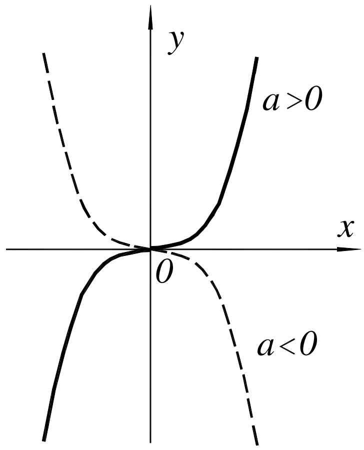
1. Parabolas

a). $y = ax^{2n}, n = 1,2,\ldots$



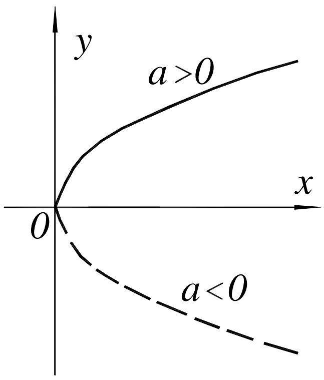
Domain of definition:
$D(y) = (-\infty,+\infty)$
Range of values
$$E(y) = \begin{cases} [0,\infty), a > 0 \\ (-\infty,0], a < 0 \end{cases}$$

b). $y = ax^{2n+1}, n = 1,2,\ldots$



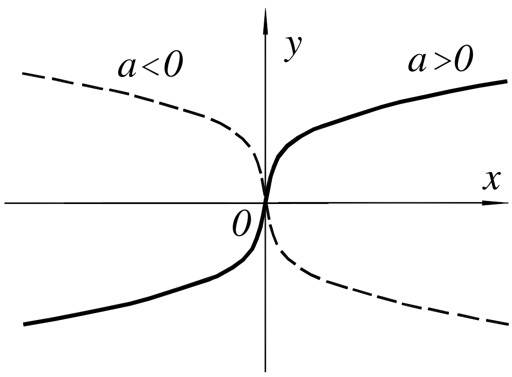
$D(y) = (-\infty,+\infty)$
$E(y) = (-\infty,+\infty)$

c). $y = a\sqrt[2n]{x}, n = 1,2,\cdots$



$D(y) = [0,+\infty)$
$$E(y) = \begin{cases} [0,+\infty), a \geq 0 \\ (-\infty,0], a \leq 0 \end{cases}$$

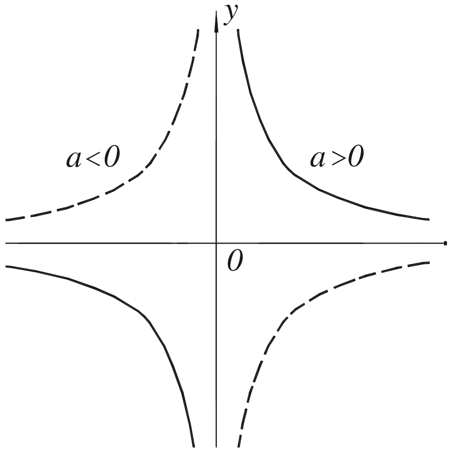
d). $y = a\sqrt[2n+1]{x}, n = 1,2,\cdots$



$D(y) = (-\infty,+\infty)$
$E(y) = (-\infty,+\infty)$

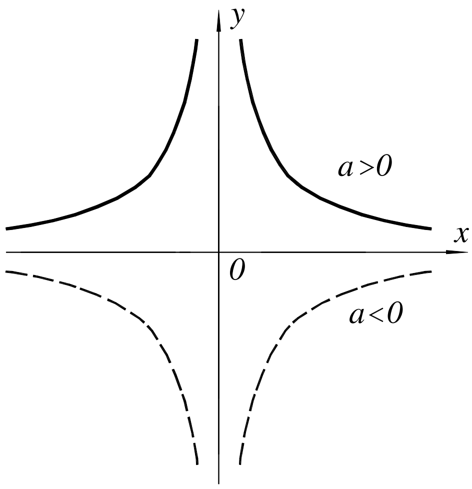
2. Hyperbolas

a). $y = \frac{a}{x^{2n-1}}, n = 1,2,\ldots$



$D(y) = (-\infty,0) \cup (0,+\infty)$
$E(y) = (-\infty,0) \cup (0,+\infty)$

b). $y = \frac{a}{x^{2n}}, n = 1,2,\ldots$

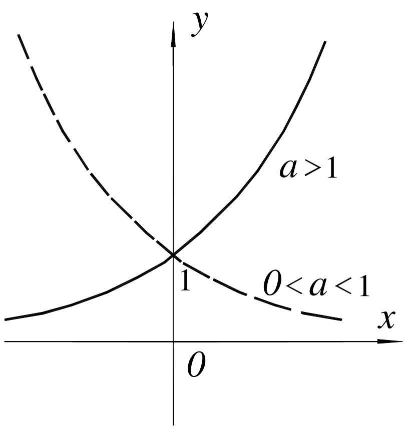


$D(y) = (-\infty,0) \cup (0,+\infty)$

$E(y) = \begin{cases} (0,+\infty), a > 0 \\ (-\infty,0), a < 0 \end{cases}$

II. Exponential Function

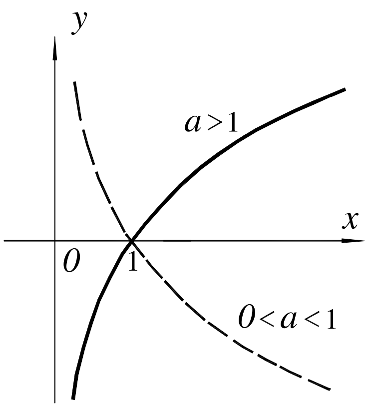
$y = a^x, a > 0, a \neq 1$



$D(y) = (-\infty,+\infty)$
$E(y) = (0,+\infty)$

III. Logarithm Function

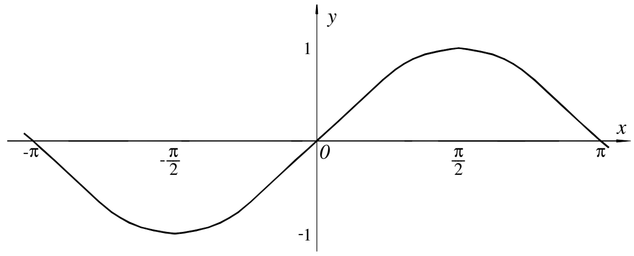
$y = \log_a x, a > 0, a \neq 1$



$D(y) = (0,+\infty)$
$E(y) = (-\infty,+\infty)$

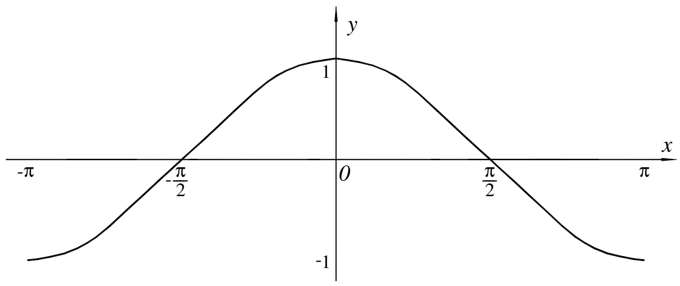
IY. Trigonometric Functions

1). Sinusoid (sine curve, harmonic curve) $y = \sin x$



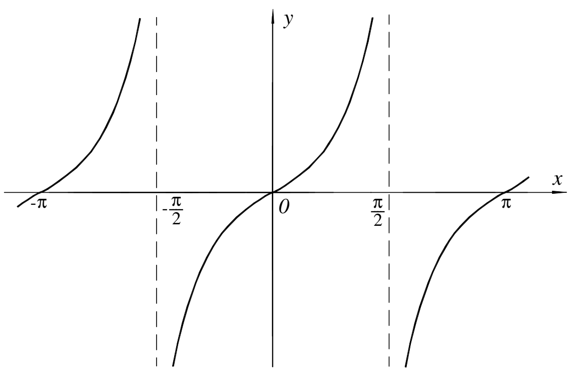
$$D(y) = (-\infty, +\infty),$$
$$E(y) = [-1, +1]$$

2). Cosine curve $y = \cos x$



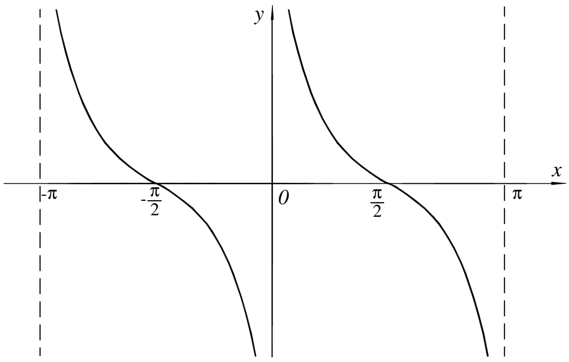
$$D(y) = (-\infty, +\infty),$$
$$E(y) = [-1, +1]$$

3). Tangent curve $y = \tan x$



$$D(y) = \left(-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi\right), k = 0, \pm 1, \pm 2, \ldots$$
$$E(y) = (-\infty, +\infty)$$

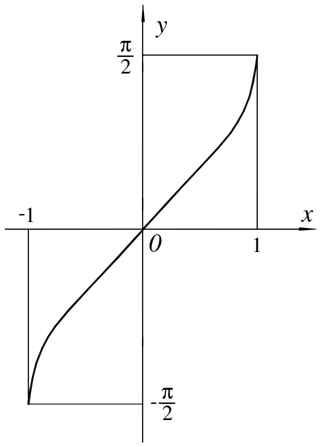
4). Cotangent curve $y = \cot x$



$$D(y) = (k\pi, (k + 1\pi)), k = 0, \pm 1, \pm 2, \ldots$$
$$E(y) = (-\infty, +\infty)$$

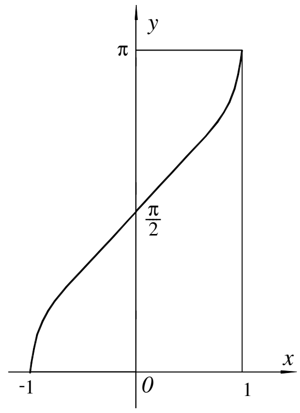
Y. Inverse Trigonometric Functions

1). $y = \sin^{-1} x = \arcsin x$



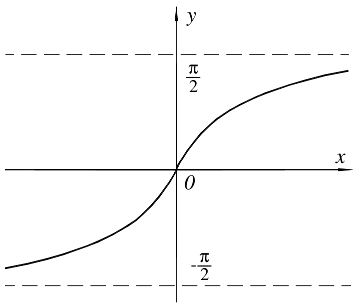
$D(y) = [-1;1]$, $E(y) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2). $y = \cos^{-1} x = \arccos x$



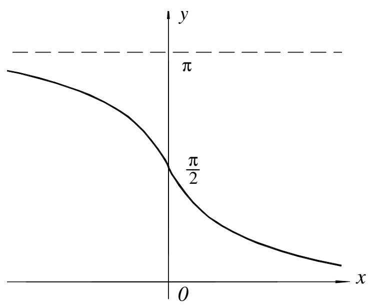
$D(y) = [-1;1]$, $E(y) = [0;\pi]$

3). $y = \tan^{-1} x = \arctan x$



$D(y) = (-\infty, +\infty)$, $E(y) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

