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# THEORY OF NOISE IMMUNITY
# OF TELECOMMUNICATION SIGNALS RECEPTION

## Module №3

**Education manual**
**on telecommunication theory**

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This education manual contains main theoretical positions of Telecommunication Theory, chapter «Theory of noise immunity of telecommunication signals reception», questions and tasks for examination of knowledge, methodical instructions and input data for course work, methodical guidelines for fulfilling laboratory works, short English-Russian and Russian-English dictionaries.

The manual is intended for students training on the direction 050903 – Telecommunications studying the Module 3 of Telecommunication Theory

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# CONTENTS

# 1 GENERAL INFORMATION ABOUT THE DEMODULATION OF DIGITAL MODULATION SIGNALS

A transmission system is considered (Fig. 1). The input transmission system receives a baseband signal $b(t)$. The modulator converts the baseband signal $b(t)$ into the modulated signal $s(t)$ for transmission over a distance through the transmission line. Transmission line is undistorted. The demodulator input receives the sum of signal $s(t)$ and noise $n(t)$: $z(t) = s(t) + n(t)$. It is believed, unless expressly agreed that the noise $n(t)$ is AWGN (more precisely, the white noise in the passband of the transmission line). On signal $z(t)$ demodulator have to restore the baseband signal $\hat{b}(t)$, the least different from the transmitted signal $b(t)$. Demodulator must take into account all characteristics of signal $s(t)$ and noise $n(t)$ at demodulation.

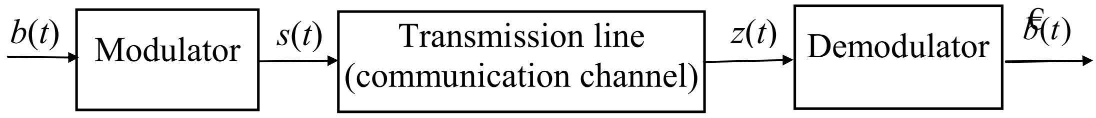



**Figure 1** – Transmission system

Depending on the nature of the baseband signal (digital or analog) transmission system is divided into digital and analog. In these systems, different modulation and demodulation are performed; quantitatively accuracy of the baseband signal recovery is estimated in different ways. Properties and signals formation of analog and digital modulation types are considered in the module 1. This module examines the problems arising in the demodulation of signals:

– algorithms (schemes) of signals demodulation;
– noise immunity of transmission systems – the ability of transmission systems to resist the action of noise.

In digital transmission systems signal $s(t)$ is a sequence determined channel symbols, which reflect the baseband digital signal, and following through clock interval $T$:

$$s(t) = \sum_{k=-\infty}^{\infty} s_i^{(k)}\left(t - kT\right), \tag{1}$$

where $s_i(t)$, $i = 0, \ldots, M - 1$ – channel symbols;
$M = 2^n$ – number of channel symbols;
$n$ – number of bits transmitting by channel symbol;
$T$ – clock interval is time, which the channel symbols passed through;
$s_i^{(k)}\left(t - kT\right)$ – $i$-th channel symbol, transmitting on $k$-th clock interval.

At demodulation the numbers $i$ of channel symbols are unknown. Channel symbols (a form of pulses, their frequency (frequencies), their phase, clock interval, moments of the reference clock intervals) are known. Each channel symbol $s_i(t)$, if it is convenient, designated as channel symbol $s_i$.

Demodulator must make a difference between channel symbols – to make decision $\hat{s}_j$ about numbers of transmitted channel symbols, and gives out a decision bits in accordance with the mapping code.

We assume that the demodulator makes a decision $\hat{s}_j$ on the transmitted channel symbol at every clock interval, independently of the decisions of other clock intervals. This method is called symbol-by-symbol demodulation. At symbol-by-symbol demodulation the work of demodulator can be considered at individual clock intervals, for example, when $k = 0$ demodulated signal has the form

$$z(t) = s_i^{(0)}(t) + n(t). \tag{2}$$

Here, number of channel symbol is unknown, and the demodulator is required to make a decision about number $i$. As on the demodulator input a distorted by noise signal is received, the decision about transmitted channel symbols will contain errors. Channel symbol error is the demodulator decision $\hat{s}_j$ when receiving on the demodulator input symbol is $s_i$ and $j \neq i$. Errors appears randomly as the random nature of noise. Wrong decision by the demodulator about channel symbol reduces to one or more incorrect bits (depending on the mapping code) on the demodulator output. Bit error probability $p$ is a quantitative measure of noise immunity of digital transmission systems. Bit error probability $p$, along with the signal rate $R$ is a basic characteristic of digital transmission system.

## 2 CRITERION OF OPTIMALITY OF DIGITAL MODULATION SIGNALS DEMODULATORS. DECISION RULES

You can offer a lot of number of algorithms for demodulation of the signal (2). Any demodulation algorithms must take into account all or part of the initial data were: description of the channel symbols $s_i(t)$, $i = 0, ..., M - 1$; a prior probabilities of channel symbols $P(s_i)$, $i = 0, \ldots, M - 1$; statistical characteristics of noise $n(t)$, in particular the probability density $p(n)$.

Our task is to find the optimal demodulation algorithms, assuming that the criterion of optimality is the minimum total error probability of demodulator decision about channel symbol. To minimize the bit error probability at the demodulator output provided the minimum error probability decision about channel symbol one optimizes the mapping code.

Obviously, the minimum total error probability of demodulator decision about channel symbol is a decision on the maximum a posterior probability of channel symbols $P(s_i/z)$. Rule of maximum a posterior probability is formulated in the following way: demodulator makes a decision about symbol transfer $\hat{s}_i$, if a system of $M - 1$ inequalities is satisfied:

$$P(s_i/z) > P(s_j/z), \quad j = 0, 1, \ldots, M - 1;\ j \neq i. \tag{3}$$

Then use the Bayes formula

$$P(s_i/z) = \frac{P(s_i)p(z/s_i)}{p(z)}, \tag{4}$$

where $p(z)$ is unconditional probability density of the signal $z(t)$;

$p(z/s_i)$ is conditional probability density of the signal $z(t)$ provided that $z(t) = s_i(t) + n(t)$.

This assumes that a prior probabilities of channel symbol $P(s_i)$, $i = 0, \ldots, M-1$ and the probability density of the noise $p(n)$ allow us to calculate the conditional probabilities density $p(z/s_i)$. Typically, channel symbols are equally probable in transmission systems, i.e.

$$P(s_i) = 1/M, \quad i = 0, \ldots, M-1. \tag{5}$$

With this in mind we rewrite the system of inequalities (3) by deleting the entries unconditional probability density $p(z)$, that is included in the right and left sides, and not dependent on the index:

$$p(z/s_i) > p(z/s_j), \quad j = 0, 1, \ldots, M-1; \; j \neq i. \tag{6}$$

The system of inequalities (6) is called usually the maximum likelihood rule, which is formulated in the following way: demodulator makes a decision about transfer symbol $\hat{s}_i$, if conditional probability density of the signal $z(t)$ provided that $z(t) = s_i(t) + n(t)$ is maximal. Conditional probability density is called also the likelihood function.

Note that the maximum likelihood rule is used for demodulators' construction, if the channel symbols are equiprobable (in this case is realized generally maximum a posterior probability) or a prior probabilities of channel symbols are unknown.

To move from decision rules (6) to the demodulation algorithm it is necessary to use representation of signals and noise in a multidimensional space. Full description of the set of channel symbols $s_i(t)$, $i = 0, \ldots, M-1$ is their representation in the $N$-dimensional space (see Module 1). They considered one-dimensional ($N$ = 1) and two-dimensional ($N$ = 2) signals as the most frequently used. For clarity of the description the signal constellations are used.

Realization of input demodulator sum of signal and noise $z(t)$ – the relation (2) – can be also represented in a multidimensional space formed by the orthonormal basis functions $\{\psi_k(t)\}$, which are used for the description of channel symbols $s_i(t)$:

$$s_i(t) = \sum_{k=0}^{N-1} a_{ik}\psi_k(t), \; i = 0, 1, \ldots, M-1; \tag{7}$$

$$z(t) = \sum_{k=0}^{N-1} z_k \psi_k(t), \tag{8}$$

where

$$z_k = \int_0^{T_s} z(t)\text{ш}_k(t)dt, \quad k = 0, 1, \ldots, N-1 \; – \tag{9}$$

is decomposition coefficients of demodulated signal. This ratio suggests that the functions $\{\psi_k(t)\}$ form an orthonormal basis with orthogonality interval $(0, T_s)$, where $T_s$ is duration of channel symbols.

The coefficients $z_k$ are uncorrelated Gaussian random values. Therefore, the conditional probability densities $p(z/s_i)$, $i = 0, ..., M-1$ are $N$-dimensional distributions, determined by the product $N$ one-dimensional distributions. The right and left sides of inequalities (6) can be rewritten as

$$\frac{1}{(\sqrt{2\pi}\sigma)^N}\exp\left(-\frac{1}{2\sigma^2}\sum_{k=0}^{N-1}(z_k-a_{ik})^2\right) > \frac{1}{(\sqrt{2\pi}\sigma)^N}\exp\left(-\frac{1}{2\sigma^2}\sum_{k=0}^{N-1}(z_k-a_{jk})^2\right), \quad (10)$$

$$j = 0,\ 1,\ \ldots, M-1;\ j \neq i,$$

where $\sigma$ is a root mean square deviation (RMS) of the coefficients $z_k$.

Verification of inequalities (10) is equivalent to checking the inequalities

$$\sum_{k=0}^{N-1}(z_k-a_{ik})^2 < \sum_{k=0}^{N-1}(z_k-a_{jk})^2, \quad j = 0,\ 1, \ldots, M-1;\ j \neq i\,. \quad (11)$$

Sums in inequality (11) are nothing other than squares of the distances between the demodulated signal and the channel symbols in the $N$-dimensional space. Last relation expressed the maximum likelihood rule for the optimal demodulation: decision about the number of channel symbols shall be in favour of the signal, the distance between that and demodulated signal is minimal. Note that there is no need to take a square root of the left and right sides of inequality (11), the square of the distance can be also compared.

Rule of the demodulation can be interpreted in a following way:

– signal space is divided into $M$ disjoint areas with the names $s_i$, $i = 0, ..., M-1$, each area $s_i$ is a set of points that are closer to the signal $s_i(t)$, rather than to other signals;

– demodulator makes a decision about transfer signal $s_i(t)$, if the point $z(t)$ in $N$-dimensional space falls in the area $s_i$.

*Notes.* 1. The criterion of minimum total error probability of demodulator decision in the Russian-language literature is also called the criterion of the ideal observer (a term introduced by V.A. Kotelnikov).

2. If the error making by demodulator differently undesirable for various channel symbol, they used the criterion of minimum average risk (criterion minimal total error probability of demodulator decision assumes that all errors are equally undesirable).

***Example 1.*** Let's find the decision rule for optimal demodulation of one-dimensional ($N = 1$) BPAM signal, whose channel symbols are described

$$s_i(t) = a_i A(t), \quad i = 0,\ 1, \quad (12)$$

where $A(t)$ is pulse, with set time and spectral characteristics, which maximum value and energy are equal to 1;

$a_i$ is factors that show the transmitted bits: $a_1 = a$, $a_0 = -a$; value $a$ determines the energy of channel symbols.

The function $A(t)$ plays the role of the basis functions $\psi_0(t)$ for the representation of the signal $s_i(t)$ in one-dimensional space.

On the demodulator input on a concrete clock interval (e.g., $k = 0$) comes

$$z(t) = a_i A(t) + n(t). \tag{13}$$

Find the coefficient submission signal $z(t)$ in the basis $A(t)$

$$z_0 = \int_{-\infty}^{\infty} z(t)A(t)dt = a_i + \zeta. \tag{14}$$

where $\zeta$ – random variable with Gaussian probability distribution, since it is the result of linear transformation noise with a Gaussian distribution.

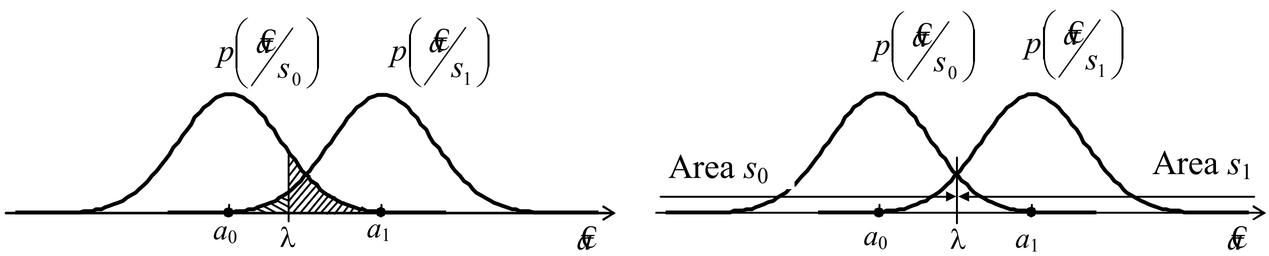
In accordance with expression (11) for the restoration of binary symbol, transfer at this clock interval, you should compare the distances $z_0 - a_1$ and $z_0 - a_0$ and decide in favor of the minimal of these:

if $z_0 - a_1 < z_0 - a_0$, then $\hat{b}_i = 1$;

if $z_0 - a_1 > z_0 - a_0$, then $\hat{b}_i = 0$.