4). $y = \cot^{-1} x = arc\cot x$



$D(y) = (-\infty, +\infty)$, $E(y) = (0, \pi)$

YI. Hyperbolic Functions

1). $y = \sinh x$ (shx)

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$D(\sinh x) = (-\infty, +\infty)$

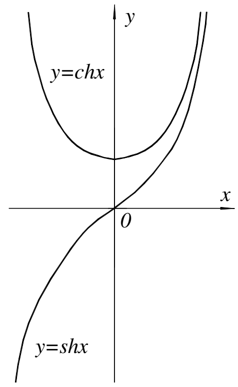
$E(\sinh x) = (-\infty, +\infty)$

2). $y = \cosh x$ (chx)

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$D(\cosh x) = (-\infty, +\infty)$

$E(\cosh x) = [1, +\infty)$



3). $y = \tanh x \ (thx)$

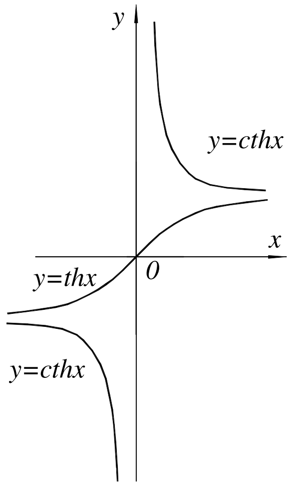
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$D(\tanh x) = (-\infty, +\infty)$$
$$E(\tanh x) = (-1, +1)$$

4). $y = \coth x \ (cthx)$

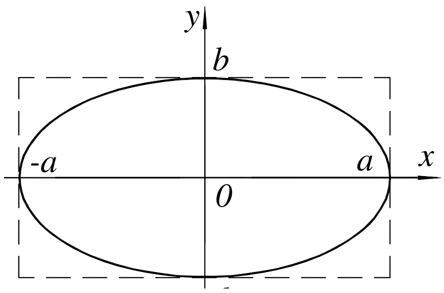
$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$D(\coth x) = (-\infty, 0) \cup (0, +\infty)$$
$$E(\coth x) = (-\infty, -1) \cup (1, \infty)$$

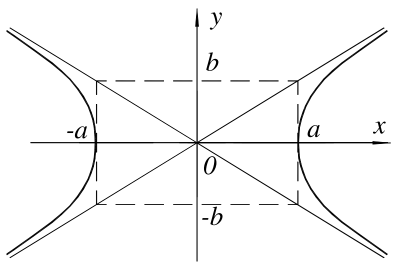


YII. Curves of the Second Order

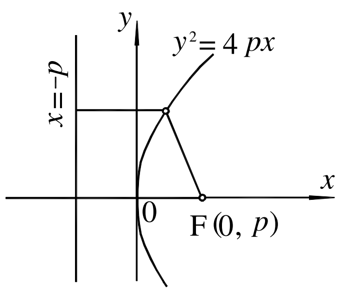
1. Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



2.Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

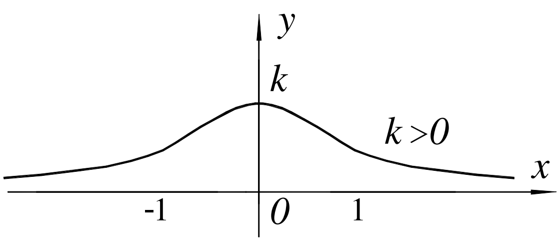


3. Parabola



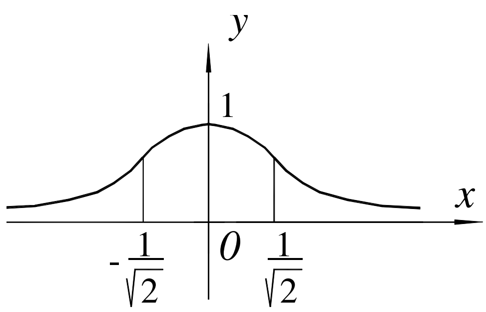
4. Witch of Agnesi:

$$y = \frac{k}{1 + x^2}$$



5.Curve of Gauss:

$$y = e^{-x^2}$$

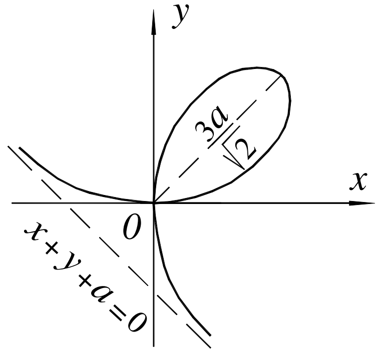


6. Loops

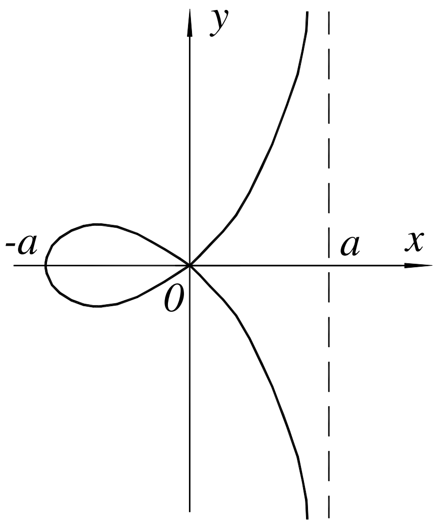
a). Folium of Descartes

$x^3 + y^3 - 3axy = 0$, or

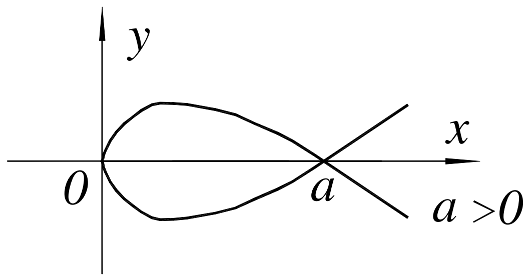
$$\begin{cases} x = \dfrac{3at}{1+t^3} \\ y = \dfrac{3at^2}{1+t^2} \end{cases}$$



b) $y^2 = x^2 \cdot \dfrac{a+x}{a-x}$



c). $a^2 y^2 = x\left(a - x^2\right), a > 0$

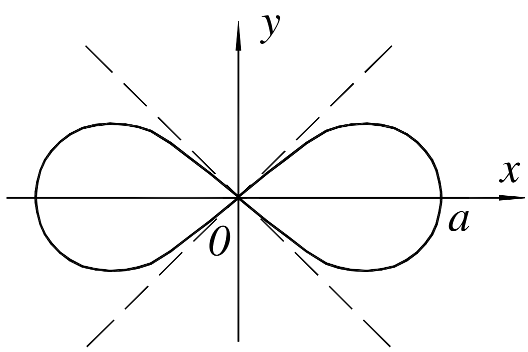


7. Lemniscate of Bernoulli

$$\left(x^2 + y^2\right)^2 = a^2\left(x^2 - y^2\right)$$

or

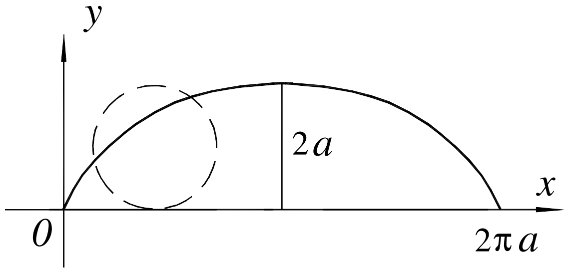
$$r^2 = a^2 \cos 2\varphi$$



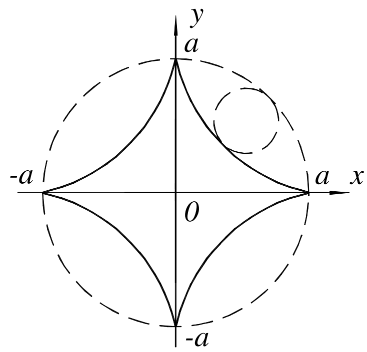
B. Curves Given by Parametric Equations

I. Cycloid:

$$\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}, a > 0$$

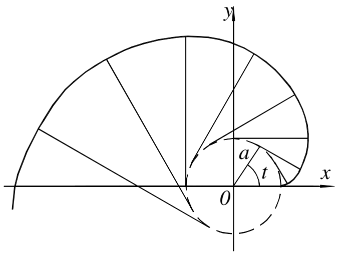


II. Astroid: $\begin{cases} x = a\cos^3 t \\ y = a\sin^3 t \end{cases}, a > 0$

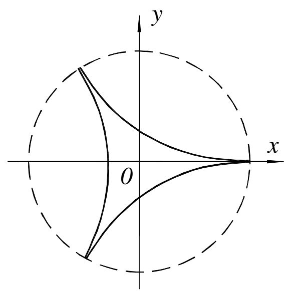


III. Evolvent of Circle:

$$\begin{cases} x = a(\cos t + t\sin t) \\ y = a(\sin t - t\cos t) \end{cases}, a > 0$$



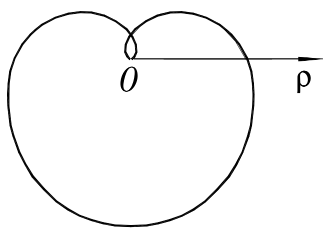
IY. $\begin{cases} x = R\cos\frac{t}{3}\cdot\left(2 + \cos\frac{t}{3}\right) \\ y = R\sin\frac{t}{3}\cdot\left(2 - \sin\frac{t}{3}\right) \end{cases}$



C. Curves in the Polar System of Coordinates

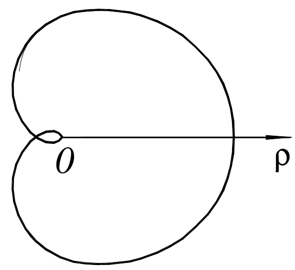
1. $\rho = a\sin^3\frac{\varphi}{3}$,

$a > 0, \varphi \in [0, 3\pi]$



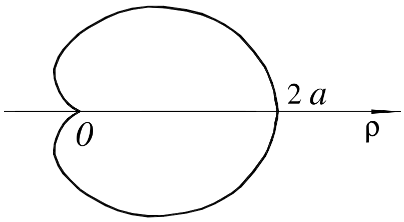
II. $\rho = a\cos^3\varphi$,

$a > 0, \varphi \in \left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$

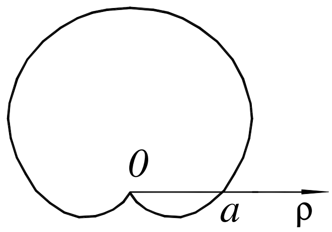


III. Cardioids

1) $\rho = a(1+\cos\varphi), a > 0$

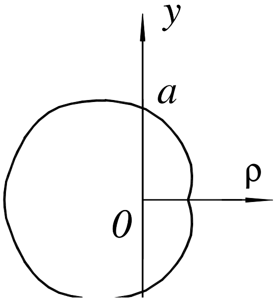


2) $\rho = a(1+\sin\varphi), a > 0$

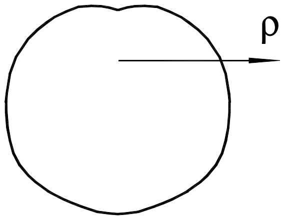


IY. Limacons

`1). $\rho = a - \cos\varphi, a > 1$

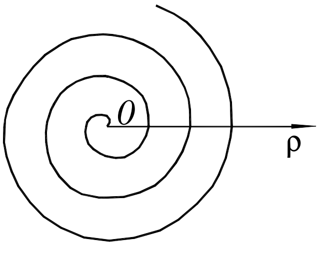


2). $\rho = a - \sin\varphi, a > 1$

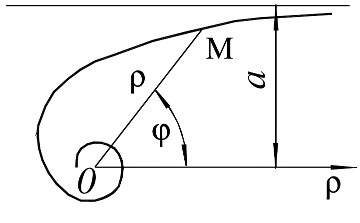


Y. Spirals

1). $\rho = a\varphi, a > 0$

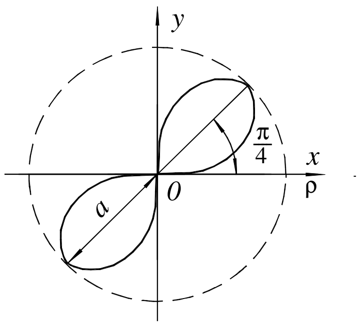


2). $\rho = \dfrac{a}{\varphi}, a > 0$

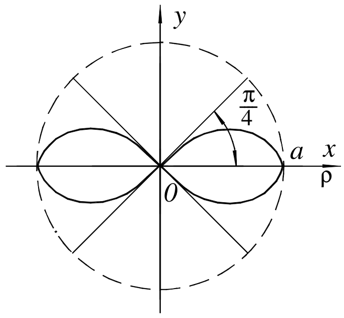


YI. Roses

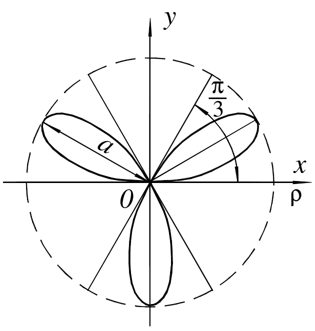
1). $\rho = a\sin 2\varphi, a > 0$



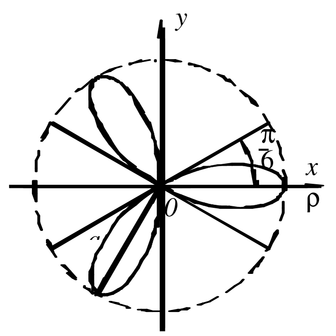
2). $\rho = a\cos 2\varphi, a > 0$



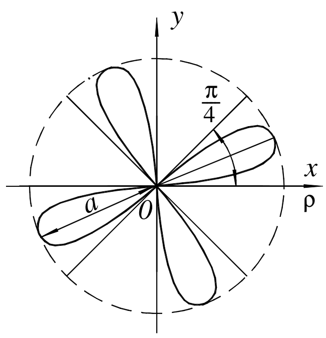
3). $\rho = a\sin 3\varphi, a > 0$



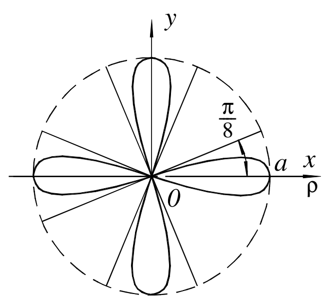
4). $\rho = a\cos 3\varphi, a > 0$



5). $\rho = a\sin 4\varphi, a > 0$



6). $\rho = a\cos 4\varphi, a > 0$



Linear Algebra
Vocabulary

addition (subtraction) of matrices – сложение (вычитание матриц)
adjoint matrix– присоединенная матрица
augmented matrix – расширенная матрица
capital letter – заглавная буква
check the equality – проверить равенство
cofactor of an element A **-** алгебраическое дополнение элемента a_{ij}
column – столбец
compatible system – совместная система
Cramer's Rule – правило Крамера
determinant - определитель
diagonal matrix – диагональная матрица
dimension – размерность
elementary operations on matrices – элементарные операции над матрицами
general case – общий случай
homogeneous system of equations – однородная система уравнений
inverse of matrix – обратная матрица
lowercase letter (small letter) – строчная буква
matrix (matrices) – матрица (матрицы)
matrix form – матричный метод
matrix of order n – матрица порядка n
minor – минор
multiplication of matrices – умножение матриц
multiplying of a matrix by a number – умножение матрицы на число
nonsingular - невырожденный
principal (main) diagonal – главная диагональ
rank of matrix – ранг матрицы
row – строка
square matrix – квадратная матрица
supplement – дописать, добавить
system of n linear equation in n unknowns – система *n* линейных уравнение с *n* неизвестными
the expanding determinant by – разложение определителя по
transformation *Z* through *X* - преобразование *Z* через *X*
transposed matrix – транспонированная матрица
triangular matrix – треугольная матрица
unique solution – единственное решение
unit-matrix – единичная матрица

Matrices, determinants, linear systems	
Titles, definitions, theorems	Algebraic form
• A rectangular array of numbers is called a **matrix** of [$m \times n$] (read m by n) dimension. • The element in the i-th row and j-th column of a matrix can be represented as $a_{ij} : A = \left(a_{ij}\right), i = \overline{1, m}, \quad j = \overline{1, n}.$	$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$
Transposed matrix	$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}^T = \begin{pmatrix} a_{11} & \cdots & a_{m1} \\ \cdots & \cdots & \cdots \\ a_{1n} & \cdots & a_{mn} \end{pmatrix}$
Square matrix of order n	$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$
Principal (main) diagonal of a square matrix	The diagonal containing $a_{11}, a_{22}, \ldots, a_{n-1n-1}, a_{nn}$
Diagonal matrix	$\begin{pmatrix} a_{12} & 0 & \cdots & 0 \\ 0 & a_{21} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix}$
Unit-matrix E or I	$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$
Let $A = \left(a_{ij}\right),\ B = \left(b_{ij}\right),\ i = \overline{1, m}, j = \overline{1, n}$ then	
Equality of matrices $A = B$	$A = B \Leftrightarrow a_{ij} = b_{ij},\ i = \overline{1, m}, j = \overline{1, n}$
Sum of matrices of the same dimension $A + B = C$	$C = \left(c_{ij}\right)$, where $c_{ij} = a_{ij} + b_{ij}$, $i = \overline{1, m}, j = \overline{1, n}$
Difference of matrices of the same dimension $A - B = D$	$D = \left(d_{ij}\right)$, where $d_{ij} = a_{ij} - b_{ij}$, $i = \overline{1, m}, j = \overline{1, n}$

Product of a matrix A by a number λ: λA	$\lambda A = \left(\lambda a_{ij}\right), i = \overline{1,m}, j = \overline{1,n}$
Determinants: • $\det a_{11}$ • $\det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ • $\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{32} & a_{32} & a_{33} \end{pmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$	• $\det a_{11} = a_{11}$ • $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$ • $\left.\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}\right. =$ $= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$ $- a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$
A minor M_{ij} of an element a_{ij} of a square matrix *A* is a determinant obtained from a given matrix deleting the *i*-th row and *j*-th column.	
A quantity $(-1)^{i+j}M_{ij}$ is called a cofactor A_{ij} of an element a_{ij}.	
The sum of the products of elements a_{ij} of any row (column) of a determinant and their cofactors is equal to one and the same number. This number is a value of the given determinant.	$\lvert A\rvert = a_{i1}A_{11} + a_{i2}A_{i2} + ... + a_{in}A_{in}$ is called the expanding determinant by the *i*-th row.
The multiplication of a matrix A[$m\times n$] by a matrix B[$n\times p$] is a matrix C[$m\times p$] whose element c_{ij} is the product of the *i*-th row of *A* and the *j*-th column of *B:*	$c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$, $i = \overline{1,m}$, $j = \overline{1,p}$
A square matrix *B* is said to be an inverse matrix of *A* if $AB = BA = I$ and it is denoted by the symbol A^{-1}. So we have $AA^{-1} = A^{-1}A = I$	
Transposed matrix of cofactors of the corresponding elements of the given matrix *A* is called the adjoint of *A*	$\text{adj}\, A = \tilde{A} = \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \cdots & \cdots & \cdots \\ A_{n1} & \cdots & A_{nn} \end{pmatrix}^T$
Method of finding the inverse matrix	1. Be sure that $\Delta = \lvert A \rvert \neq 0$. 2. Construct the matrix of corresponding cofactors and transpose it $(\tilde{A})$. 3. Divide the matrix $\tilde{A}$ by $\lvert A \rvert$: $A^{-1} = \frac{1}{\det A}\tilde{A}$.

The system of n linear equation with n unknowns	$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases}$
Matrix form of a system: $AX = B$	$AX = B$, where $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{pmatrix}$.
Methods of solution of linear systems	
Matrix method solution of a square linear system	$X = A^{-1}B$
Cramer's Rule :If the determinant of the system does not equal zero then this system has a unique solution .	$x_1 = \frac{\Delta_1}{\Delta}, \ x_2 = \frac{\Delta_2}{\Delta}, \ \ldots, \ x_n = \frac{\Delta_n}{\Delta}$, where $\Delta = \lvert A \rvert$. Δ_i is the determinant formed by replacing the ith column of the matrix A with the column of constant terms $b_1, b_2, b_3, \ldots, b_n$, $i = \overline{1, n}$
System of linear equations in the general Case $\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x + \ldots + a_{2n}x_2 = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}$ **(1)**	• Matrix of a system: $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$ • Augmented matrix: $B = \left(\begin{array}{cccc\|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right)$

Let an arbitrary matrix *A* dimension of which is $[m\times n]$ be given Let us strike out *k* rows and *k* columns in this matrix. Then elements a_{ij} found at the intersection of these rows and columns form the matrix of order *k*. Determinant of this matrix is said to be minor of the *k* order of the matrix *A*.
The highest order of the minor of matrix *A* different from zero is called the rank of this matrix and denoted *r(A)*.
Matrix *A* is equivalent to matrix *B* if their ranks are equal: *A* ~*B* if *r(A) = r(B)*.
A system **(1)** is compatible if and only if the rank of matrix *A* is equal to the rank of matrix *B*: $r(A) = r(B)$
• Gauss Method. The goal of this method is to reduce an augmented matrix to triangular form. After that it is able to answer the next questions: 1. Is the given linear system compatible or not? 2. How many solutions has this system? 3. If the system is compatible you can find its solution.
• Homogeneous System of Equations is a linear system of equations for which the constant terms are zeroes: $\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0 \end{cases}$
Homogeneous system is compatible as the solution $(0,0,...,0)$ is always the solution of this system. If a homogeneous system has non-zero solutions you can find them using the Gauss method

Example. Solve the system of linear equation using Gauss method.

$$\begin{cases} 2x_1 + x_2 + x_3 + 2x_4 = 8, \\ x_1 - x_2 + 3x_3 + x_4 = 10, \\ x_1 + x_2 + x_4 = 5. \end{cases}$$

Solution.