As follows from relation (14) $z_0$ is nothing short of the estimate $\hat{a}$ of the coefficient $a_i$. Let us discuss the decision on a comparison of conditional probabilities densities (6) (rule of maximum likelihood): demodulator makes a decision about the transmitted channel symbol $\hat{s}_1$, if $p(\hat{a}/s_1) > p(\hat{a}/s_0)$, and the decision $\hat{s}_0$, if $p(\hat{a}/s_1) < p(\hat{s}/s_0)$.

In Fig. 2 the BPAM signal constellation and the conditional probability density estimates of the coefficient, which describes demodulated signal are shown. This figure shows that instead of comparing the conditional probability density demodulator can make a decision on the result of comparison of estimates € with threshold value $\lambda$ according to the rule: if $\hat{a} > \lambda$, then a signal $s_1(t)$ was transmitted, and if $\hat{a} < \lambda$, then a signal $s_0(t)$ was transmitted.




*a* *b*

**Figure 2 -** BPAM signal constellation and the conditional probability density estimates: *a* – not optimal setup of threshold; *b* – optimal setup of threshold

The error probability in the transmission $s_1(t)$

$$P_{\text{er}}(s_1) = \int_{-\infty}^{\lambda} p(\hat{a}/s_1)d\hat{a}. \tag{15}$$

Similarly, we define the error probability in the transmission $s_0(t)$

$$P_{\text{er}}(s_0) = \int_{\lambda}^{\infty} p\left(\hat{a}/s_0\right) d\hat{a}. \tag{16}$$

The unconditional error probability of signal and bit

$$p = P(s_1)P_{\text{er}}(s_1) + P(s_0)P_{\text{er}}(s_0) = 0{,}5[P_{\text{er}}(s_1) + P_{\text{er}}(s_0)]. \tag{17}$$

From Fig. 2, *a* can be seen that the probabilities in square brackets equal to the shaded areas. It is easy to see that the total area will be minimal when the threshold is midway between $a_1$ and $a_0$:

$$\lambda = 0{,}5(a_1 + a_0). \tag{18}$$

This value is shown in Fig. 2, *b*. It is seen that $P_{\text{er}}(s_1) = P_{\text{er}}(s_0)$ in this case. It also shows the partition of the signals space (in this example, the number axis) in the signals areas: the range of values $\hat{a}$, where $p(\hat{a}/s_1) > p(\hat{a}/s_0)$, is an area of the symbol $s_1$, and the range of values $\hat{a}$, where $p(\hat{a}/s_1) < p(\hat{a}/s_0)$ is the area of symbol $s_0$.

*Note.* Figure 2 shows that the probability of error depends on the values of RMS $\sigma$ of estimate $\hat{a}$ – the less RMS, the less error probability. It will be shown that the calculation of estimates for the algorithm (14) provides a minimum value of RMS estimates.

## 3 ALGORITHM FOR OPTIMAL DEMODULATION OF DIGITAL MODULATION SIGNALS (GENERAL CASE)

The received above maximum likelihood rule is reflected in the form of *M*-ary signals optimal demodulator scheme (Fig. 3). Individual circuit blocks perform the following functions:

1. Determination of signal $z(t)$ coordinates in the space of channel symbols, on the basis of (9).
2. Determination of the squares of the distances between $z(t)$ and $s_i(t)$ in the space of channel symbols based on the ratio

$$d^2(z, s_i) = \sum_{k=0}^{N-1} (z_k - a_{ik})^2, \quad i = 0,\ 1, \ldots, M-1. \tag{19}$$

3. Comparison of the squares of distances (or distances), identification number $j$, which corresponds to the minimum value of $d^2(z, s_j)$, delivery of solutions $\hat{s}_j$.
4. Presentation channel symbol $s_j$ by bits in accordance with the mapping code.

On the next clock interval mentioned actions are repeated.

The scheme of optimal demodulator (Fig. 3) can be used to demodulate the signal of an arbitrary specified type of modulation – types of modulation have different values *N* and *M*, form of channel symbols. There is one limit. Channel symbols are equally probable, but this restriction is usually performed in practice.

Depending on the modulation type, by considering properties of channel symbols, coordinates of the signal $z(t)$ can calculate in different ways in the space of channel symbols. This diversity of ways of demodulators constructing, which will be devoted a significant place in the following sections. The remaining blocks of the demodulator: calculation of the square distances between $z(t)$ and $s_i(t)$, a decision on the basis of minimum distance value, decoding based on the mapping code are the standard for the demodulator, and thereafter will be merged into one unit which will be called "decision".

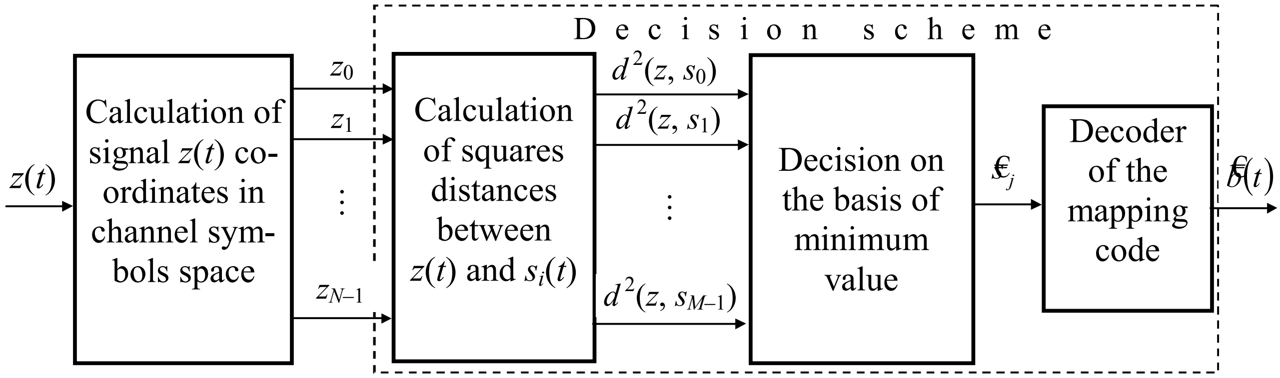



**Figure 3 –** Optimal demodulator of $M$-ary signal

Calculation of signal $z(t)$ coordinates in the space of channel symbols based on the ratio (9) can be satisfied by correlators schemes (Fig. 4). The scheme of the correlator includes a signal generator $\psi_k(t)$ – a replica of the $k$-th basis function of channel symbol, multiplier and integrator with reset. Switch takes sample at the end of the signal $\psi_k(t)$, and the integrator is given in the zero state to be ready for the next signal processing.

The term "correlator" is due to the fact that the scheme calculates the value of cross-correlation function between signals $z(t)$ and $\psi_k(t)$.

The calculation corresponding to the relation (12), can be performed by linear electric circuit with a specially chosen impulse response $g_k(t)$. In the general case, the output signal $y(t)$ and the input signal $z(t)$ linear circuit are connected by the relation, called the Duhamel integral

$$y(t) = \int_{-\infty}^{\infty} z(\tau) g(t-\tau) d\tau, \tag{20}$$

where $g(t)$ is the impulse response of a circuit.

Let

$$g_k(t) = \psi_k(T_s - t). \tag{21}$$

Since the signal $\psi_k(t)$ exists on the interval $(0, T_s)$, then at the same interval there exists a function $\psi_k(T_s - t)$. Therefore, the limits of integration are 0 and $T_s$. Let's find the value $y_k(T_s)$

$$y_k(T_s) = \int_0^{T_s} z(\tau)\psi_k(T_s - T_s + \tau) d\tau = \int_0^{T_s} z(t)\psi_k(t) dt = z_k. \tag{22}$$

Linear electric circuit with the impulse response (21) is called a matched filter with the signal $\psi_k(t)$ (the impulse response is a mirror reflection of a signal).

Thus, the calculation of expansion coefficients of the signal $z(t)$ can be accomplished with the help of correlator or matched filter (Fig. 5). Accordingly, the demodulator scheme will contain $N$ correlators or $N$ matched filter and decision scheme (Fig. 6).

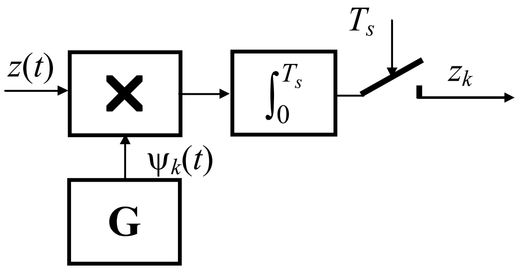



**Figure 4** – Calculation of the coefficient $z_k$ by correlator

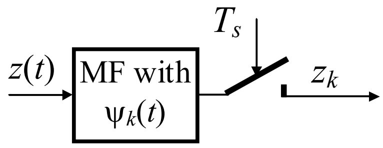



**Figure 5 –** Calculation of the coefficient $z_k$ by matched filter

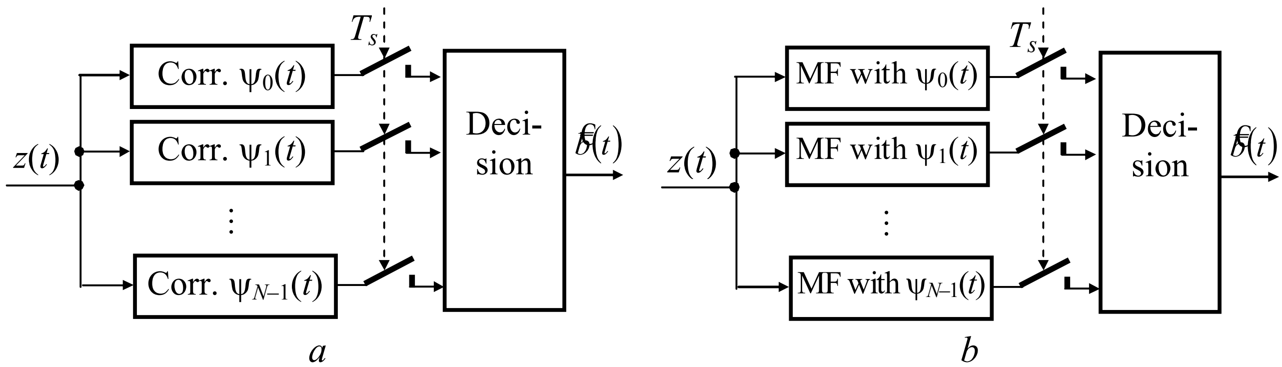



**Figure 6 –** Optimum demodulators:
*a* – on the basis of correlators; *b* – on the basis of matched filters

## 4 MATCHED FILTER

In section 3 the term "matched filter" (MF) was introduced as a device for coefficient calculation representation of demodulated signal in orthonormal basis. MF is more widely used in transmission systems equipment. Therefore we consider below MF with common positions.

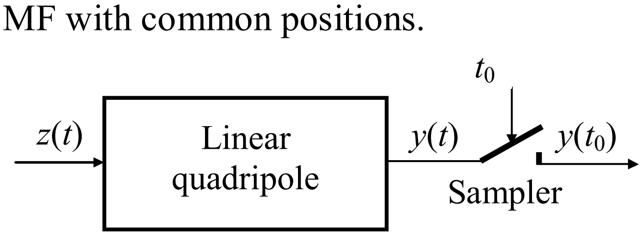



**Figure 7** – Inclusion of quadripole and sampler

There is a linear quadripole (filter) with transfer function $H(j\omega)$. Its input is the sum of deterministic pulse signal $s(t)$ and noise $n(t)$: $z(t) = s(t) + n(t)$. There is a sum of responses to the signal and noise $y(t) = y_s(t) + y_n(t)$ on the quadripole output. Sampler is connected to the quadripole output to sample at the time $t_0$ (Fig. 7). This device is used to reduce noise and to sample in order to determine the maximum value of the pulse.

The filter is called matched with a signal $s(t)$, if at submission on its input of the sum of the signal $s(t)$ and a noise $n(t)$ on its output at the certain moment (designated $t_0$) the maximum ratio of the instantaneous power of a signal $y_s^2(t_0)$ to the average power of noise $P_{n\,\text{out}}$ takes place: $\rho_{\text{peak}} = y_s^2(t_0)/P_{n\,\text{out}}$.

Matched filter is used not only for signal/noise ratio maximization, but for other important signal and noise transformations also. Therefore we will consider properties of MF.

1. Let's find the transfer function $H(j\omega)$. Signal $s(t)$ is specified and the noise $n(t)$ is white noise with power spectral density $N_0/2$.