Reduce the augmented matrix to triangular form:

$$B=\left(\begin{array}{cccc|c}2&1&1&2&8\\1&-1&3&1&10\\1&1&0&1&5\end{array}\right)\begin{array}{l}\\\\R_3\to R_1\end{array}\sim\left(\begin{array}{cccc|c}1&1&0&1&5\\2&1&1&2&8\\1&-1&3&1&10\end{array}\right)\begin{array}{l}\\R_2-2R_1\\R_2-R_1\end{array}\sim$$

$$\sim\left(\begin{array}{cccc|c}1&1&0&1&5\\0&-1&1&0&-2\\0&-2&3&0&5\end{array}\right)\begin{array}{l}\\\\R_3-2R_2\end{array}\sim\left(\begin{array}{cccc|c}1&1&0&1&5\\0&-1&1&0&-2\\0&0&1&0&9\end{array}\right)$$

As we can see $r(A)=r(B)=3$. But $n = 4 > r = 3$. It means that the general solution depends on $n-r$ arbitrary constants. Here is one constant in this case. Let $x_4=C$, then the equivalent system is

$$\begin{cases}x_1+x_2=5-C\\-x_2+x_3=-2\\x_3=9\end{cases}\Rightarrow\begin{cases}x_3=9,\\-x_2+9=-2\Rightarrow x_2=11,\\x_1+11=5-C\Rightarrow x_1=-6-C.\end{cases}$$

Check the solution: $AX=B$.

$$AX=\begin{pmatrix}2&1&1&2\\1&-1&3&1\\1&1&0&1\end{pmatrix}\cdot\begin{pmatrix}-6-C\\11\\9\\C\end{pmatrix}=\begin{pmatrix}\text{-12-2}C+11+9+2C\\-6-C-11+27+C\\-6\text{-}C+11+C\end{pmatrix}=\begin{pmatrix}8\\10\\5\end{pmatrix}=B.$$

Answer: $X=\begin{pmatrix}-6-C\\11\\9\\C\end{pmatrix}$.

VECTORS

Vocabulary

accordingly – соответственно
be observed - наблюдается
clockwise – по часовой стрелке
collinear - коллинеарный
coplanar - компланарный
counterclockwise – против часовой стрелки
directed – направленный
direction cosines – направляющие косинусы
expansion of the vector $\overline{a}$ through the base – разложение вектора по базису
initial point - начальная точка
left- handed triple – левая тройка
middle point of a segment – точка, делящая отрезок пополам
ordered triple –упорядоченная тройка
ort of a vector (unit vector) – единичный вектор
orthogonal – ортогональный
right -handed triple – правая тройка
rotation – поворот
scalar – скаляр
scalar product (dot product) – скалярное произведение
terminal (end) point – конечная точка
test of collinearity of vectors – признак коллинеарности векторов
test of coplanarity of vectors – признак компланарности векторов
test of orthogonality of vectors – признак ортогональности векторов
tetrahedron – тетраэдр
torque – момент (силы)
triangle – треугольник
triple scalar product – смешанное произведение
vector – вектор
vector product (cross product) – векторное произведение
volume of the parallelepiped – объем параллелепипеда

- Vector is a line segment. $\overline{a} = \overline{AB}$, *A* is the initial point. *B* is the terminal (end) point.
- Let $\overline{i}, \overline{j}, \overline{k}$ be the unit and orthogonal vectors giving the direction of *x*-axis, *y*-axis and *z*-axis accordingly: $|\overline{i}| = |\overline{j}| = |\overline{k}| = 1$, $\overline{i} \perp \overline{j} \perp \overline{k}$. Then vector $\overline{a} = a_x \overline{i} + a_y \overline{j} + a_z \overline{k}$, where $a_x, a_y, a_z -$ are the projections of the vector $\overline{a}$ onto the vectors $\overline{i}, \overline{j}$ *and* $\overline{k}$ accordingly and are called the coordinates of the vector.

• $\overline{a} = (a_x, a_y, a_z)$ – the coordinate form of the vector $\overline{a}$;

• $\overline{a} = a_x\overline{i} + \overline{a}_y\overline{j} + a_z\overline{k}$ – the vector form of the vector $\overline{a}$, or the expansion of the vector $\overline{a}$ through the base $\overline{i}, \overline{j}, \overline{k}$.

№	Theorems, definitions	Formulas for Calculation
Let the coordinates of the points A and B and vectors $\overline{a}, \overline{b}, \overline{c}$ be given: $A(x_A, y_A, z_A)$, $B(x_B, y_B, z_B)$, $\overline{a} = (a_x, a_y, a_z)$, $\overline{b} = (b_x, b_y, b_z)$, $\overline{c} = (c_x, c_y, c_z)$ then		
1	Vector's coordinates through the coordinates of initial and end points	$\overline{AB} = (x_B - x_A; y_B - y_A, z_B - z_A)$
2	The length of a vector	$\lvert \overline{a} \rvert = \sqrt{a_x^2 + a_y^2 + a_z^2}$
3	Equal vectors $(\overline{a} = \overline{b})$ have equal corresponding coordinates	$\overline{a} = \overline{b} \Leftrightarrow a_x = b_x, a_y = b_y, a_z = b_z$
4	Sum and difference of vectors	$\overline{a} \pm \overline{b} = (a_x \pm b_x; a_y \pm b_y; a_z \pm b_z)$
5	A product of a vector $\overline{a}$ by a scalar (number) λ	$\lambda\overline{a} = (\lambda a_x; \lambda a_y; \lambda a_z)$
6	A scalar product (dot product) of vectors $\overline{a}$ and $\overline{b}$ is the number equal to the product of the moduli of these vectors and the cosine of the angle φ between them: $(\overline{a}, \overline{b}) = \lvert\overline{a}\rvert \cdot \lvert\overline{b}\rvert \cos\varphi$	$(\overline{a}, \overline{b}) = a_x b_x + a_y b_y + a_z b_z$
7	Cosine of the angle φ between vectors $\varphi = (\overline{a}, \wedge\overline{b})$	$\cos\varphi = \dfrac{(\overline{a}, \overline{b})}{\lvert\overline{a}\rvert \cdot \lvert\overline{b}\rvert}$
8	The scalar product is the work.	$A = (\overline{F}, \overline{S})$
9	The ordered triple of vectors is called a right (left)-handed triple if the shortest rotation of the first vector to the second one is observed from the end point of the third vector in the counterclockwise (clockwise)	

10	Vector product (cross product) of two vectors $\bar{a}$ and $\bar{b}$ is the vector $\bar{S}$ such that 1) $\mid \bar{S} \mid = \mid \bar{a} \mid \mid \bar{b} \mid \sin\varphi$, where φ is the angle between $\bar{a}$ and $\bar{b}$; 2) $\bar{S}\perp\bar{a}, \bar{S}\perp\bar{b}$ (the vector $\bar{S}$ is orthogonal to both of the vectors $\bar{a}$ and $\bar{b}$); 3) $\bar{a},\bar{b},\bar{S}$ is the right-handed triple of vectors.	$\bar{S} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$
11	The area of the parallelogram constructed on the vectors $\bar{a}$ and $\bar{b}$	$S = \left\| \left[\bar{a},\bar{b}\right] \right\|$
12	The area of a triangle constructed on the vectors $\bar{a}$ and $\bar{b}$	$S_\Delta = \frac{1}{2}\left\| \left[\bar{a},\bar{b}\right] \right\|$
13	The vector product is the torque of the force	$\left[\bar{r},\bar{F}\right] = L_0$
14	The triple scalar product: $\left(\bar{a},\bar{b},\bar{c}\right) = \left(\left[\bar{a},\bar{b}\right],\bar{c}\right) \left(\bar{a},\left[\bar{b},\bar{c}\right]\right)$	$\left(\bar{a},\bar{b},\bar{c}\right) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$
15	The volume of the parallelepiped determined by vectors $\bar{a},\bar{b},\bar{c}$ is equal to the module of the triple scalar product of these vectors	$V = \left\| \left(\bar{a},\bar{b},\bar{c}\right) \right\|$
16	The volume of a tetrahedron determined by vectors $\bar{a},\bar{b},\bar{c}$	$V_{\text{tet.}} = \frac{1}{6}\left\| \left(\bar{a},\bar{b},\bar{c}\right) \right\|$
17	Tests of collinearity of vectors $\bar{a} \parallel \bar{b}$	• $\bar{a} \parallel \bar{b} \Leftrightarrow \frac{a_x}{b_x} = \frac{a_y}{b_y} = \frac{a_z}{b_z}$ • $\bar{a} \parallel \bar{b} \Leftrightarrow \bar{a} = \lambda\bar{b}$ • $\bar{a} \parallel \bar{b} \Leftrightarrow \left[\bar{a},\bar{b}\right] = 0$

18	Test of orthogonality of vectors $\overline{a}\perp\overline{b}$	$\overline{a}\perp\overline{b} \Leftrightarrow \left(\overline{a},\overline{b}\right)=0$
19	Test of coplanarity	$\overline{a}$, $\overline{b}$ and $\overline{c}$ are coplanar if and only if $\left(\overline{a},\overline{b},\overline{c}\right)=0$
20	Coordinates of a point M dividing a segment AB in the given ratio $\lambda:\left(\frac{AM}{MB}=\lambda\right)$	$x_M = \frac{x_A+\lambda x_B}{1+\lambda}$, $y_M = \frac{y_A+\lambda y_B}{1+\lambda}$, $z_M = \frac{z_A+\lambda z_B}{1+\lambda}$.
21	Coordinates of a middle point of a segment AB ($AC = CB$)	$x_C = \frac{x_A+x_B}{2}$, $y_C = \frac{y_A+y_B}{2}$, $z_C = \frac{z_A+z_B}{2}$.
22	Direction cosines ($\alpha=\overline{a}\wedge\overline{i}, \beta=\overline{a}\wedge\overline{j}, \gamma=\overline{a}\wedge\overline{k}$)	$\cos\alpha=\frac{a_x}{\lvert\overline{a}\rvert}$, $\cos\beta=\frac{a_y}{\lvert\overline{a}\rvert}$, $\cos\gamma=\frac{a_z}{\lvert\overline{a}\rvert}$.
23	Ort of a vector $\overline{a}$ (the unit vector): $\overline{a^0}$	$\overline{a}^0=\left(\cos\alpha;\cos\beta;\cos\gamma\right)$

ANALYTIC GEOMETRY

Vocabulary

angular relations – угловые соотношения
asymptote - асимптота
axis of the parabola - ось параболы
canonical equations – канонические уравнения
directrix - директриса
distance from a point to – расстояние от точки до
eccentricity – эксцентриситет
ellipse - эллипс
ellipse's focal axis – фокальные оси эллипса
ellipsoid - эллипсоид
elliptic cone – эллиптический конус
elliptic cylinder – эллиптический цилиндр
elliptic paraboloid – эллиптический параболоид
general equation – общее уравнение
hyperbolic cylinder – гиперболический цилиндр
hyperbolic paraboloid – гиперболический параболоид
intercept form of the equation of a plane – уравнение плоскости в отрезках
major axis – большая ось
normal vector - нормальный вектор
one-sheeted hyperboloid – однополостный гиперболоид
parabola – парабола
parabolic cylinder – параболический цилиндр
position vector - направляющий вектор
quadric surface – поверхность второго порядка
second order curves – кривые второго порядка
semimajor axis - большая полуось
slope equation - уравнение с угловым коэффициентом
standard parametric equations – параметрические уравнения
two-points equation of a straight line – уравнение прямой через две точки
two-sheeted hyperboloid – двуполостный гиперболоид
vector equation – векторное уравнение
vertex of the parabola – вершина параболы