Let

$$S(j\omega) = \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt \; - \tag{23}$$

spectral density of a signal $s(t)$. Then spectral density of the output signal $y_s(t)$ is defined as

$$S_{\text{out}}(j\omega) = S(j\omega) H(j\omega). \tag{24}$$

Sampling value of signal $y_s(t_0)$ is the inverse Fourier transforms from $S_{\text{out}}(j\omega)$

$$y_s(t_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(j\omega) H(j\omega) e^{j\omega t_0} d\omega. \tag{25}$$

Noise power on the filter output and average square of noise sample $y_n(t_0)$ is defined as

$$P_{n\,\text{out}} = \overline{y_n^2(t_0)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{N_0}{2} |H(j\omega)|^2 d\omega. \tag{26}$$

Signal to noise ratio is a signal power $y_s^2(t_0)$ to average square of noise sample ratio $P_{n\,\text{out}}$ in the sampling moment

$$\rho = \frac{\left(\frac{1}{2\pi}\right)^2 \left[\int_{-\infty}^{\infty} S(j\omega) H(j\omega) e^{j\omega t_0} d\omega\right]^2}{\frac{1}{2\pi} \cdot \frac{N_0}{2} \int_{-\omega}^{\infty} |H(j\omega)|^2 d\omega}. \tag{27}$$

We seek a transfer function $H(j\omega)$, in which there is a maximum value of the numerator in the ratio (27). For the next calculation it is necessary to take advantage of that the integral in the numerator – scalar product of two functions $S^*(j\omega)$ and $H(j\omega)e^{j\omega t_0}$ $S^*(j\omega)$ is complex conjugate function with the function $S(j\omega)$). Scalar product is maximum if these functions coincide with accuracy to any positive coefficient $c$, i.e. $H(j\omega)e^{j\omega t_0} = c \cdot S \cdot (j\omega)$. Hence, the maximum of the numerator (27) takes place when the transfer function

$$H(j\omega) = c \cdot S^*(j\omega) e^{-j\omega t_0}. \tag{28}$$

After substituting expression (28) in relation (27) we obtain

$$\mathrm{c} = \frac{\left(\frac{1}{2\pi}\right)^2 \left[ c^2 \int\limits_{-\infty}^{\infty} |S(j\omega)|^2 d\omega \right]^2}{\frac{c^2}{2\pi} \cdot \frac{N_0}{2} \int\limits_{-\omega}^{\infty} |S(j\omega)|^2 d\omega} = \frac{2E_s}{N_0}. \tag{29}$$

Here it is used, that energy of a signal is defined as

$$E_s = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} |S(j\omega)|^2 d\omega. \tag{30}$$

We see that ratio (28) provides not only a maximum of the numerator of the SNR (27), but the maximum of this ratio (the value of ρ does not depend on the specific form of the transfer function $H(j\omega)$, included in the denominator). In such a way, the problem of determining the transfer function of MF $H(j\omega)$ is solved.

2. Value (29) determines the maximum possible signal to noise ratio at the output of the filter in the sampling moment. This ratio is called peak

$$\mathrm{c}_{\text{peak}} = \frac{2E_s}{N_0}. \tag{31}$$

Let's define the gain in the signal/noise ratio, which shows how many times increases the signal/noise ratio provided with the MF,

$$g_{\text{MF}} = \frac{\mathrm{c}_{\text{peak}}}{P_s / P_n} = \frac{2E_s \cdot P_n}{N_0 \cdot P_s} = \frac{2P_s T_s N_0 F_{\text{n}}}{N_0 \cdot P_s} = 2F_{\text{n}} T_s, \tag{32}$$

where $F_n$ is band of noise on filter input;

$T_s$ is signal $s(t)$ duration;

$P_s$ and $P_n$ are average powers of signal and noise on filter input.

From the expression (32) evidently, that at certain correlations between the noise band and duration of signal gain can take on large values.

3. Let's find amplitude response and phase response of MF. Transfer function of a linear circuit defines AR and PR circuit

$$H(j\omega) = H(\omega) \exp(j\varphi(\omega)), \tag{33}$$

where $H(\omega)$ is AR circuit, $\varphi(\omega)$ is PR circuit.

Let's present the spectral density of a signal through the module and the argument

$$S(j\omega) = S(\omega) \exp(j\psi(\omega)), \tag{34}$$

where $S(\omega)$ is an amplitude spectrum of a signal, $\psi(\omega)$ is a phase spectrum of a signal.

After substituting (33) and (34) in (28) we find that the frequency response of MF

$$H(\omega) = cS(\omega) \tag{35}$$

up to an arbitrary factor coincides with the amplitude of the signal the filter is matched with. MF transmission coefficient is bigger at those frequencies at which signal $s(t)$components is bigger.

Equality arguments of left and right sides of (28) gives

$$\varphi(\omega) = -\psi(\omega) - \omega t_0, \tag{36}$$

that is interpreted as follows: PR of MF up to a linear term is opposite in sign to the phase spectrum signal the filter is matched with.

For finding out of physical essence of MF PR will consider some signal harmonic of frequency $f_i$: $A_i\cos(2\pi f_i + \psi_i)$. This harmonic on the MF output is determined:

$$A_iH(f_i)\cos(2\pi f_i\, t + \psi_i + \varphi(f_i)) = A_iH(f_i)\cos(2\pi f_i\, t + \psi_i - \psi_i - 2\pi f_i\, t_0).$$

The full phase of oscillation is equal $2\pi f_i\,(t - t_0)$. At a moment $t = t_0$ the full phase of oscillation is equal to the zero independently of frequency. At this moment all harmonics are in a phase and at addition give the maximally possible value of response.

4. Let's find the impulse response of MF as the inverse Fourier transform of the transfer function

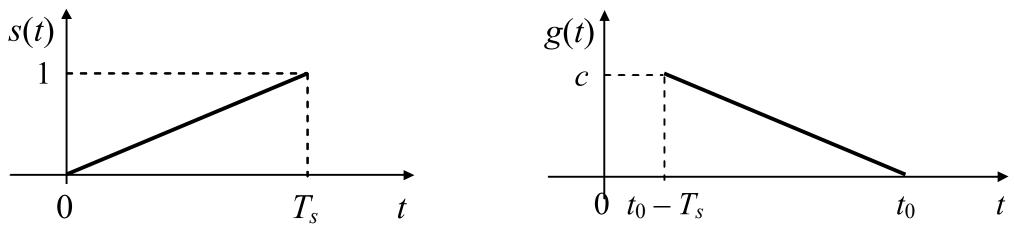
$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)e^{j\omega t}dt = \frac{c}{2\pi}\int_{-\infty}^{\infty} S(-j\omega)e^{-j\omega t_0}e^{j\omega t}dt = \frac{c}{2\pi}\int_{-\infty}^{\infty} S(j\omega)e^{j\omega(t_0 - t)}dt = cs(t_0 - t).\tag{37}$$

We see, that impulse response of matched filter is mirror reflection of a signal which the filter is matched with, about the point $t_0$ in scale $c$.

**Example 2**. We will build the graph of impulse response of filter, matched with a signal

$$s(t) = \begin{cases} 1 - t/T_s, & 0 \le t \le T_s, \\ 0, & t < 0,\ t > T_s. \end{cases}$$

Condition of a physical realization of a circuit is $g\,(t) \equiv 0$ at $t < 0$. The matched filter accumulates all of the components of a signal and in sampling moment forms of them the greatest possible value. It is obvious, that the filter accumulated all components of a signal, sampling moment should not be less than the duration of a signal: $t_0 \ge T_s$, where $T_s$ is a signal $s(t)$ duration.




*a* *b*

**Figure 8** – Graphs: *a* – signal,
*b* – impulse response of filter matched with signal

5. Let on the MF input operates arbitrary signal $z(t)$. The response of the filter is determined by the Duhamel integral

$$y(t)=\int_{-\infty}^{\infty} z(\tau)g(t-\tau)d\tau = c\int_{-\infty}^{\infty} z(\tau)s(t_0-t+\tau)d\tau = cK_{zs}(t-t_0), \tag{38}$$

where $K_{zs}(\tau)$ – cross-correlation function of signals $z(t)$ and $s(t)$.

From the expression (38) follows that the shape of the signal on the MF output is determined by cross-correlation function of the input signal and the signal which the filter is matched with, namely, it repeats the cross-correlation function in the scale $c$ and shifted to the right on $t_0$.

If in the ratio (38) put $c = 1$ and $t_0 = T_s$, it is easy to see that $y(T_s)$ gives the value of the scalar product of signals $z(t)$ and $s(t)$. This property of MF was used to determine the expansion coefficients in the ratio (22).

6. Let on an input of MF be the signal $s(t)$ with which the filter is matched. Then, on foundation (38) will write down

$$y(t) = cK_s(t-t_0), \tag{39}$$

where $K_s(\tau)$ – correlation function of signal $s(t)$.

Thus, if on the MF input we have signal, which filter is matched with, the response of the filter is determined by the correlation function of signal, namely, it repeats the correlation function in the scale $c$ and shifted to the right on $t_0$.

**Exercise 1.** Let's illustrate the properties of MF considered by the example of a filter, matched with a rectangular pulse of amplitude $A$ and duration $T_s$. Let $c = 1/A$ and $t_0 = T_s$. Impulse response of filter, matched with rectangular pulse have rectangular shape, amplitude 1 and duration $T_s$, i.e. impulse response coincides with the signal (Fig. 9, *a*).

The spectral density of the rectangular impulse is defined with the aid of Fourier transform

$$S_{\text{r}}(j\omega) = \int_0^{T_s} A\mathrm{e}^{-j\omega t}dt = \frac{A}{-j\omega}\left[\mathrm{e}^{-j\omega T_s}-1\right] = \frac{2A}{\omega}\sin\frac{\omega T_s}{2}\cdot \mathrm{e}^{-j\omega T_s/2}. \tag{40}$$

According to the ratio (28) we will get the expression for transfer function of filter, matched with rectangular pulse, if $c = 1/A$ and $t_0 = T_s$

$$H(j\omega) = \frac{1}{j\omega}\left[1-\mathrm{e}^{-j\omega T_s}\right]. \tag{41}$$

Hence this relationship follows that the scheme of the filter, matched with the rectangular impulse consists of the integrator (with transfer function $1/j\omega$), delay device for the period of $T_s$ (with transfer function $\exp(-j\omega T_s)$) and the subtracter (Fig. 9, *c*). In this figure the numbers denote the individual points of scheme to discuss its work.

It is easy to receive expression for AR of the filter, matched with the rectangular impulse. Final expression for AR after transition to a variable $f$ looks like function $\sin(x)/x$

$$H_{\mathrm{r}}(f) = T_s \frac{\sin \pi f T_s}{\pi f T_s}. \tag{42}$$

AR of MF and amplitude spectrum of the signal is shown in Fig. 9, *b*.

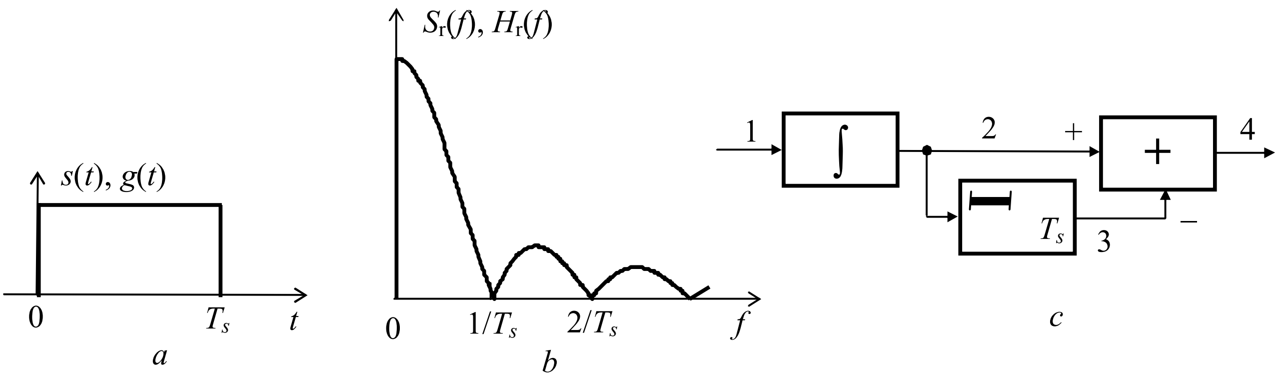



**Figure 9** – Characteristics of the filter matched with rectangular pulse: *a* – impulse response, *b* – AFC, *c* – MF scheme

Figure 10, *a* shows the processes taking place in MF for its input δ-function. There is the impulse response on the output. Fig. 10, *b* shows the process taking place in MF for its input pulse which filter is matched with. On the scheme output is observed a response, which coincides with the correlation function of the rectangular pulse duration $T_s$ (see module 1).

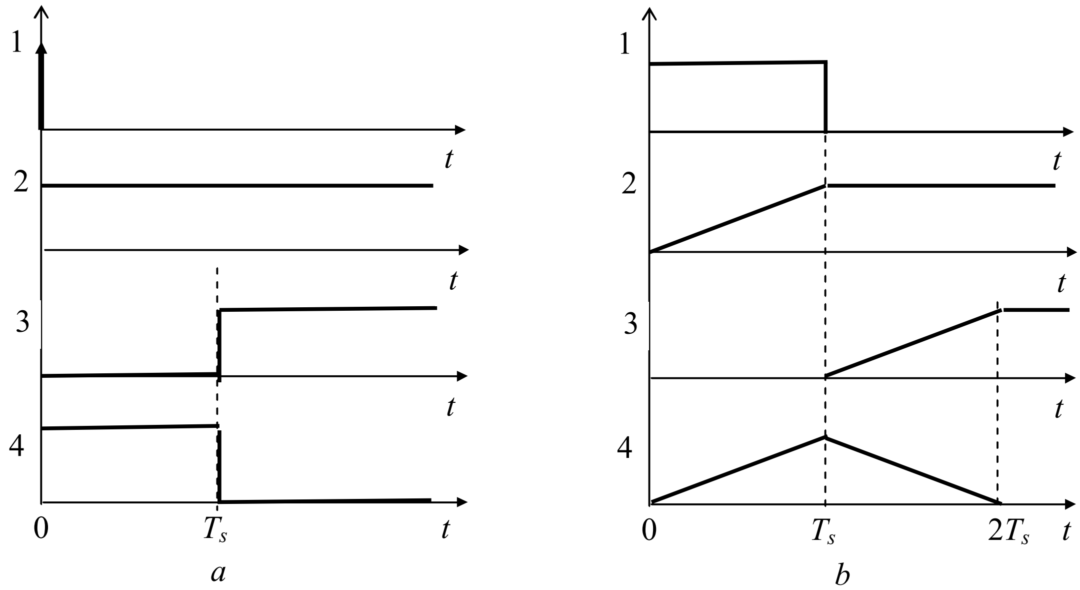



**Figure 10** – Time diagrams, which illustrated work of the filter matched with rectangular pulse: *a* – δ-function, *b* – rectangular pulse on the input

## 5 APPLICATION OF MATCHED FILTER IN THE *M*PAM SIGNAL DEMODULATOR

Consider the joint scheme of the *M*PAM signal modulator and demodulator (Fig. 11). Scheme of the modulator is based on the description of *M*PAM signals (discussed in Module 1)

$$s_i(t) = a_i A(t), \quad i = 0, 1, \ldots, M-1, \tag{43}$$

where $A(t)$ – pulse with a certain frequency and temporal characteristics;
$a_i$ – coefficient that map transferred bits.
In terms of signals space $M$PAM is a one-dimensional space, $A(t)$ is basic function, $a_i$ are decomposition coefficients.

Mapper generates the coefficients $a_i$ by the input digital signal. On each clock interval $n = \log_2 M$ bits block is assigned to coefficient $a_i$. This coefficient applied to the shaping filter (SF) input by a signal $a_i\delta(t)$. SF generates a pulse $a_iA(t)$.

Demodulator scheme is based on earlier sections material. The input filter receives a sum of signal and noise $a_iA(t) + n(t)$. Matched filter reduces noise, and on its output there is a useful signal $a_iP(t)$ and noise $\zeta(t)$. Sampler takes the sample and gives out an estimate $\hat{a}$ of the coefficient $a_i$. The maximum value of pulse $P(t)$ at the sample time equals to 1, $\hat{a} = a_i + \zeta$ (estimate $\hat{a}$ can be considered as the coefficient $z_0$ submission signal $z(t)$ in one-dimensional space on the basic function $A(t)$). The Sampler is controlled by a sequence of pulses from a clock recovery scheme (ClR), which ensures the samples in the time of maximum signal/noise ratio. Based on information received from the sampler estimate $\hat{a}$ decision scheme makes a decision about the number of transmitted channel symbol and gives the solution of binary symbols in accordance with the modulation code.

Since shaping filter is excited by δ-function, then the amplitude spectrum of pulse $A(t)$ is equal to the AR of SF

$$S_A(f) = H_{SF}(f). \tag{44}$$

Amplitude spectrum of pulse $P(t)$ s defined as

$$S_P(f) = S_A(f) \cdot H_{MF}(f), \tag{45}$$

where $H_{MF}(f)$ is an amplitude response of filter matched with pulse $A(t)$.

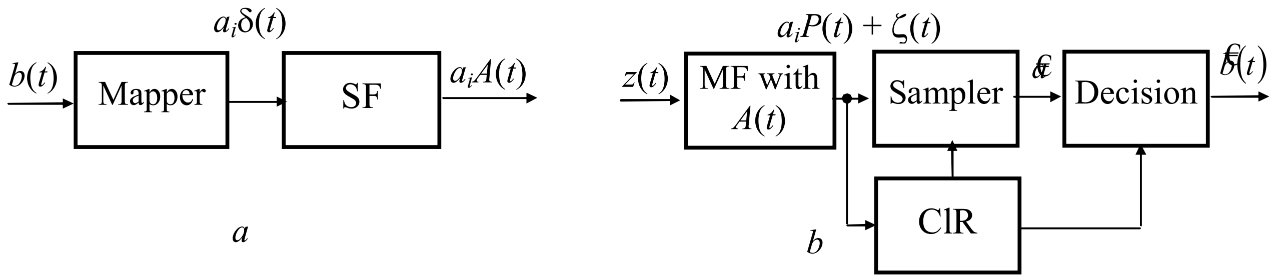



**Figure 11** – Schemes: *a* –*M*PAM signals modulator;
*b* – *M*PAM signals demodulator

Pulse on the MF output $P(t)$ must satisfy the absence of ISI, so we require that the spectrum $S_P(f)$ will be a Nyquist spectrum $N(f)$.

$$S_P(f) = N(f). \tag{46}$$

We use the property of MF: its amplitude response coincides with the amplitude spectrum of the signal, matched with (if $c = 1$)

$$H_{MF}(f) = S_A(f). \tag{47}$$

Given the equality (44) - (47) we conclude that

$$H_{SF}(f) = H_{MF}(f) = \sqrt{N(f)}. \tag{48}$$

As they say the AR of MF and SF is described by the "square root of the Nyquist spectrum" dependence.

Usually Nyquist spectrum is described by the "raised cosine" dependence.

$$N(f) = \begin{cases} T, & 0 \le |f| \le (1-\alpha) f_N, \\ 0{,}5T\left[1 + \sin\left(\frac{\pi}{2\alpha}\left(1 - \frac{|f|}{f_N}\right)\right)\right], & (1-\alpha) f_N < |f| < (1+\alpha) f_N, \\ 0, & |f| \ge (1+\alpha) f_N, \end{cases} \tag{49}$$

where $T$ – clock interval;

$f_N = 1/(2T)$ – Nyquist frequency;

$\alpha$ – roll-off factor.

Dependence "square root of the Nyquist spectrum" is described as

$$\sqrt{N(f)} = \begin{cases} \sqrt{T}, & |f| \le (1-\alpha) f_N, \\ \sqrt{T}\sin\left[\frac{\pi}{4\alpha}\left(1 + \alpha - \frac{|f|}{f_N}\right)\right], & (1-\alpha) f_N < |f| < (1+\alpha) f_N, \\ 0, & |f| \ge (1+\alpha) f_N. \end{cases} \tag{50}$$

Figure 12 shows the dependences $N(f)$ and $\sqrt{N(f)}$ for $\alpha$ = 0,4. Figure 12, *b* shows that the shaping and matched filters are a lowpass filters, but with a special response. If as MF and SF filters of Butterworth, Chebyshev, etc. are used, synthesized with a view to bringing them as close to the AR of rectangular, it will not be fulfilled the condition of absence of ISI.

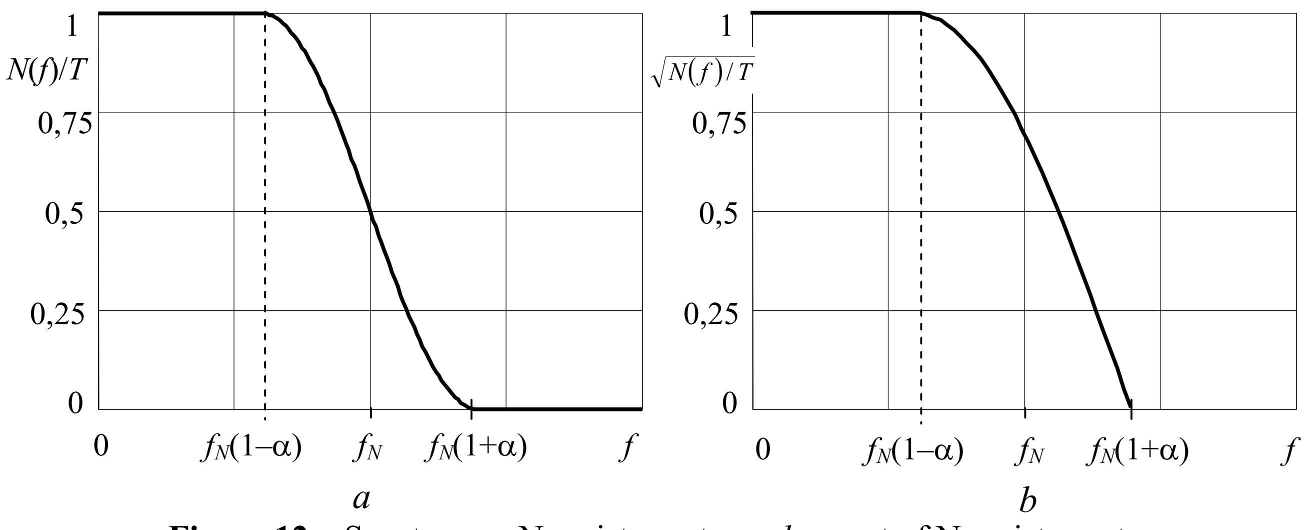



**Figure 12** – Spectra: *a* – Nyquist spectrum, *b* – root of Nyquist spectrum

The expression for the pulse $A(t)$ can be obtained as the inverse Fourier transform of dependence $\sqrt{N(f)}$, assuming that the phase spectrum is identically equal to zero:

$$A(t)=\frac{\pi}{8\alpha+2\pi(1-\alpha)}\left\{\frac{8\alpha}{\pi(1-(8\alpha f_N t)^2)}\cdot\left[\cos(2\pi(1+\alpha)f_N t)+8\alpha f_N t\cdot\sin(2\pi(1-\alpha)f_N t)\right]+\right.$$
$$\left.+2(1-\alpha)\frac{\sin(2\pi(1-\alpha)f_N t)}{2\pi(1-\alpha)f_N t}\right., \qquad -\infty<t<\infty. \tag{51}$$

$P(t)$ function can be obtained as the inverse Fourier transform of $N(f)$, assuming that the phase spectrum is identically equal to zero:

$$P(t)=\frac{\sin\ 2\pi f_N t}{2\pi f_N t}\cdot\frac{\cos\ 2\pi\alpha\cdot f_N t}{1-(4\alpha\cdot f_N t)^2}. \tag{52}$$

Figure 13 shows graphs of pulses $A(t)$ and $P(t)$. From graphs $P(t)$ shows that its peak value is 1. This means that when the pulse $a_iA(t)$ transfer sample on the output of the sampler is $a_i$. Figure 13 shows that the pulse $P(t)$ takes zero value at $t = \pm kT$ ($k$ = 1, 2, 3,…), i.e. satisfies the sampling condition. Impulse $A(t)$ doesn't take zero values at $t = \pm kT$ ($k$ = 1, 2, 3,…).

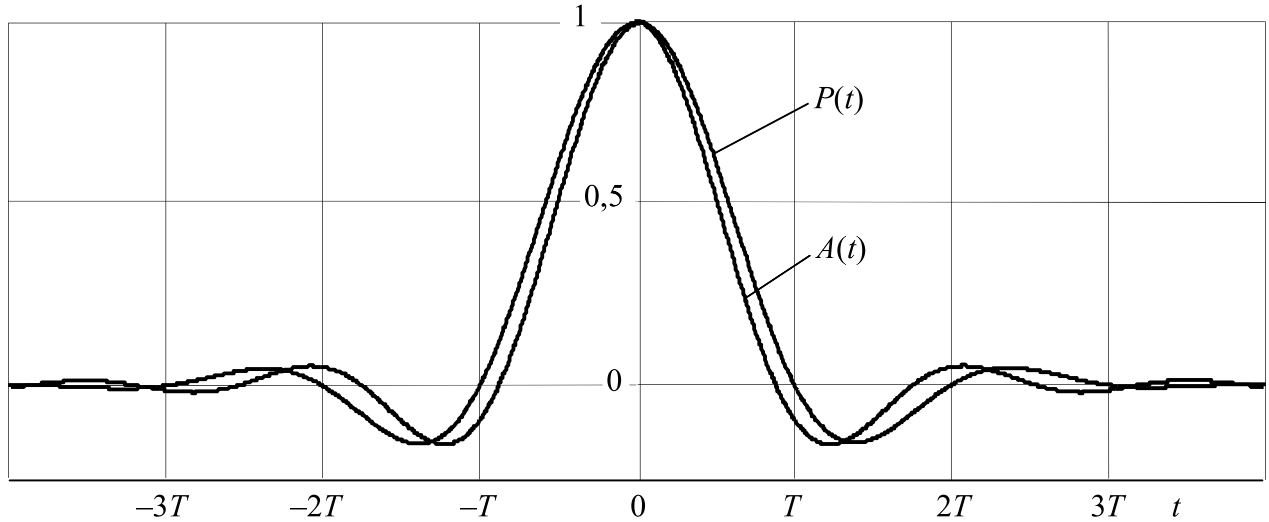



**Figure 13** – $A(t)$ and $P(t)$ pulses

Define the value of the pulse $A(t)$ energy which require in further analysis,

$$E_A=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_A^2(\omega)d\omega=2\int_0^{\infty}S_A^2(f)df=2\int_0^{\infty}(\sqrt{N(f)})^2df=2\int_0^{\infty}N(f)df=1. \tag{53}$$

The result was obtained, based on the fact that the integral equals the area under the curve described by the integrand (Fig. 12, *a*). Since the function $N(f)$ has antisymmetric slope, then this area is the area of a rectangle of height $T$ and the base $f_N = 1/(2T)$. This value allows you to easily determine the signal energy $s_i(t)=a_iA(t)$:

$$E_i=a_i^2. \tag{54}$$

For further analysis the value of noise average power is also required on the MF output, which response are described by the dependence $\sqrt{N(f)}$,

$$P_{n\ \mathrm{out}} = 2\int_0^{\infty} \frac{N_0}{2} H_{MF}^2(f)df = N_0 \int_0^{\infty} N(f)df = \frac{N_0}{2}. \tag{55}$$

When integrating the same approach as in the calculation of the integral (53). The value of RMS noise on the MF output is equal to

$$\sigma = \sqrt{N_0/2}. \tag{56}$$

Given (54) and (56) is easily seen that the signal to noise ratio at the sampling moment

$$\frac{a_i}{\sigma} = \sqrt{\frac{2E_i}{N_0}} \tag{57}$$

corresponds to a matched filter property (31).