№	Names, definitions, theorems	Equations and formulas
A plane in a space		
1.	The equation for the plane through the point $M(x_0; y_0; z_0)$ normal to the vector $\overline{n}=(A, B, C)$	$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$
2	Standard equation of a plane π	$\pi: Ax+By+Cz+D=0 \Rightarrow$ $\overline{n}=(A; B; C)\perp\pi$
3	Equation of a plane passing through points $P_1(x_1; y_1; z_1)$, $P_2(x_2; y_2; z_2)$, $P_3(x_3; y_3; z_3)$.	$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$
4	Intercept form of the equation of a plane	$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
5	The distance from the point $P(x_0; y_0; z_0)$ to the plane $\pi: Ax+By+Cz+D=0$	$d(P,\pi)=\frac{\lvert Ax_0+By_0+Cz_0+D\rvert}{\sqrt{A^2+B^2+C^2}}$
Let $\overline{n}_1=(A_1; B_1; C_1)$ - normal vector of a plane π_1, and $\overline{n}_2=(A_2; B_2; C_2)$ - normal vector of a plane π_2, then there are relations:		
6	Conditions of the perpendicularity of the planes: $\pi_1\perp\pi_2 \Leftrightarrow \overline{n}_1\perp\overline{n}_2$	$\pi_1\perp\pi_2 \Leftrightarrow A_1A_2+B_1B_2+C_1C_2$
7	Conditions of the parallelism of the planes: $\pi_1 \mid\mid \pi_2 \Leftrightarrow \overline{n}_1 \mid\mid \overline{n}_2$	$\pi_1 \mid\mid \pi_2 \Leftrightarrow \frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$
8	The angle between the planes $(\pi_1 \wedge \pi_2)$	$\cos(\pi_1 \wedge \pi_2)=\lvert\cos(\overline{n}_1, \wedge \overline{n}_2)\rvert$
A straight line in a space		
Let a straight line ℓ passes the point $M(x_0; y_0; z_0)$ parallel to a vector $\overline{s}=(m; n; p)$, ($\overline{s}$ – directed vector of ℓ) then:		
9	Vector equation of a straight line ℓ	$\overline{r}=\overline{r_0}+t\overline{s}$
10	Canonical equations of a straight line ℓ	$\ell: \frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$

11	Standard parametric equations of a straight line ℓ	$\ell : \begin{cases} x = x_0 + mt \\ y = y_0 + nt \\ z = z_0 + pt \end{cases}$
Let $\bar{s}_1 = (m_1; n_1; p_1)$ - direction vector of a straight line ℓ_1, and $\bar{s}_2 = (m_2; n_2; p_2)$ - direction vector of a straight line ℓ_2, then there are relations:		
12	Conditions of the parallelism of the straight lines: $\ell_1 \| \ell_2 \Leftrightarrow \bar{s}_1 \| \bar{s}_2$	$\ell_1 \| \ell_2 \Leftrightarrow \frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{p_1}{p_2}$
13	Conditions of the perpendicularity of the straight lines: $\ell_1 \perp \ell_2 \Leftrightarrow \bar{s}_1 \perp \bar{s}_2$	$\ell_1 \perp \ell_2 \Leftrightarrow m_1 m_2 + n_1 n_2 + p_1 p_2 = 0$
14	The angle between the straight lines: $\cos(\ell_1 \wedge \ell_2) = \lvert\cos(\bar{s}_1, \wedge \bar{s}_2)\rvert$	$\cos(\ell_1 \wedge \ell_2) = = \left\lvert \frac{m_1 m_2 + n_1 n_2 + p_1 p_2}{\sqrt{m_1^2 + n_1^2 + p_1^2}\sqrt{m_2^2 + n_2^2 + p_2^2}} \right\rvert$
A plane and a straight line in a space		
Let $\ell : \frac{x - x_0}{m} = \frac{y - y_0}{n} = \frac{z - z_0}{p}$ and $\pi : Ax + By + Cz + D = 0$ be given, then:		
15	Parallelism of a straight line ℓ and a plane π: $\pi \| \ell \Leftrightarrow \bar{n} \perp \bar{s}$	$\pi \| \ell \Leftrightarrow Am + Bn + Cp = 0$
16	Conditions of the perpendicularity of a straight line ℓ and a plane π: $\pi \perp \ell \Leftrightarrow \bar{n} \| \bar{s}$	$\pi \perp \ell \Leftrightarrow \frac{A}{m} = \frac{B}{n} = \frac{C}{p}$
17	Angle φ between a straight line ℓ and a plane π: $\sin(\pi \wedge \ell) = \lvert\cos(\bar{n}, \wedge \bar{s})\rvert$	$\sin(\pi \wedge \ell) = = \left\lvert \frac{Am + Bn + Cp}{\sqrt{A^2 + B^2 + C^2}\sqrt{m^2 + n^2 + p^2}} \right\rvert$

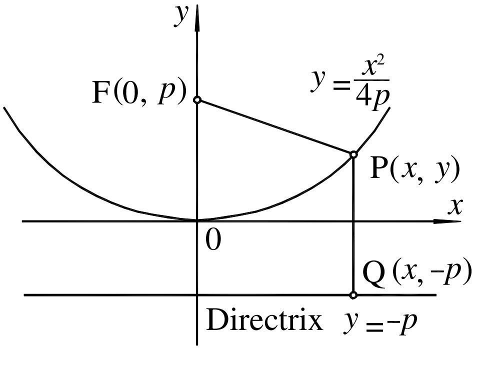
A straight line in a plane		
The first group of equations of a straight line (with normal vector $\overline{n}=(A,B)$)		
18	The equation of a straight line through the point $P_0(x_0;y_0)$ with the normal vector $\overline{n}=(A;B)$.	$A(x-x_0)+B(y-y_0)=0$
19	A general equation of a straight line	$Ax+By+C=0$
20	A distance from a point $P_0(x_0;y_0)$ to a straight line $Ax+By+C=0$	$d=\dfrac{\lvert Ax_0+By_0+C\rvert}{\sqrt{A^2+B^2}}$
21	Intercept form of the equation of a straight line	$\dfrac{x}{a}+\dfrac{y}{b}=1$
The second group of equations of a straight line (with direction vector $\overline{s}=(m,n)$)		
22	A vector equation	$\overline{r}=\overline{r_0}+t\overline{s}$,
23	A parametric equations	$\begin{cases} x=x_0+mt \\ y=y_0+nt \end{cases}$
24	A canonical equation	$\dfrac{x-x_0}{m}=\dfrac{y-y_0}{n}$
25	A two-points equation	$\dfrac{x-x_1}{x_2-x_1}=\dfrac{y-y_1}{y_2-y_1}$
The third group of equations of a straight line in a plane: the straight line with a slope ($k=\tan\varphi$, $\varphi=\ell\wedge Ox$)		
26	the point – slope equation of a straight line	$y-y_0=k(x-x_0)$
27	the slope-intercept equation of a straight line	$y=kx+b$

Angular relations between straight lines ℓ_1 and ℓ_2		
28	Conditions of parallelism of straight lines ℓ_1 and ℓ_2	$\ell_1 \,\|\, \ell_2 \Leftrightarrow \begin{cases} \dfrac{A_1}{A_2} = \dfrac{B_1}{B_2} \\ \dfrac{m_1}{m_2} = \dfrac{n_1}{n_2} \\ k_1 = k_2 \end{cases}$
29	Conditions of the perpendicularity of lines ℓ_1 and ℓ_2	$\ell_1 \perp \ell_2 \Leftrightarrow \begin{cases} A_1A_2 + B_1B_2 = 0 \\ m_1m_2 + n_1n_2 = 0 \\ k_1 = -\dfrac{1}{k_2} \end{cases}$
30	Finding an angle θ between straight lines ℓ_1 and ℓ_2: $\begin{cases} \cos\theta = \left\|\cos\left(\overline{n_1} \wedge \overline{n_2}\right)\right\| \\ \cos\theta = \left\|\cos\left(\overline{s_1} \wedge \overline{s_2}\right)\right\| \\ \tan\theta = \left\|\dfrac{\tan\varphi_1 - \tan\varphi_2}{1 + \tan\varphi_1 \tan\varphi_2}\right\| \end{cases}$	$\begin{cases} \cos\theta = \left\|\dfrac{A_1A_2 + B_1B_2}{\sqrt{A_1^2 + B_1^2}\sqrt{A_2^2 + B_2^2}}\right\| \\ \cos\theta = \left\|\dfrac{m_1m_2 + n_1n_2}{\sqrt{m_1^2 + n_1^2}\sqrt{m_2^2 + n_2^2}}\right\| \\ \tan\theta = \left\|\dfrac{k_1 - k_2}{1 + k_1k_2}\right\| \end{cases}$

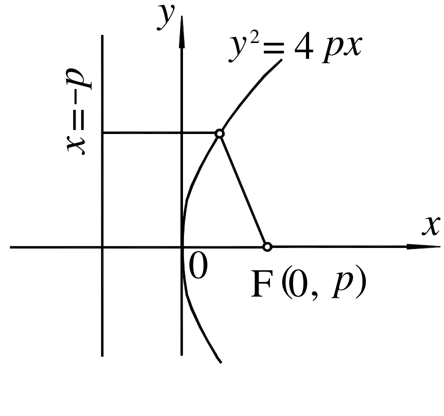
SECOND ORDER CURVES

Parabolas

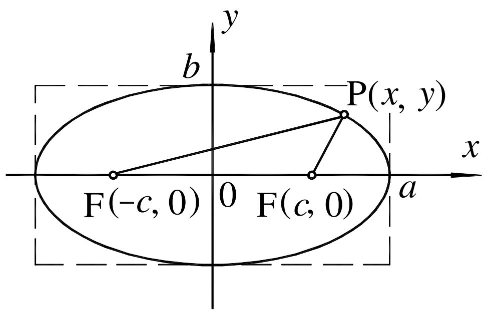
$x^2 = 4py$ $\qquad\qquad$ $y^2 = 4px$



Elli pse:



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

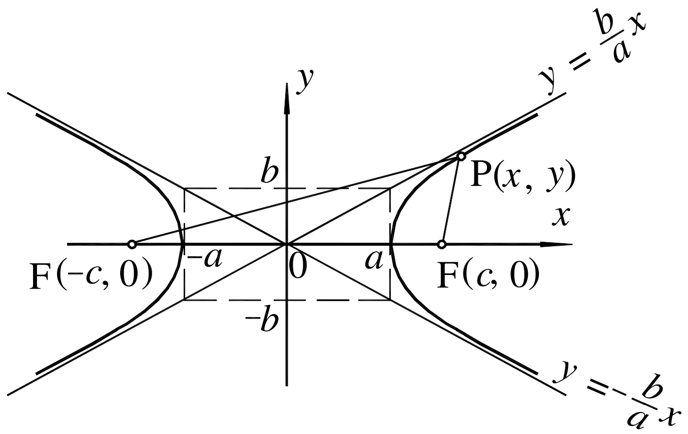


foci $F_1(-c,0)$ and $F_2(c,0)$, where

$c=\sqrt{a^2-b^2}$

eccentricity: $e=\frac{\sqrt{a^2-b^2}}{a}$

Hyperbola: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$



Foci $F_1(-c,0)$ and $F_2(c,0)$, where

$c=\sqrt{a^2+b^2}$

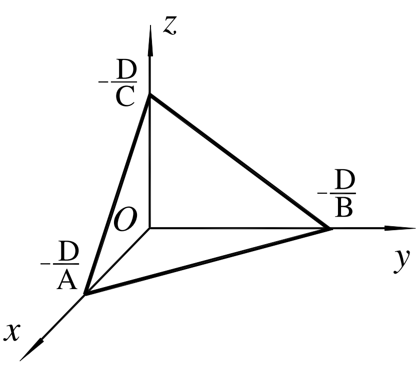
Eccentricity: $e=\frac{c}{a}=\frac{\sqrt{a^2+b^2}}{a}$

Asymptotes $y=\pm\frac{b}{a}x$

1. Planes

1) A General Equation of a Plane

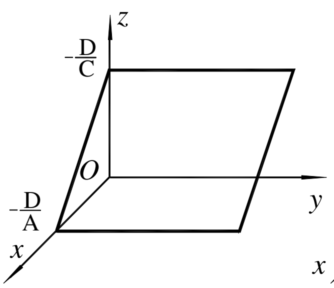
Ax + By + Cz + D = 0

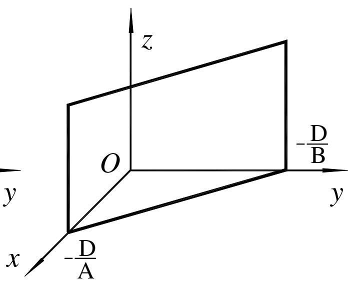


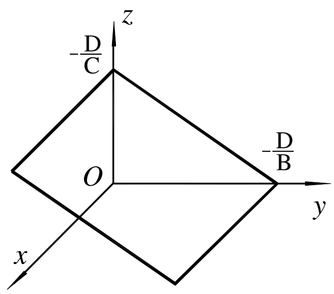
2) Planes which are Parallel to One of the Coordinat Axis

a) $\pi_1 \| Oy$ b) $\pi_2 \| Oz$ c) $\pi_3 \| Ox$

$\pi_1 : Ax + Cz + D = 0$ $\pi_2 : Ax + By + D = 0$ $\pi_3 : By + Cz + D = 0$



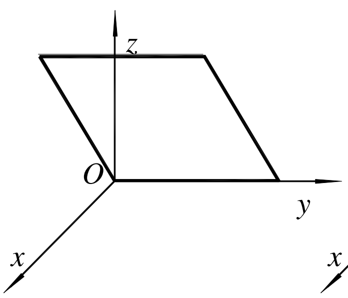


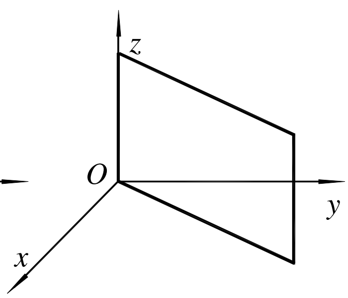


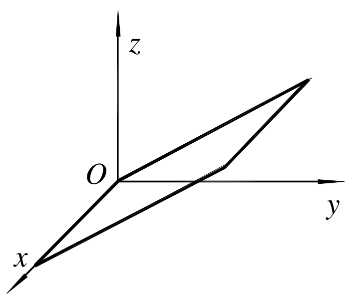
3) Planes which are Passing through One of the Axis of Coordinates

a) $Oy \subset \pi_1$ b) $\pi_2 \supset Oz$ c) $\pi_3 \supset Ox$

$\pi_1 : Ax + Cz = 0$ $\pi_2 : Ax + By = 0$ $\pi_3 : By + Cz = 0$



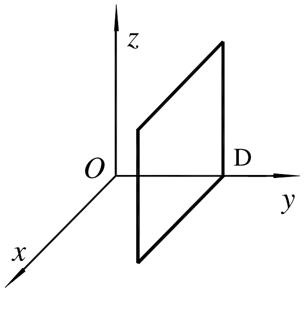


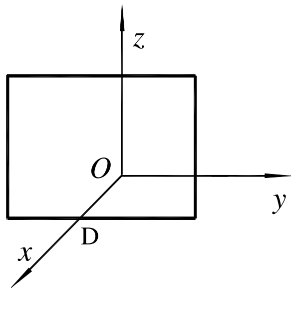


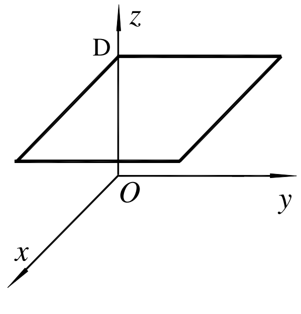
4) Planes which are Parallel to One of the Coordinat Planes

a) $\pi_1 \| xOz$ b) $\pi_2 \| yOz$ c) $\pi_3 \| xOy$

$\pi_1 : y = D$ $\pi_2 : x = D$ $\pi_3 : z = D$







2. Quadric Surfaces

Elliptic Cylynder

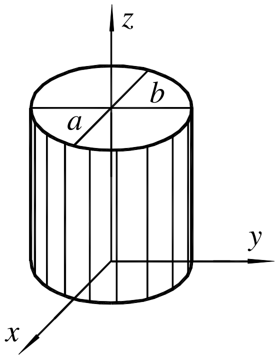
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

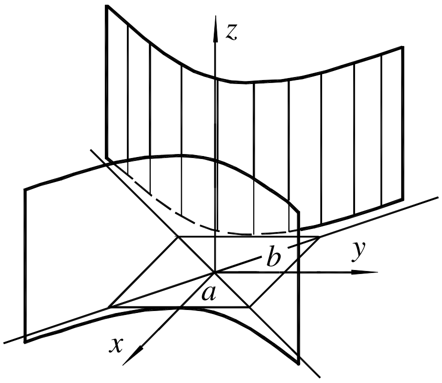
Hyperbolic Cylinder

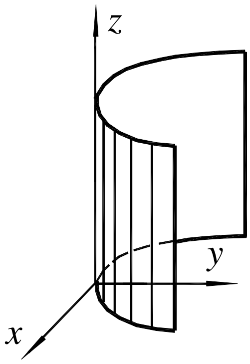
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Parabolic Cylinder

$$x^2=2py$$

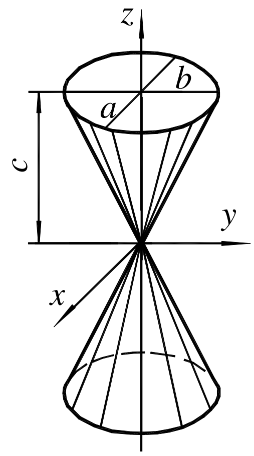






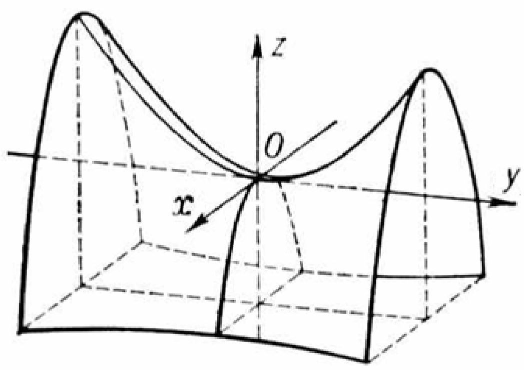
Elliptic Cone

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$



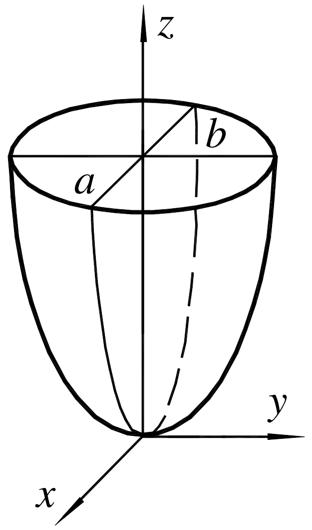
Hyperbolic Paraboloid

$$\frac{x^2}{p}-\frac{y^2}{q}=2z$$



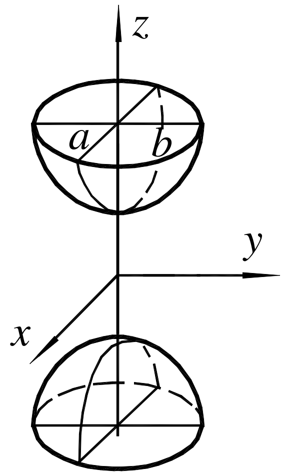
Elliptic Paraboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=z$$



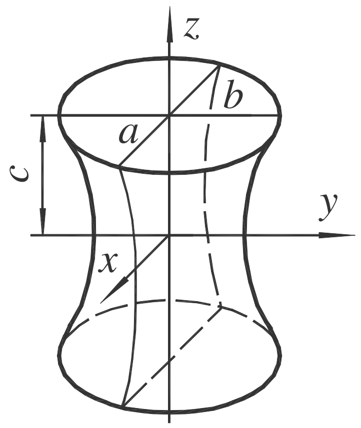
Two-sheeted Hyperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$



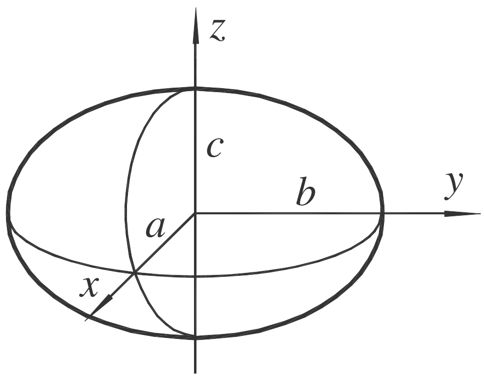
One-sheeted Hyperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$



Ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

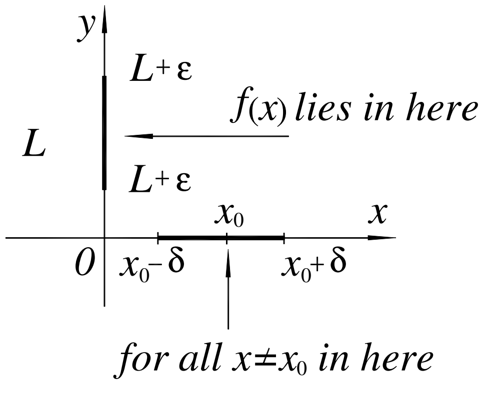


LIMITS

Vocabulary

approach x_0 - достигать x_0
arbitrarily number – произвольное число
comparison of infinitesimals – сравнение бесконечно малых
deleted neighborhood – выколотая окрестность
equivalent infinitesimals – эквивалентные бесконечно малые
however small – как угодно малый
indeterminacy - неопределенность
infinitely large function – бесконечно большая функция
infinitesimal – бесконечно малая
infinitesimal of higher order – бесконечно малая более высокого порядка
infinitesimal of the same order – бесконечно малые одного порядка
left-hand limit - левосторонний предел
limit of a sequence – предел последовательности
neighborhood – окрестность
one-sided limits – односторонние пределы
reciprocal – обратный по величине
remarkable limit – замечательный предел
right-hand limit - правосторонний предел
strip - полоса
tend to x_0 **-** стремиться к x_0

Definition of a Limit of a Function



$\lim\limits_{x\to x_0} f(x)=L$**:** L is the limit of the function $f(x)$ as x tends to x_0 (approaches x_0)

1. **Remember:** $\lim\limits_{\substack{x\to x_0\\ x_0\in D}} f(x)=f(x_0)$

2. $\lim\limits_{x\to x_0} f(x)=L \Leftrightarrow \forall\varepsilon>0\exists\dot U_\delta(x_0):$
$\forall x\in\dot U_\delta(x_0)\Rightarrow |f(x)-L|<\varepsilon$

where $\dot U_\delta(x_0)$ is deleted neighborhood of a point x_0.