## 6 CORRELATOR

In analyzing the properties of the matched filter has been related between the output signal $y(t)$ and input signal $z(t)$ – relation (38). Let $c = 1$, $t_0 = T_s$ and under these conditions, the relation (38) define $y(T_s)$. At the same time we take into account that the signal $s(t)$ exists on the interval $(0, T_s)$:

$$y(T_s) = \int_0^{T_s} z(t)s(t)dt. \tag{58}$$

Hence the last relation follows that you can make an optimal signal processing with the noise $z(t) = s(t) + n(t)$ by scheme, which work is described by the relation (58). The device, which work is described by the relation (58) is called a correlator and it is considered in Section. 3.

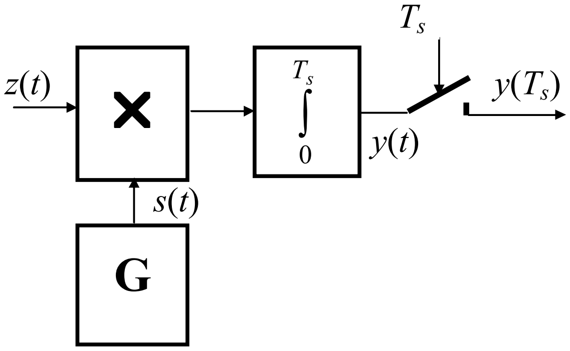



**Figure 14** – Correlator

Figure 14 shows the scheme of the correlator. It contains the signal $s(t)$ generator (a replica of the processed signal), multiplier and integrator with reset. At the end of the signal $s(t)$ sampler takes the sample, and the integrator is given in the zero state to be ready to process the next signal.

From the description of the scheme it is understandable that have not been shown circuit of synchronization signal generator $s(t)$, the control circuit of the sampler and the integrator reset.

Equivalence of signal processing with the noise by matched filter and correlator in both cases is

$$\mathrm{c}_{\mathrm{out}} = \frac{2E_s}{N_0}. \tag{59}$$

However, the processes taking place in the schemes of matched filter and correlator are different. Illustrate this for an example of the radio pulse $s(t)$ (Fig. 15, *a*). The output of matched filter response is observed in the form of the correlation function of the signal $s(t)$ (Fig. 15, *b*). From the output of the filter is sample $y_s(T_s)$ taken. Figure 15, *c* shows the signal at the output of the integrator with the reset of the correlator $y_s(t)$ and the sample value from the output of the correlator $y_s(T_s)$. Despite the fact that the processes are different, the outputs of both circuits of the signal/noise ratio equal

$$\frac{y(T_s)}{y} = \sqrt{\frac{2E_s}{N_0}}, \tag{60}$$

where σ is RMS of sample $y(t)$ .

Relationships (59) and (60) are valid for the correlator, if the signal processing time by correlator equals to the duration of the signal. Situation is different in the processing of pulses with a substantially limited range, for example, Nyquist pulses. In this case, the pulse duration can take values $T_s = (8\ldots20)T$, where $T$ – clock interval duration. Processing time by correlator can not be more than $T$ at digital modulation signal demodulation.

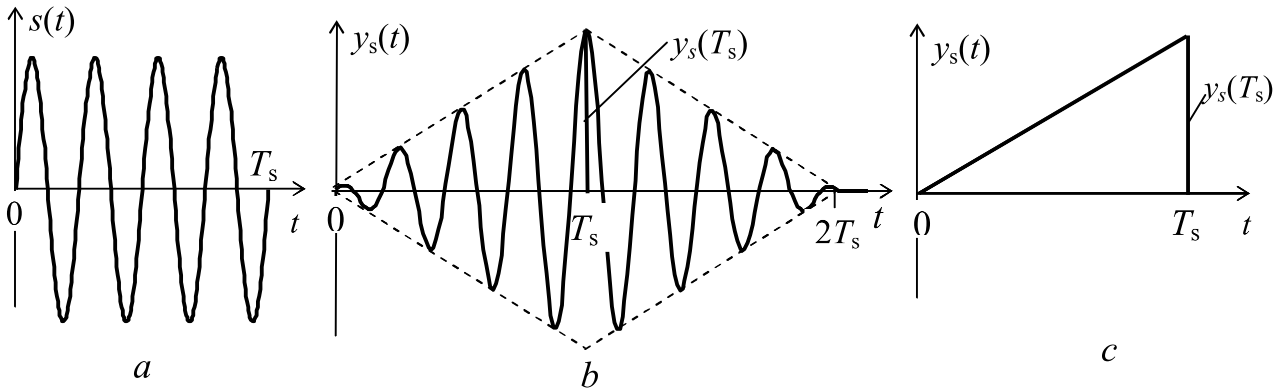



**Figure 15 –** Signals: *a* – on the processing device input, *b* – on the matched filter output; *c* – on the integrator output of the correlator

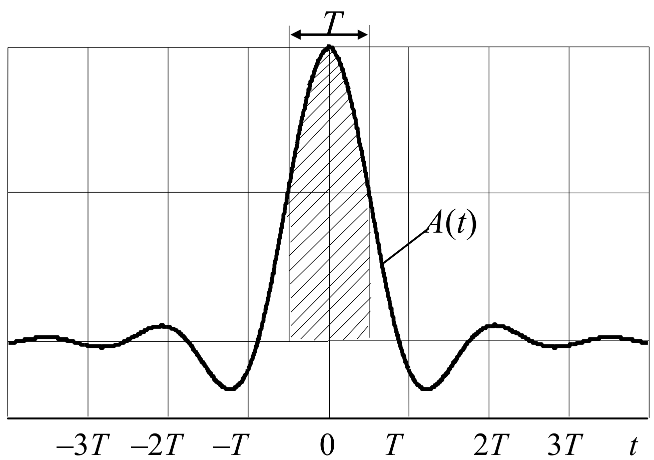



**Figure 16** – Processing of the pulse by correlator

In Fig 16 pulse $A(t)$ is shown. The shaded area under the curve $A(t)$ on the interval ($-T/2$, $T/2$) shows the result of integrating by correlator – in the processing of the pulse are not using the pulse value at intervals ($-\infty$, $-T/2$) and ($T/2$, $\infty$). Therefore, signal/noise ratio at the correlator output is less than the processing of a matched filter. Processing time of matched filter is equal to the duration of its impulse response, and it is equal to the duration of the signal $T_s$, and all the signal values used in processing. For this reason, the correlator was used in the digital modulation demodulator using weakly filtered rectangular pulses, when the duration of the signal is practically equal to the duration of the clock interval. Matched filters are applied when pulses with a substantially limited range are used.

## 7 MATCHED FILTERING OF RADIO PULSES

In the case of bandpass signals of digital modulation ($M$APSK, $M$PSK, $M$QAM etc.) channel symbols are based on pulse-carrier

$$s(t) = \sqrt{2}A(t)\cos 2\pi f_0 t\,. \tag{61}$$

Signal $s(t)$ is bandpass. Its spectrum is centered round the frequency $f_0$. We must perform an optimal filtering of the signal $s(t)$, coming together with the noise $n(t)$, and take sample. We suppose that the noise is white noise, its spectrum is concentrated in the bandwidth of the communication channel.

First method. Used bandpass filter which amplitude response is described by the relation (35). Engineering practice has shown that the bandpass filters have a low accuracy of implementation.

Second method. First is the coherent (synchronous) detection of the signal and noise sum $z(t) = s(t) + n(t)$, and then pulse filtering $A(t)$ with noise by low-pass matched filter (Fig. 17). Carrier recovery (CR) scheme produces an oscillation $\sqrt{2}\cos 2\pi f_0 t$, which is necessary for the detector.

For the analysis of noise transmission through the synchronous detector we represent bandpass noise by quadrature components

$$n(t) = N_c(t)\sqrt{2}\cos 2\pi f_0 t + N_s(t)\sqrt{2}\sin 2\pi f_0 t, \tag{62}$$

where $N_c(t)$ is amplitude of the cosine component of the noise;

$N_s(t)$ is amplitude of the sine component of the noise.

The output multiplier we obtain

$$u_{\text{mult}}(t) = [A(t) + N_c(t)] + [A(t) + N_c(t)]\cos 2\pi 2 f_0 t. \tag{63}$$

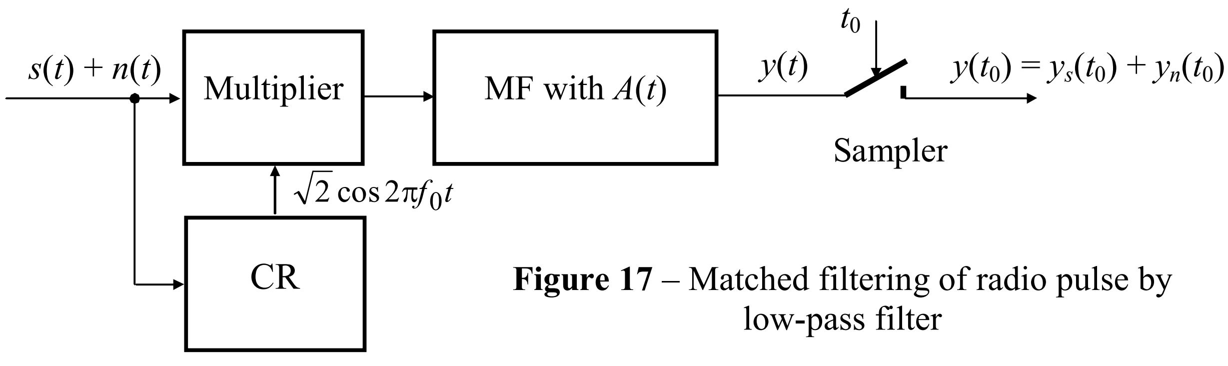



**Figure 17** – Matched filtering of radio pulse by low-pass filter

The first term in (63) is low-frequency component, and the second term is DSB-SC signal with a carrier frequency $2f_0$. In transmission systems, carrier frequency is substantially greater than the maximum frequency of the signal spectrum

$A(t)$, and the spectra of two terms in (63) do not overlap. For signal $A(t)$ after the multiplier one should include low-pass filter, transmitting $A(t)$ and weakening $A(t)\cos 2\pi 2f_0t$. I.e. requires a low-pass filter with a cut off frequency is higher than the maximum frequency in the spectrum of the signal $A(t)$. The same filter after the multiplier (in fact LPF) performs optimal filtering of the signal $A(t)$. On the MF output due to the pulse $A(t)$ we obtain the pulse $P(t)$ (see section 5).

Let us discuss the passing of noise through the synchronous detector and matched filter. Noise $n(t)$ is white in the bandwidth of the communication channel $F_{\text{ch}}$ with average frequency $f_0$. Its average power is determined $P_n = N_0F_{\text{ch}}$. This power is divided equally between the cosine and sine components. Thus, the power of cosine component

$$\overline{\left(N_c(t)\sqrt{2}\cos 2\pi f_0 t\right)^2} = P_n/2. \tag{64}$$

Performing the averaging over the left-hand side of equation (64), we obtain

$$\overline{N_c^2(t)} = P_n/2. \tag{65}$$

Noise $N_c(t)$ is white in the frequency band $(0, F_{\text{ch}}/2)$. Its power density is

$$\frac{P_n/2}{F_{\text{ch}}/2} = \frac{P_n}{F_{\text{ch}}} = N_0. \tag{66}$$

An analysis of transformations of the signal and noise synchronous detector can be seen that the input filter, matched with $A(t)$, signal $A(t)$ and white noise with power density $N_0$ act. This filtering is considered in the previous section. MF provides the ratio of the instantaneous signal power to average noise power

$$\text{c}_{\text{peak}} = \frac{2E_A}{N_0}. \tag{67}$$

Define the signal energy $s(t)$

$$E_s = \int_{-\infty}^{\infty} s^2(t)dt = \int_{-\infty}^{\infty} \left(A(t)\sqrt{2}\cos 2\pi f_0 t\right)^2 dt = \int_{-\infty}^{\infty} A^2(t)dt = E_A. \tag{68}$$

Thus, the circuit shown in Fig. 17, provides the signal/noise ratio

$$\text{c}_{\text{peak}} = \frac{2E_s}{N_0}, \tag{69}$$

then there is a filter, matched with the bandpass signal $s(t)$.

This is precisely the scheme used in the demodulator of two-dimensional digital modulation signals for optimum signal filtering, as the inclusion of two such schemes with the reference oscillations $\cos 2\pi f_0 t$ and $\sin 2\pi f_0 t$ a division of cosine and sine of radio pulse and their subsequent separate processing performed.

## 8 OPTIMAL DEMODULATOR FOR BANDPASS SIGNALS

Let's consider the one-dimensional *M*ASK and BPSK signals. Channel symbols are described as

$$s_i(t) = a_i \sqrt{2} A(t) \cos 2\pi f_0 t, \quad i = 0, 1, ..., M - 1, \tag{70}$$

where $\sqrt{2}A(t)\cos 2\pi f_0 t$ is radio pulse, with time and spectral characteristics, whose energy is equal to 1, the maximum value is equal to $\sqrt{2}$;

$a_i$ – coefficients that reflect $n$ = $\log_2 M$ transmitted bits in accordance with mapping code.

Above it was found that the main elements of the optimal demodulator are: a matched filter, sampler, and decision. We have said that the optimal filtering of radio pulse should perform circuit containing a synchronous detector and low-frequency matched filter. To synchronous detector it is necessary to use oscillation $\sqrt{2}\cos 2\pi f_0 t$ that generates by a carrier recovery scheme (CR). Coming from the clock recovery scheme (ClR) impulses controls the work of the sampler. Therefore, the demodulator circuit has the form shown in Fig. 18.

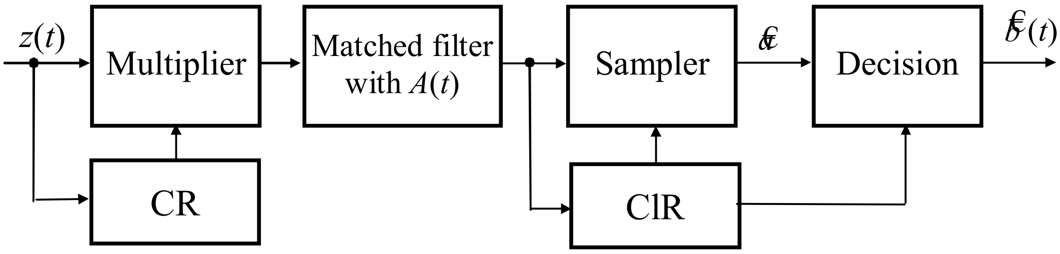



**Figure 18 –** Optimal demodulator of *M*ASK and BPSK signals

Rule making decisions formulated on the basis of partitioning the space of signals into $M$ disjoint areas $s_i$, $i = 0, \ldots, M - 1$; each area $s_i$ is a set of points that are closer to the signal $s_i(t)$, rather than to other signals. Decision scheme gives solutions bits in accordance with the mapping code.

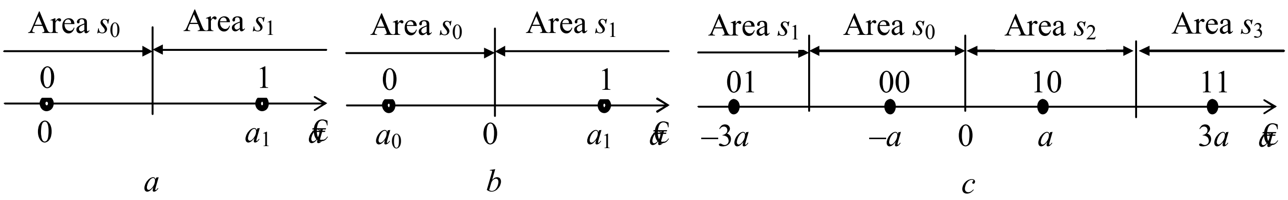



**Figure 19 –** Partitioning of signals space on the signals areas:
*a* – BASK, *b* – BPSK, *c*– QASK

In the case of two-dimensional signals *M*PSK ($M \geq 4$), *M*APSK, *M*QAM channel symbols are described as

$$s_i(t) = a_{ci}\sqrt{2}A(t)\cos 2\pi f_0 t + a_{si}\sqrt{2}A(t)\sin 2\pi f_0 t, \quad i = 0,\ 1, ...,\ M - 1, \tag{71}$$

where $a_{ci}$ and $a_{si}$ – pair of coefficients that reflect $n$ = $\log_2 M$ transmitted bits in accordance with mapping code.

Channel symbols consist of cosine and sinus impulses. They should be divided. This is done by two synchronous detectors, supporting different variations – if the reference oscillation $\sqrt{2}\cos 2\pi f_0 t$, that the detector does not respond to the sinus component of the input signal, if the reference oscillation $\sqrt{2}\sin 2\pi f_0 t$, that the detector does not respond to the cosine component of the input signal. Therefore, two detectors divide cosine and sine pulses and their subsequent separate processing in the two subchannel demodulator (Fig. 20): matched filtering and sampling. Estimates coefficients $\hat{a}_c$ and $\hat{a}_s$ act at the decision scheme.

Just as in the case of one-dimensional signals, generally make decisions formulated on the basis of partitioning the space of signals into $M$ disjoint areas $s_i$, $i = 0, \ldots, M-1$; each area $s_i$ is a set of points that are closer to the signal $s_i(t)$, rather than to other signals. The difference lies in the fact that the space is two-dimensional. If the point $(\hat{a}_c, \hat{a}_s)$ appears in the region signal $s_i$, then decides that the transmitted signal $s_i$. Decision scheme gives out solutions bits in accordance with the mapping code.

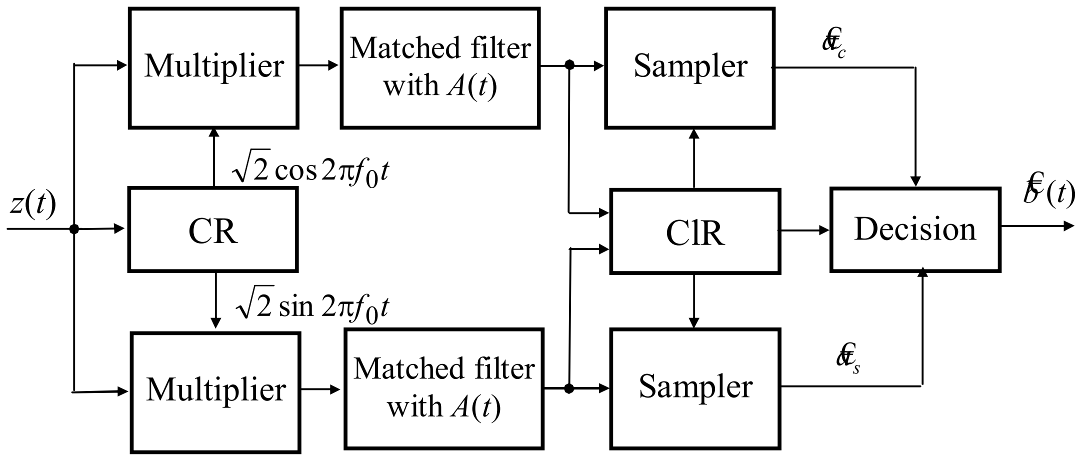



**Figure 20 –** Optimal demodulator of $M$PSK ($M \geq 4$), $M$APSK, $M$QAM signals

Figure 21 shows a partition of signal space on the signals areas. Boundaries of the areas shown in heavy lines. In the case of QPSK signal the signals areas are 4 quadrant, in the case of 8PSK signal the signals areas are 8 sectors, in the case of 16QAM signal the signals area are formed by vertical and horizontal lines. Due to the simplicity of two-dimensional $M$QAM signal space partitioning on the signals areas they have received the widest distributed among the $M$APSK signals.

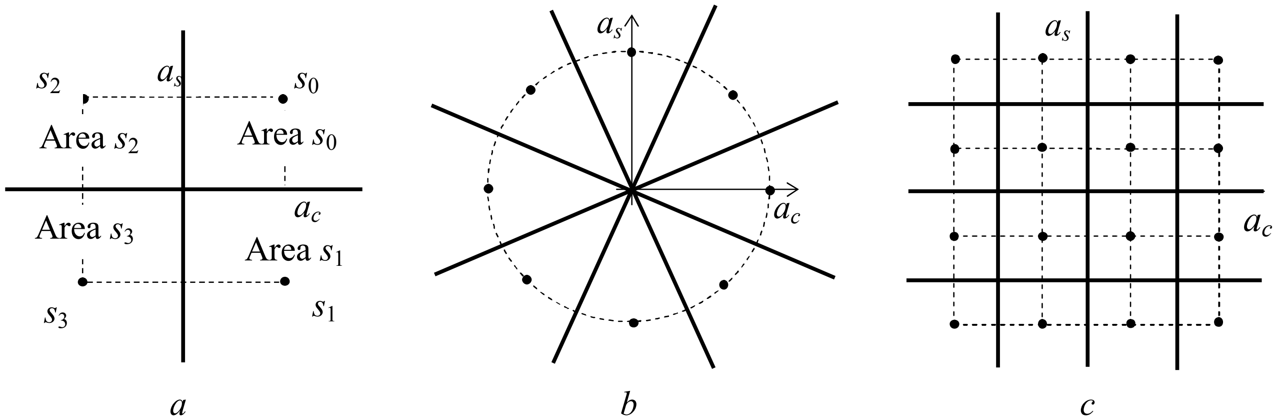



**Figure 21 –** Partitioning of signals space on the signals areas:
*a* – QPSK; *b* – 8PSK; *c* – 16QAM

BFSK signals referred to two-dimensional signals also. Channel symbols are described as

$$s_0(t) = a\sqrt{2}A(t)\cos\left(2\pi(f_0 - \Delta f/2)\right)t,\quad s_1(t) = a\sqrt{2}A(t)\cos\ \left(2\pi(f_0 + \Delta f/2)\right)t, \tag{72}$$

where $\Delta f$ – frequency spacing.

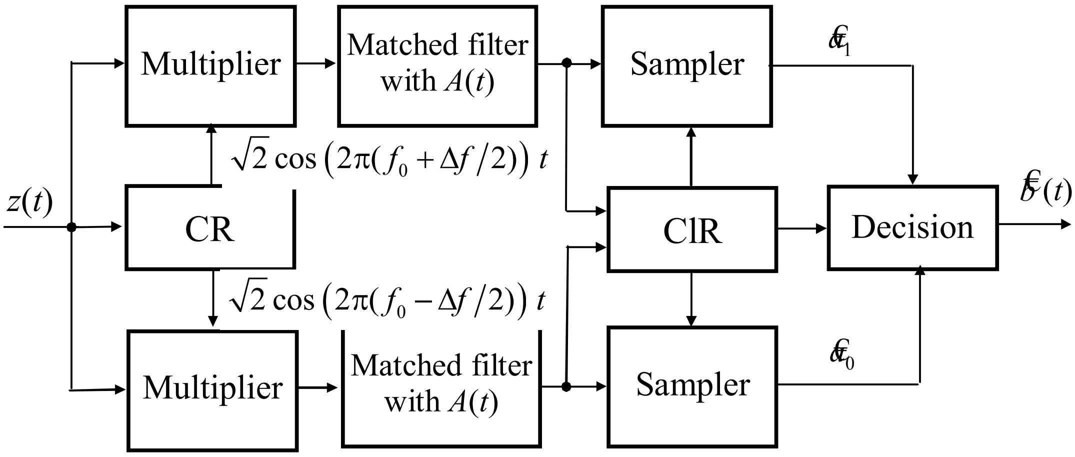



**Figure 22 –** Optimal demodulator of BFSK signal

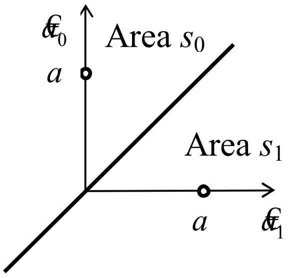



**Figure 23 –** Partitioning of BFSK signals space on the signals areas

If frequency spacing satisfies $\Delta f = k/(2T)$, where $k$ = 1, 2, …, the signals are orthogonal, and they can be separated by a synchronous detector (similar to the separation of cosine and sine pulses). Scheme of optimal demodulator of BFSK signal is similar to the scheme shown in Fig. 21, but differs in supporting synchronous oscillations in the detectors (Fig. 22).

Figure 23 shows a partition of the BFSK signal space on signals areas. A heavy line shows the border area. This figure shows that it is enough to compare estimates of the signal amplitudes $s_0(t)$ and $s_1(t)$ for a decision on the transmitted signal: if $\hat{a}_1 > \hat{a}_0$ demodulator decides $\hat{s}_1$ and on the contrary.

## 9 THE ERROR PROBABILITY AT THE OPTIMUM DEMODULATION

Bit error probability $p$ is a quantitative measure of noise immunity of digital transmission systems. Analysis of bit error probability will start from the consideration of one-dimensional signals ($M$PAM, $M$ASK, BPSK) – see Fig. 2, $b$ and relations (15)–(18).

The conditional probability density $p(\hat{a}/s_0)$ has a normal probability distribution with average equals to $a_0$. With this in mind (16) we can write

$$P_{\text{er}}\left(s_0\right) = \int_{\lambda}^{\infty} p(\hat{a}/s_0)d\hat{a} = Q\left(\frac{\lambda - a_0}{\sigma_\zeta}\right), \tag{73}$$

where $\sigma_\zeta$ – noise RMS on the matched filter output, defined earlier by the ratio (56).