Let $\lim\limits_{x\to x_0} f(x)=L_1$ and $\lim\limits_{x\to x_0} g(x)=L_2$, then		
1	Sum rule and difference rule	$\lim\limits_{x\to x_0}(f(x)\pm g(x))=L_1\pm L_2$
2	Constant multiple rule	$\lim\limits_{x\to x_0}(kf(x))=kL_1$ for any number k
3	Product rule	$\lim\limits_{x\to x_0}(f(x)\cdot g(x))=L_1\cdot L_2$
4	Quotient rule	$\lim\limits_{x\to x_0}\left(\frac{f(x)}{g(x)}\right)=\frac{L_1}{L_2}$ if $L_2\neq 0$
5	Infinitesimal function	Function limit of which equals 0 when $x\to x_0$,
6	Infinitely large function	Function limit of which equals ∞, when $x\to x_0$
7	If $\lim\limits_{x\to x_0}\alpha(x)=0$	then $\lim\limits_{x\to x_0}\frac{1}{\alpha(x)}=\infty$
8	If $\lim\limits_{x\to x_0} f(x)=\infty$	then $\lim\limits_{x\to x_0}\frac{1}{f(x)}=0$
9	Equivalent infinitesimal functions (infinite functions) for $x\to x_0$:	$\lim\limits_{x\to x_0}\frac{\alpha(x)}{\beta(x)}=1\Rightarrow\alpha(x)\sim\beta(x)$, if $x\to x_0$.
10	Ratio of infinitesimal functions (infinite functions), difference of infinite functions, powers with bases tending to 1 and infinitely large exponents are called inderteminacies	$\left[\frac{0}{0}\right]$, $\left[\frac{\infty}{\infty}\right]$, $[\infty - \infty]$, $\left[1^{\infty}\right]$
11	The first remarkable limit	$\lim\limits_{u\to 0}\frac{\sin u}{u}=\lim\limits_{u\to 0}\frac{u}{\sin u}=1$
12	Equivalent infinitesimal functions	$\sin u\sim\ tgu\sim\ \arcsin u\sim\ arctg\, u\sim\ u$ if $u\to 0$

13	The second remarkable limit	$\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$, where $e \approx 2{,}718$ – irrational. $\lim\limits_{x\to\infty}\left(1+\frac{a}{bx+c}\right)^{kx+m} = e^{ak/b}$
14	Polynomial is equivalent to its leading monomial as $x \to \infty$	$P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \sim a_n x^n$ if $x \to \infty$.
	Some recommends for calculating limits	
	Type of indeterminacy	Recommends
15.	$\lim\limits_{x\to x_0}\frac{P_n(x)}{Q_n(x)} = \left[\frac{0}{0}\right]$	Devide the numarator and the dinominator by $(x - x_0)$
16.	$\lim\limits_{x\to x_0}\frac{f(x)}{g(x)} = \left[\frac{0}{0}\right]$, where $f(x)$, $g(x)$- are irrational	To carry irrationality from one part of the fraction to another knowing that • $(\sqrt{a}-\sqrt{b})\cdot(\sqrt{a}+\sqrt{b}) = a-b$ • $(\sqrt[3]{a}\pm\sqrt[3]{b})\cdot(\sqrt[3]{a^2}\mp\sqrt[3]{ab}+\sqrt[3]{b^2}) =$ $= a \pm b$
17.	$\lim\limits_{x\to x_0}\frac{f(x)}{g(x)} = \left[\frac{0}{0}\right]$, where $f(x)$, $g(x)$ contain trigonometric or inverse trigonometric functions	To transform $\frac{f(x)}{g(x)}$ in such a way to use (12)
18	$\lim\limits_{x\to\infty}\frac{f(x)}{g(x)} = \left[\frac{\infty}{\infty}\right]$	Use (14)
19	$\lim\limits_{x\to\infty}(f(x))^{g(x)} = [1^\infty]$	Transform $(f(x))^{g(x)}$ in such a way to use (13)

- Any monotone and bounded sequence has a limit as $n \to +\infty$
- A sum of any finite number of infinitesimal functions is an infinitesimal
- The product of a bounded function and an infinitesimal is an infinitesimal
- A product of any finite number of infinitesimal functions is an infinitesimal

DIFFERENTIAL CALCULUS

Vocabulary

acceleration – ускорение
angular velocity – угловая скорость
concavity – вогнутый
convexity – выпуклый
critical (stationary) point – критическая (стационарная) точка
decrease – убывать
decreasing – убывающий
derivative - производная
derivatives of y **with respect to** x – производная y по x
differentiable - дифференцируемый
differential - дифференциал
differentiate - дифференцировать
directional derivative – производная по направлению
exstremum – экстремум
force of current – сила тока
function of several variables – функция многих переменных
gradient – градиент
inclined asymptote – наклонная асимптота
increase – возрастать
increasing – возрастающий
increment of a function – приращение функции
increment of an argument – приращение аргумента
inflection point – точка перегиба
kinetic energy – кинетическая энергия
low of motion – закон движения
maximum – максимум
minimum – минимум
normal – нормаль
partial derivative – частная производная
rectilinear motion – прямолинейное движение
second derivative – производная второго порядка
tangent line – касательная
tangent plane – касательная плоскость
total differential – полный дифференциал
velocity – скорость
vertical asymptote – вертикальная асимптота

Table of Derivatives

I

$$\begin{cases} C' = 0 \\ x' = 1 \\ (u \pm v)' = u' \pm v' \\ (uv)' = u'v + uv' \\ (Cu)' = Cu' \\ \left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2} \\ \left(\dfrac{C}{v}\right)' = -\dfrac{Cv'}{v^2} \end{cases}$$

II

$$\begin{cases} \left(u^a\right)' = au^{a-1}u' \\ \left(\sqrt{u}\right)' = \dfrac{1}{2\sqrt{u}}u' \\ \left(\sqrt[n]{u}\right)' = \dfrac{1}{n \cdot \sqrt[n]{u^{n-1}}}u' \end{cases}$$

III

$$\begin{cases} \left(a^u\right)' = a^u u' \ln a \\ \left(e^u\right)' = e^u u' \\ \left(u^v\right)' = u^v v' \ln u + vu^{v-1}u' \end{cases}$$

IY

$$\begin{cases} (\ell og_a u)' = \dfrac{u'}{u \ell n a} \\ (\ell n u)' = \dfrac{u'}{u} \end{cases}$$

YI

$$\begin{cases} (\arcsin u)' = \dfrac{u'}{\sqrt{1-u^2}} \\ (\arccos u)' = -\dfrac{u'}{\sqrt{1-u^2}} \\ (\arctan u)' = \dfrac{u'}{1+u^2} \\ (arc\cot u)' = -\dfrac{u'}{1+u^2} \end{cases}$$

Y

$$\begin{cases} (\sin u)' = \cos u \cdot u' \\ (\cos u)' = -\sin u \cdot u' \\ (\tan u)' = \dfrac{u'}{\cos^2 u} \\ (\cot u)' = -\dfrac{u'}{\sin^2 u} \end{cases}$$

YII*

$$\begin{cases} (\sinh u)' = \cosh u \cdot u' \\ (\cosh u)' = \sinh u \cdot u' \\ (\tanh u)' = \dfrac{u'}{\cosh^2 u} \end{cases}$$

where C, e, a, α, n – are constants, $u = u(x), v = v(x)$ – are functions.

№	Name	Formula, equation
1	Increment of an argument Δx	$\Delta x = x - x_0$
2	Increment of a function $y = f(x)$: Δy	$\Delta y = f(x) - f(x_0)$
3	Definition of derivative of a function $y(x)$ at a point x_0	$y' = \lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$
Using of a derivative		
4	Equation of a tangent line at a point $M_0(x_0, y_0)$ belonging to the graph of a function $y = f(x)$	$y = f'(x_0) \cdot (x - x_0) + y_0$
5	Equation of a normal at $x = x_0$	$y = -\frac{1}{j'(x_0)} \cdot (x - x_0) + y_0$
6	L'Hospitale's Rule	$\ell im_{x \to x_0} \frac{f(x)}{g(x)} = \left[\frac{0}{0}, \frac{\infty}{\infty}\right] = \ell im_{x \to x_0} \frac{f'(x)}{g'(x)}$
7	$x_0 \in D(y)$ is called critical point	$f'(x_0) = 0$, or does not exist
8	$y = f(x)$ increases if	$f'(x) > 0$
9	$y = f(x)$ decreases if	$f'(x) < 0$
10	Critical point x_0 is the point of maximum of a function $y = f(x)$ if	1). $f'(x)$ changes its sign from "+"to "– " at x_0 2). $f''(x_0) < 0$
11	Critical point x_0 is the point of minimum of a function $y = f(x)$ if	1). $f'(x)$ changes its sign from "– " to "+" at x_0 2). $f''(x_0) > 0$
12	x_0 is the inflection point of a function $y = f(x)$ if	$f''(x)$ changes its sign at a point x_0
13	Inflection points can be only points where the function exists and its second derivative is zero or does not exist.	
14	Inflection points split a domain of a function into intervals of concavity or of convexity.	
15	Arc of the curve $y = f(x)$ is concave if	$f''(x) > 0$
16	Arc of the curve $y = f(x)$ is convex if	$f''(x) < 0$

17	A straight line *Γ* is called an asymptote of a curve *L* if the distance between the moving point of the curve line L and the line Γ tends to zero as the distance between this point and the origin increases infinitely	
18	Equation of vertical asymptote of the curve $y = f(x)$	$x = x_0$, where x_0 is such that $\lim\limits_{x \to x_0} f(x) = \pm\infty$
19	Equations of inclined asymptote of the curve $y = f(x)$	$y = kx + b$, where $k = \lim\limits_{x \to \pm\infty} \frac{f(x)}{x}$, and $b = \lim\limits_{x \to \pm\infty} (f(x) - kx)$
20	Investigation of functions and constructing their graphs	1. Find $D(y), E(y)$, asymptotes, roots, intervals of constant sign. Define if the function is a) periodic or not, b) even or odd. 2. Find critical points, intervals of increasing and decreasing of the function, extremums. 3. Find inflection points, intervals of concavity and convexity.
21	Differential of a function $y = y(x)$	$dy = y' dx$
22	Differentiation of a function giving in parametric form: $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$	$\frac{dy}{dx} = \frac{y'_t}{x'_t}$
Functions of Several Variables		
Let a function $u = f(x, y, z)$ be given		
23	Partial derivatives of the first order	$\frac{\partial u}{\partial x} = f'_x \Big\|_{\substack{y=C \\ z=C}}$; $\frac{\partial u}{\partial y} = f'_y \Big\|_{\substack{x=C \\ z=C}}$; $\frac{\partial u}{\partial z} = f'_z \Big\|_{\substack{x=C \\ y=C}}$

24	A total differential of $u = f(x, y, z)$ at a point $P_0(x_0; y_0; z_0)$	$du = \frac{\partial u(P_0)}{\partial x}dx + \frac{\partial u(P_0)}{\partial y}dy + \frac{\partial u(P_0)}{\partial z}dz$
25	Gradient of $u = f(x, y, z)$ at a point $P_0(x_0; y_0; z_0)$	$\overline{gradu}(P_0) = \left(\frac{\partial u(P_0)}{\partial x}; \frac{\partial u(P_0)}{\partial y}; \frac{\partial u(P_0)}{\partial z}\right)$
26	Directional derivative in the direction of the vector $\bar{a} = (a_x; a_y; a_z)$ $\bar{a}^0 = (\cos\alpha; \cos\beta; \cos\gamma)$, where $\cos\alpha = \frac{a_x}{\lvert\bar{a}\rvert} = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$, $\cos\beta = \frac{a_y}{\lvert\bar{a}\rvert} = \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$, $\cos\gamma = \frac{a_z}{\lvert\bar{a}\rvert} = \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$	$\frac{\partial u(P_0)}{\partial \bar{a}} = \left(\overline{gradu}(P_0), \bar{a}^0\right) = \frac{\partial u(P_o)}{\partial x}\cos\alpha + \frac{\partial u(P_o)}{\partial y}\cos\beta + \frac{\partial u(P_o)}{\partial z}\cos\gamma$
Let a function $z = f(x, y)$ be given		
27	Partial derivatives of the second order of a function $z = f(x, y,)$	• $\frac{\partial^2 z}{\partial x^2} = \frac{\partial\left(\frac{\partial z}{\partial x}\right)}{\partial x}$ • $\frac{\partial^2 z}{\partial y^2} = \frac{\partial\left(\frac{\partial z}{\partial y}\right)}{\partial y}$ • $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial\left(\frac{\partial z}{\partial x}\right)}{\partial y}$ • $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial\left(\frac{\partial z}{\partial y}\right)}{\partial x}$

28	Equation of a tangent plane to a surface $z = f(x, y)$ at a point $P_0(x_0; y_0; z_0)$	$z - z_0 = f'_x(P)(x - x_0) + f'_y(P_0)(y - y_0)$
29	Equation of a normal to a surface $z = f(x, y)$ at a point $P_0(x_0, y_0, z_0)$	$\dfrac{x - x_0}{f'_x(P_0)} = \dfrac{y - y_0}{f'_y(P_0)} = \dfrac{z - z_0}{-1}$
Exstremum of Function of Two Variables $z = f(x, y)$		
30	Stationary point $P_0(x_0, y_0)$:	$\begin{cases} \dfrac{\partial z}{\partial x} = 0 \\ \dfrac{\partial z}{\partial y} = 0 \end{cases}$
31	Let $\dfrac{\partial^2 z(P_0)}{\partial x^2} = A$, $\dfrac{\partial^2 z(P_0)}{\partial x \partial y} = B$, and $\dfrac{\partial^2 z(P_0)}{\partial y^2} = C$, and $\Delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = AC - B^2$	
32	Sufficient Conditions of Exstremum at a point $P_0(x_0, y_0)$	1) $\Delta > 0$ then extremum exists, and if a) $A > 0$ $(B > 0)$ it is **minimum**, b) $A < 0$ $(B < 0)$ it is **maximum**, 2) $\Delta < 0$ extremum doesn't exist, 3) $\Delta = 0$ we can say nothing.

RUSSIAN - ENGLISH VOCABULARY

А

абсолютная величина – absolute value

алгебраическое дополнение элемента a_{ij} – cofactor of an element a_{ij}

арккосинус – anticosine, arccosine

арккотангенс – anticotangent, arccotangent

арксинус – antisine, arcsine

арктангенс – antitangent, arctangent

асимптота – asymptote

астроида – Astroid

Б

большая ось – major axis

большая полуось – semimajor axis

больше или равно – greater than or equal to

В

вектор – vector

векторное произведение – vector product (cross product)
векторное уравнение – vector equation
вершина параболы – vertex of the parabola
взаимно-однозначная функция – one-to-one function
возрастающий – to be increasing
вычислять – evaluate
вычитаемое – subtrahend
вычитание – subtraction
вычитание матриц –subtraction of matrices
вычитать – subtract
Г
геометрическая интерпритация – geometrical representation
гипербола – hyperbola
гиперболические функции – hyperbolic Functions
гиперболический косинус – hyperbolic cosine
гиперболический котангенс – hyperbolic cotangent
гиперболический параболоид – hyperbolic paraboloid
гиперболический синус – hyperbolic sin
гиперболический тангенс – hyperbolic tangent
гиперболический цилиндр – hyperbolic cylinder
главная диагональ – principal (main) diagonal
главный аргумент – principal argument
график функции – graph of a function
Д
дважды – twice
двуполостный гиперболоид – two-sheeted hyperboloid
действительная ось – real axis
действительная часть – real part
действительные числа – real number
деление – division
деление в столбик – long division
делимое – dividend
делимость – divisibility
делитель – divisor
делить – divide (by)
делиться на – be divisible by
десятичная дробь – decimal
десятичный логарифм $(\log_{10} x)$ – common logarithm.
диагональная матрица – diagonal matrix
директриса – directrix
дискриминант – discriminant
для того, чтобы (найти) – to (find)
дописать, добавить – supplement
дробь – fraction

Е
единичная матрица – unit-matrix
единичный вектор – ort of a vector (unit vector)
единственное решение – unique solution
З
заглавная буква – capital letter
закрытый интервал – closed interval
знаменатель – denominator
И
известный – known
интервал – interval
иррациональные числа – irrational numbers
К
канонические уравнения – canonical equations
кардиоида – cardioid
квадратичная функция – quadratic function
квадратная матрица – square matrix
коллинеарный – collinear
компланарный – coplanar
комплексное число – complex number
конечная десятичная дробь – terminating decimal
конечная точка – terminal (end) point
корень или ноль функции – root or a zero of a function
корень энной степени – nth root of
корень, радикал – radical
косинусоида – cosine curve
котангенсоида – cotangent curve
коэффициент при старшей степени – leading coefficient
кривая Гаусса – curve of Gauss
кривые второго порядка – second order curves
кубическая парабола – cube parabola
Л
левая тройка – left-handed triple
лемниската Бернулли – lemniscate of Bernoulli
линейная функция – linear function
лист Декарта – Folium of Descartes
логарифмическая функция – logarithm function
локон Аньези – witch of Agnesi
М
максимум – maximum
матрица (матрицы) – matrix (matrices)
матрица порядка *n* – matrix of order n
меньше или равно – less than or equal to
минимум – minimum

минор – minor
мнимая единица – imaginary unit
мнимая ось – imaginary axis
мнимая часть – imaginary part
многочлен – polynomial
множитель – factor
момент (силы) – torque
Н
наблюдается – is observed
наибольший общий делитель – Greatest Common Factor
наименьшее общее кратное – Least Common Multiple
наименьший общий знаменатель – the least common denominator
направленный – directed
направляющие косинусы – direction cosines
направляющий вектор – position vector
натуральные числа – counting numbers
натуральный логарифм $(\ln x)$ – natural logarithm
начальная точка – initial point
не определено – be undefined
невырожденный – nonsingular
неизвестный – unknown
неправильная дробь – improper fraction
нечетная функция – odd function
нормальный вектор normal vector
О
область значений – range
область определения – domain
обратная матрица – inverse of matrix
обратная функция – inverse function
обратные тригонометрические функции – inverse trigonometric functions
общее уравнение – general equation
общий случай – general case
объединение – union
объем параллелепипеда – volume of the parallelepiped
один раз – once
однополостный гиперболоид – one-cheeted hyperboloid
однородная система уравнений – homogeneous system of equations
одночлен – monomial
определитель – determinant
ортогональный – orthogonal
основание – base
остаток – remainder
острый – acute
ось параболы – axis of the parabola
открытый интервал – open interval

отрицательный – negative

П

парабола – parabola
параболический цилиндр – parabolic cylinder
параметрические уравнения – standard parametric equations
перевести неправильную дробь в смешанное число – to change an improper fraction to the mixed number
перевести смешанное число в неправильную дробь – to change a mixed number to the improper fraction
переодическая функция – periodic function
пересечение – intersection
пересечение с осью *ОУ* – *y*-intercept point
периодическая дробь – repeating decimal
петля – loop
по часовой стрелке – clockwise
поверхность второго порядка – quadric surface
поворот – rotation
подкоренное выражение – radicand
подмножество – subset
показатель степени – exponent
показательная форма – exponential form
показательная функция – exponential function
полином – polynomial
положительный – positive
правая тройка – right-handed triple
правило Крамера – Cramer's Rule
правильная дробь – proper fraction
преобразование *Z* через *X* – transformation *Z* through *X*
прибавлять – add
привести подобные члены – combine like terms
признак коллинеарности векторов – test of collinearity of vectors
признак компланарности векторов – test of coplanarity of vectors
признак ортогональности векторов – test of orthogonality of vectors
прилежащий – adjacent
пример – example
присоединенная матрица –adjoint matrix
проверить равенство – check the equality
произведение – product
простое число – prime number
против часовой стрелки – counterclockwise
прямая – straight line
пустое множество – empty set

Р

разложение вектора $\bar{a}$ по базису – expansion of the vector $\bar{a}$ through the base
разложение на простые множители – prime factorization

разложение определителя по – the expanding determinant by
размерность – dimension
разность – difference
ранг матрицы – rank of matrix
расстояние – distance
расстояние от точки до – distance from a point to
расширенная матрица – augmented matrix
рациональные числа – rational numbers
розы (*n*-лепестковая роза) – roses (*n*-leafed rose)
С
свободный член – constant term
синусоида – Sinusoid
система n линейных уравнение с *n* неизвестными – system of n linear equation in *n* unknowns
скаляр – scalar
скалярное произведение – scalar product (dot product)
сколько – how much
слагаемое – summand
сложение – addition
сложение матриц – addition of matrices
сложная функция – composite function
смежный – adjacent
смешанное произведение – triple scalar product
смешанное число – mixed number
совместная система – compatible system
соответственно – accordingly
сопряженное комплексное число – conjugate complex number
составное число – composite
спираль – spiral степенная функция – power function
столбец – column
строка – row
строчная буква – lowercase letter (small letter)
сумма – sum
Т
тангенсоида – tangent curve
тетраэдр – tetrahedron
точка, делящая отрезок пополам – middle point of a segment
транспонированная матрица – transposed matrix
треугольная матрица – triangular matrix
треугольник – triangle
трижды – three times
тупой – obtuse
У
убывающий – to be decreasing
увеличивать – increase

угловой коэффициент – slope
угловые соотношения – angular relations
улитки Паскаля – Limacons
уменьшаемое – minuend
уменьшать – decrease
умножать – multiply (by)
умножение – multiplication
умножение матриц – multiplication of matrices
умножение матрицы на число – multiplying of a matrix by a number
упорядоченная тройка – ordered triple
уравнение плоскости в отрезках – intercept form of the equation of a plane
уравнение прямой через две точки – two-points equation of a straight line
уравнение с угловым коэффициентом – slope equation
Ф
фокальные оси эллипса – ellipse's focal axis
функция – function
Ц
целые корни комплексных чисел – integral roots of complex numbers
целые степени комплексных чисел – integral powers and roots of complex
целые числа – integers (whole numbers)
циклоида – cycloid
Ч
частное (от деления) – quotient
четверть – quadrant
четная функция – even function
числитель – numerator
Э
эвольвента окружности – evolvent of circle
экспонента $\left(e^{x}\right)$ – natural exponential function
эксцентриситет – eccentricity
элемент – element
элементарные операции над матрицами – elementary operations on matrices
эллипс – ellipse
эллипсоид – ellipsoid
эллиптический конус – elliptic cone
эллиптический параболоид – elliptic paraboloid
эллиптический цилиндр – elliptic cylinder
энная степень числа – the nth power of a number