We take into consideration the distance between signals

$$d = (a_1 - a_0). \tag{74}$$

Express the threshold λ through distance $d$ (Fig. 2):

$$\lambda = a_0 + 0{,}5d. \tag{75}$$

In view of (56) and (75) ratio (73) can be rewritten

$$P_{\text{er}}(2) = Q\left(\frac{d}{\sqrt{2N_0}}\right), \tag{76}$$

where $P_{\text{er}}(2)$ – signal error probability in the binary transmission system.

The definition of Gaussian $Q$-function $Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^{\infty} e^{-t^2/2}dt$ follows that for the larger value of the argument is the lower value of the function $Q(z)$. Signal error probability (76) will be decreased with distance $d$ between signals increasing and the noise power density at the demodulator input $N_0$ decreasing.

It's easy (73) stated in terms of errors, when the line connecting the signal point is parallel to the axis $a_s$ or the axis $a_c$. If the line connecting the signal point is not parallel to anyone of the axes, then the condition of the error is stated differently. But we can show that in such cases, the probability of error signal in the binary transmission system $P_{\text{er}}$ (2) is defined by (76).

The task is to express the distance $d$ through the physical parameters of the signal at the demodulator input – the average signal power $P_s$ and the digital signal transmitted rate $R$. This problem is solved in Module 1. To compare the noise immunity of different types of modulation signals is conveniently expressed in terms of distance $E_b$ – average value of signal energy to transfer one bit.

For binary transmission systems $p = P_{\text{er}}$. Formulas are given in Table 1.

**Table 1** – Bit error probability in binary transmission systems

| Modulation method | BPAM | BASK | BPSK | BFSK |
|---|---|---|---|---|
| Distance | $2\sqrt{E_b}$ | $\sqrt{2E_b}$ | $2\sqrt{E_b}$ | $\sqrt{2E_b}$ |
| Bit error probability | $p = Q(\sqrt{2}h_b)$ | $p = Q(h_b)$ | $p = Q(\sqrt{2}h_b)$ | $p = Q(h_b)$ |

In a table $h_b^2 = E_b / N_0$ is signal/noise ratio;

$E_b = P_s \cdot T_b = P_s / R$.

We pass on now to multilevel transmission systems, i.e. $M > 2$. If the channel symbols are equiprobable, the signal error probability is determined

$$P_{\text{er}} = \frac{1}{M}\sum_{i=0}^{M-1}\sum_{\substack{j=0 \\ j\neq i}}^{M-1} P_{\text{er}}(s_i, s_j), \tag{77}$$

where $P_{\text{er}}(s_i,\ s_j)$ is error probability in binary transmission systems, using signals $s_i$ and $s_j$, error appears when the demodulator makes a decision on $s_j$, if $s_i$ was transferred.

To simplify the calculations take into account only the transitions in the nearest signals. Table 2 shows the formulas of error probability for certain multilevel modulation types when mapping Gray code is used.

**Table 2** – Bit error probability in multilevel transmission systems

| Type of modulation | QPAM | QPSK | 8PSK | 16QAM |
|---|---|---|---|---|
| Bit error probability | $p = Q(\sqrt{2/5}\, h_b)$ | $p = Q(\sqrt{2}h_b)$ | $p = \frac{2}{3}Q(0{,}94h_b)$ | $p = Q(0{,}89h_b)$ |

## 10 CARRIER RECOVERY SYSTEM

Carrier recovery system (CR) of bandpass digital modulation signals demodulators is intended for the formation of the reference harmonic oscillation whose phase coincides with the phase of the carrier on which demodulated signal is formed.

Already in the 30-th years of the last century became clear that the BPSK signals have highest noise immunity. It was necessary to solve the problem of carrier recovery in demodulator for application of these signals in the transmission systems. The reference oscillation is necessary for the synchronous detector (Fig. 24). Let on the detector input BPSK signal acts. Channel symbol is described as

$$s_i(t) = a_i\sqrt{2}A(t)\cos 2\pi f_0 t. \tag{78}$$

If the oscillations phase from the generator

$$u_{\text{ref}}(t) = \sqrt{2}\cos(2\pi f_0 t + \Delta\varphi). \tag{79}$$

differs from the carrier phase of the input signal by a value $\Delta\varphi$, the signal at the output of the synchronous detector receives the factor $\cos\Delta\varphi$:

$$u_{\text{out}}(t) = a_i A(t)\cos\Delta\varphi. \tag{80}$$

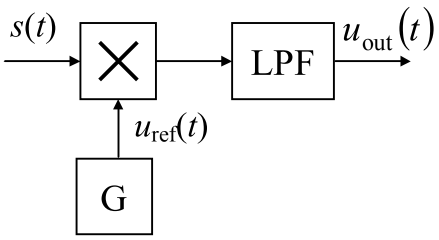



**Figure 24** – Synchronous detector

As the maximum value of the cosine is one and is achieved only in the case $\Delta\varphi = 0$, presence of the phase difference leads to a decrease in the level of the signal at the detector output. If $\Delta\varphi =$ $= \pi/2$, the signal at the detector output is absent: $u_{\text{out}}(t) \equiv 0$. At $\Delta\varphi=\pi$ $u_{\text{out}}(t) = -a_i A(t)$, i.e. changes its sign. The task is $\Delta\varphi = 0$.

First the carrier recovery scheme with frequency multiplication on 2 has been proposed (Fig. 25).

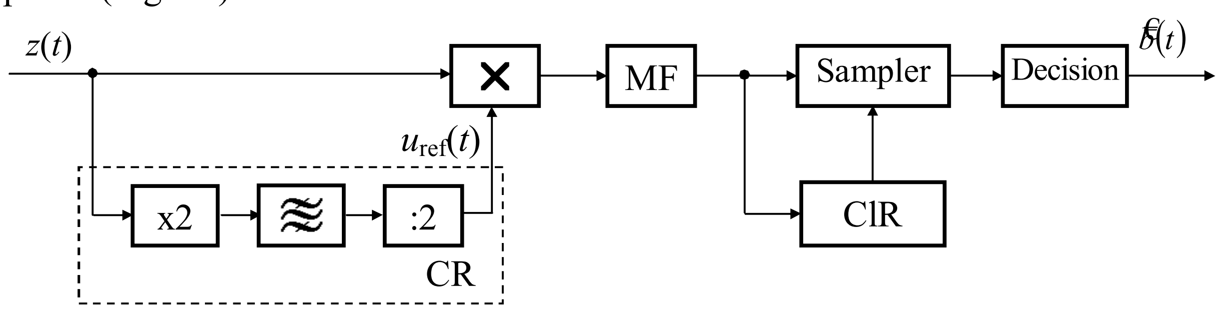



**Figure 25** – Carrier recovery scheme with frequency multiplication on 2

BPSK signal constellation is given in Fig. 19, *b*. Channel symbols are described by the formula (78):

$$s_1(t) = aA(t)\cos 2\pi f_0 t; \;\; s_0(t) = aA(t)\cos(2\pi f_0 t + \pi). \tag{81}$$

For many decades "weak" filtered pulses $A(t)$ are used, they were similar in shape to the rectangular pulse at the interval duration $T$

$$A(t) = \begin{cases} 1, & 0 \le t \le T, \\ 0, & t < 0, t > T. \end{cases} \tag{82}$$

After multiplying the frequency by 2, the signal $s_1(t)$ and signal $s_0(t)$ give $a^2\cos 2p2f_0t$. Narrow band filter has a center frequency $2f_0$. It is designed to reduce noise. Frequency divider 2 can issue one of two possible reference oscillations:

– case 1: $u_{\text{ref1}}(t) = \cos 2pf_0t$;
– case 2: $u_{\text{ref2}}(t) = \cos(2pf_0t + \pi)$.

Both variants are possible, because the result depends on what the initial conditions arise in the scheme of the divisor. It is said that the reference oscillation has a phase uncertainty of the second order.

In case 1, the algorithm of optimal BPSK signal demodulation is implemented. In case 2, on the multiplier output, and then on the matched filter and the sampler will be tensions, opposite to those which occur in case 1. The scheme will make the inverse decision: instead of 1 will return 0 and vice versa. This phenomenon is called the inverse (opposite) work of demodulator. It turned out that in the demodulation process can occur random cuts from fluctuation $u_{\text{ref1}}(t)$ to oscillation $u_{\text{ref2}}(t)$ and vice versa.

In the QPSK signal demodulator must use a frequency multiplier on 4, a filter with a center frequency $4f_0$ and the frequency divider on 4. After the frequency divider arises one of the reference oscillations, which differ in phase with the step 90°. There is phase uncertainty of the reference oscillation of fourth order.

Differential coding can be used for elimination of phase uncertainty of the reference fluctuations in the demodulator. Such transfer methods are called differential encoded phase modulation (see section 11).

Examined system with CR with exponentiation works well when the pulse amplitude $A(t)$ is close to a rectangular shape. Now, Nyquist pulses are used (pulses with substantially smoothed form $A(t)$). In this pulse form the CR system with exponentiation works bad.

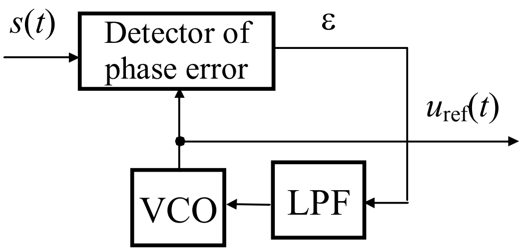



**Figure 26** – PLL scheme

Now CR system is a system of phase locked-loop (PLL) (Fig. 26) with a special detector of phase error, which is able to operate in the absence of a carrier in the signal spectrum. Here VCO is voltage-controlled oscillator. In the presence of phase error voltage ε adjusts the frequency and phase oscillations, which is produced by VCO, so as to reduce the amount of phase error.

Let's consider the construction of detector of phase error in the case of BPSK signal. Scheme of the detector contains one additional synchronous detector, the reference oscillation that is $\sqrt{2}\sin(2\pi f_0 t + \Delta\varphi)$. Recall that the work of the synchronous detector can be regarded as the calculation of the projection of $s(t)$ on $u_{\text{ref}}(t)$. Two synchronous detectors differ by supporting oscillations shifted in phase by 90°. Therefore, the voltages received from the outputs of synchronous detectors are the quadrature components of the detected signal.

Figure 27 shows the signal constellation of demodulated BPSK signal and the calculated quadrature components in the sample time provided that the channel symbol with the amplitude $a$ is demodulated: $I$ – in-phase component, $Q$ – quadrature component. In Fig. 29, *a* phase error of the reference oscillation $\Delta\varphi = 0$; and synchronous detector calculates $I = a$, $Q = 0$. In Fig. 27, *b* phase error of the reference oscillation $\Delta\varphi > 0$; and synchronous detector calculates $I = a\cdot\cos\Delta\varphi$, $Q < 0$. In Fig. 27, *c* phase error of the reference oscillation $\Delta\varphi < 0$; and synchronous detector calculates $I = a\cdot\cos\Delta\varphi$, $Q > 0$.

We see that the sign for $Q$ corresponds to the error phase: namely, if $Q < 0$, then $\Delta\varphi > 0$ and it is necessary to decrease the frequency and phase of the VCO, if the $Q > 0$, then $\Delta\varphi < 0$ and it is necessary to increase the frequency and phase of the VCO. Thus, the value of $Q$ can be taken as the phase error ε. But the situation with the $Q$ sign is opposite if channel symbol with the amplitude $-a$ is demodulated.

Costas suggested calculate a phase error of reference oscillation in BPSK signal demodulators

$$\varepsilon = Q \cdot \operatorname{sign}(I). \tag{83}$$

Figure 28 shows the scheme of BPSK signal demodulator with an open circuit of the carrier recovery.

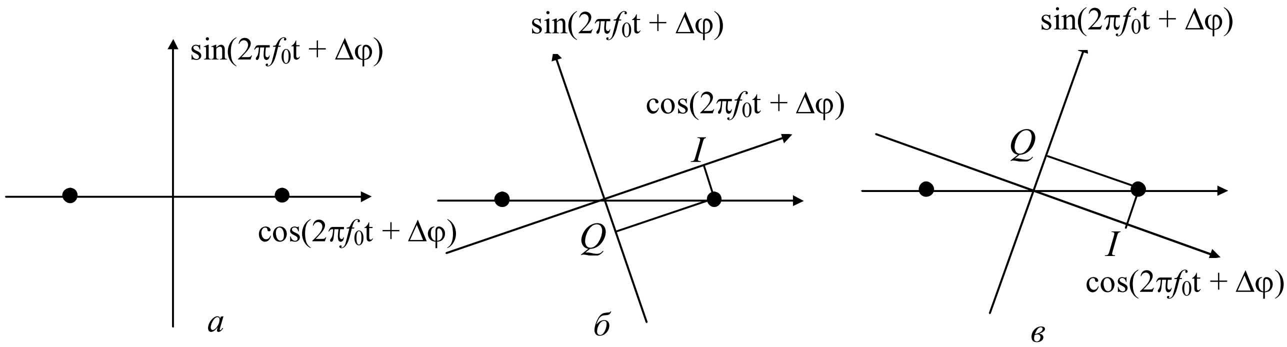



**Figure 27** – Calculation of projections of demodulated signal at the reference oscillation: $a - \Delta\varphi = 0$; $b - \Delta\varphi > 0$; $c - \Delta\varphi < 0$

To construct the CR system of QPSK signal demodulator phase error detector is used, which is calculated by the Costas algorithm

$$\varepsilon = Q \cdot \operatorname{sign}(I) - I \cdot \operatorname{sign}(Q). \tag{84}$$

Here is a sign of the phase error of the reference oscillation is inequality of quadrature components *I* and *Q*. The same algorithm for calculating the phase error is used in the *M*QAM signals demodulators.

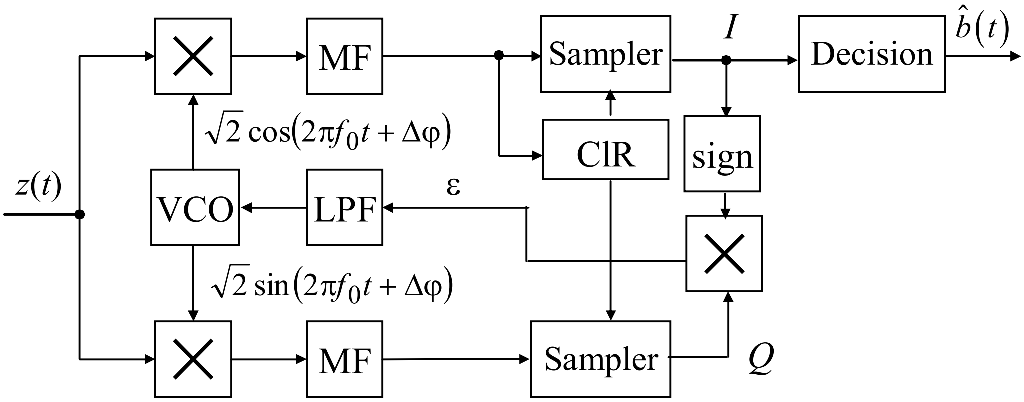



**Figure 28** – BPSK signal demodulator with the CR system

## 11 DIFFERENTIALLY ENCODED PHASE MODULATION

Solve the problem of inverse work of BPSK demodulator allows the transition to the difference method of transmission, in which the transmitted binary symbols (bits) appearing not in the initial phases of radio pulses (as in BPSK), but in the phase difference between adjacent radio pulses on time. The formation of modulated signal in this way is called the differentially encoded phase shift keying (DPSK).

The principle of modulation and demodulation of BDPSK signals is displayed in Fig. 29. BDPSK modulator signal consists of the differential coder (DC) and the BPSK signal modulator, and the demodulator BDPSK signal consists of BPSK signal demodulator and differential decoder (DD).

Differential coder of BDPSK modem works on rule

$$b_k^{\mathrm{d}} = b_k \oplus b_{k-1}^{\mathrm{d}}, \tag{85}$$

where $b_k$ – bit 1 or 0 on coder input on $k$-th clock interval;

$b_k^{\mathrm{d}}$ – symbol 1 or 0 on coder output on $k$-th clock interval;

$\oplus$ – mod 2 plus sign.

Differential decoder of BDPSK modem works on rule

$$\hat{b}_k = \hat{b}_k^{\mathrm{d}} \oplus \hat{b}_{k-1}^{\mathrm{d}}, \tag{86}$$

where $\hat{b}_k^{\mathrm{d}}$ – symbol 1 or 0 on differential decoder input on $k$-th clock interval;

$\hat{b}_k$ – bit 1 or 0 on differential decoder output on $k$-th clock interval.

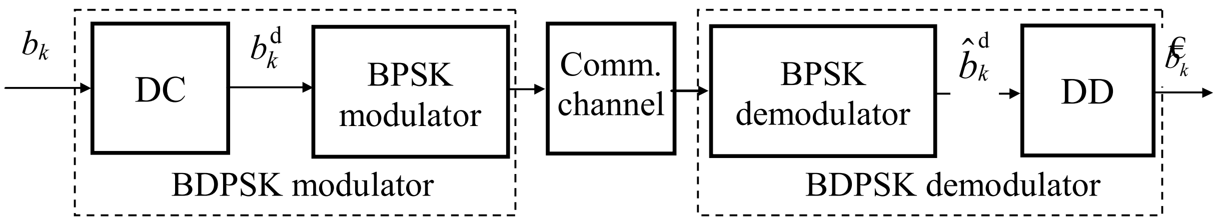



**Figure 29 –** The principle of BDPSK signal modulation and demodulation

The initial phase of the recovered carrying oscillation at the demodulator can coincide with the initial phase of the demodulated BPSK signal or differ from it in the angle π. In general terms, we can write that the phase of the reference signal is $p \cdot \pi$, where $p = 0$ or 1 is a value, which describe the phase shift. Assuming that there is no noise in the communication channel, the symbols in the output of BPSK demodulator will be determined by the ratio

$$\hat{b}_k^{\mathrm{d}} = b_k^{\mathrm{d}} \oplus p \tag{87}$$

for all $k$. Substituting (87) into formula (85), is easy to see that bit $\hat{b}_k$ does not depend on $p$.

Example of coding and decoding of an arbitrary sequence of bits is shown in Table 3. Table illustrates the coding, starting with $k = 1$. As coding on the $k$-th interval of the pulse participates previous encoded symbol, in the second row of arbitrary decision $b_0^{\mathrm{d}} = 0$. Row 3 repeats the row 2 – demodulation without inverse work. The result of decoding is shown in row 4. Row 5 contains the inversion row 2 – demodulation inverse work. After decoding the reconstructed signal (row 6) coincides with the original signal (row 1). Thus, the transfer with a differential coding eliminates inverse work of BPSK demodulator.

**Table 3 –** Example of difference encoding and decoding

| row № | | $k$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | $b_k$ | | 1 | 1 | 0 | 0 | 1 | 0 |
| 2 | $b_k^{\mathrm{d}}$ | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 3 | $\hat{b}_k^{\mathrm{d}}$ | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 4 | $\hat{b}_k$ | | 1 | 1 | 0 | 0 | 1 | 0 |
| 5 | $\hat{b}_k^{\mathrm{d}}$ | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 6 | $\hat{b}_k$ | | 1 | 1 | 0 | 0 | 1 | 0 |

In a case of $M$-ary phase modulation for the same reason as in the case of BPSK, there is phase uncertainty of $M$-th order. Similarly BDPSK information appears in the phase difference between adjacent pulses and it is called $M$DPSK. At $M > 2$ differential encoder and decoder work with $M$-ary symbols. The transition from bit of digital signal to the $M$-ary channel symbols takes place in the mapper.

Four elementary signals are used at QPSK signals transmission

$$s_i(t) = aA(t)\sqrt{2}\sin\left(2\pi f_0 t + q_i \frac{\pi}{2} + \frac{\pi}{4}\right), \tag{88}$$

where $q_i$ are quaternary symbols that take values 0, 1, 2, 3;

$q_i \frac{\pi}{2} + \frac{\pi}{4}$ are initial phases of elementary signals that take values π/4, 3π/4, 5π/4, 7π/4.

The initial phase of recovery reference oscillation in the demodulator cannot be defined uniquely; it is determined to π/2. This is due to the symmetry of the QPSK signal constellation: demodulator does not know which of the four signals considered "zero".

To eliminate the influence of the phase uncertainty of the reference oscillations at QPSK signal demodulation, go to modulation QDPSK. The principle of modulation and demodulation of signals QDPSK is displayed in Fig. 32. QDPSK signal modulator consists of a mapper, differential encoder and the QPSK signal modulator, and the QDPSK signal demodulator consists of a QPSK signal demodulator, differential decoder and the mapping decoder. In this figure, and hereafter the subscript $k$ specifies the number of clock interval, and the shift $q$ can take on a value 0, 1, 2 and 3.

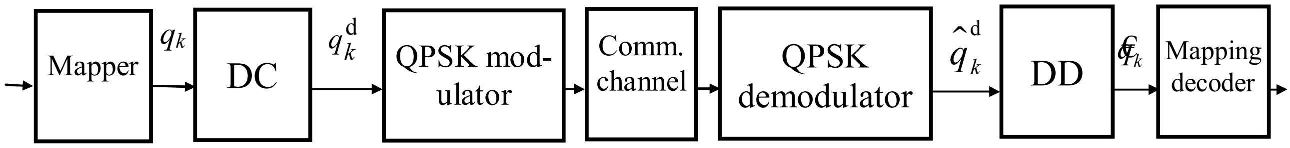



**Figure 30 –** Transmission system with QDPSK signal

Differential QDPSK coder implements the coding rule of quaternary symbols:

$$q_k^{\mathrm{d}} = q_k \oplus q_{k-1}^{\mathrm{d}}, \tag{89}$$

where $\oplus$ – mod 4 plus sign (remainder of division by 4 arithmetic sum of the parts).

Differential decoder implements the decoding rule of quaternary symbols:

$$\hat{q}_k = \hat{q}_k^{\mathrm{d}} \ominus \hat{q}_{k-1}^{\mathrm{d}}, \tag{90}$$

where $\ominus$ – mod 4 minus sign (remainder of division the difference by 4).

The initial phase of the recovered carrying oscillation at the demodulator can coincide with the initial phase of the demodulated QPSK signal or differ from it in the angle $p \cdot \pi/2$ ($p$ = 0, 1, 2 or 3 is a value which describe phase shift). Assuming that there is no noise in the communication channel, the symbols in the output of QPSK demodulator will be determined by the ratio

$$\hat{q}_k^{\mathrm{d}} = q_k^{\mathrm{d}} \oplus p \tag{91}$$

for all $k$. Thus, because of the phase uncertainty of coherent oscillations in the QPSK signal demodulators all the symbols $\hat{q}_k^{\mathrm{d}}$ get increment $p$. Substituting (91) into formula (90), is easy to see that symbol $\hat{b}_k$ does not depend on $p$.

At QDPSK because of subtraction in the decoder, the value $p$ (i.e. phase uncertainty) is excluded. As for the exclusion phase uncertainty value $\hat{q}_k$ defined as the difference between two next symbols, the encoding value $q_k^{\mathrm{d}}$ formed as the sum of the previous value $q_{k-1}^{\mathrm{d}}$ and transmitted symbol $q_k$.

Rules of addition on mod 4 are shown in table 4, and the rules of subtraction on mod 4 are shown in table 5.

Digital QPSK signal transmission the transition from pair of bits $b_1b_2$ to quaternary symbols $q$ at each clock interval the modulation carried out according to Gray code, an example of which is shown in table 6.

Since at the QPSK signal demodulation the most likely error is its transitions to the nearest signal, then at Gray code using such transitions result in error in only one bit and it minimizes the bit error probability.

**Table 4 –** Addition on mod 4 ($a \oplus b$)

| ***a*** | ***b*** | | | |
|---|---|---|---|---|
| | 0 | 1 | 2 | 3 |
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 2 | 3 | 0 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 0 | 1 | 2 |

**Table 5 –** Subtraction on mod 4 ($a \ominus b$)

| ***a*** | ***b*** | | | |
|---|---|---|---|---|
| | 0 | 1 | 2 | 3 |
| 0 | 0 | 3 | 2 | 1 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 1 | 0 | 3 |
| 3 | 3 | 2 | 1 | 0 |

**Table 6 –** Mapping Gray code

| Pair of bits $b_1b_2$ | Quaternary symbol $q$ | Initial signal phase |
|---|---|---|
| 00 | 0 | 45° |
| 10 | 1 | 135° |
| 11 | 2 | 225° |
| 01 | 3 | 315° |

In table 7 an example of coding and decoding digital signal transmission by QDPSK method is shown. The transition from pair of bits to quaternary symbols is carried out according to table 6. It is assumed that the value $p$ = 3, and $q_0^{\mathrm{d}} = 1$. From the data in table 7 that received bits coincide with the transmitted bits.

**Table 7 –** Example of difference encoding and decoding of quaternary symbols

| Number of clock interval $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Sequence of transmitted bits | | 01 | 00 | 11 | 01 | 10 | 11 | 10 | 00 |
| Sequence of symbols $q_k$ | | 3 | 0 | 2 | 3 | 1 | 2 | 1 | 0 |
| Sequence of symbols $q_k^{\mathrm{d}}$ | 1 | 0 | 0 | 2 | 1 | 2 | 0 | 1 | 1 |
| Sequence of symbols $\hat{q}_k^{\mathrm{d}}$ | 0 | 3 | 3 | 1 | 0 | 1 | 3 | 0 | 0 |
| Sequence of symbols $\hat{q}_k$ | | 3 | 0 | 2 | 3 | 1 | 2 | 1 | 0 |
| Sequence of received bits | | 01 | 00 | 11 | 01 | 10 | 11 | 10 | 00 |

Assume that because of noise BPSK demodulator makes a incorrect decision on the $k$-th clock interval. Each symbol that comes to the differential decoder input, at the decoding is used twice – on the $k$-th and ($k$ + 1)-th clock interval. Therefore, if the demodulator decisions on ($k$ – 1)-th and ($k$ + 1)-th clock intervals are correct, then on the differential decoder output will be two erroneous bits. Thus, the difference decoder multiplies errors.

The bit error probability of the BDPSK and QDPSK transmission methods and for small values of error probability ($p$ << 1) can be written

$$p_{\mathrm{BDPSK}} = p_{\mathrm{QDPSK}} = 2Q(\sqrt{2}h_b)\,. \tag{92}$$

## 12 INCOHERENT DEMODULATION OF DIGITAL MODULATION SIGNALS

Until now the demodulators schemes that implements coherent demodulation are considered. Demodulation of signals is called incoherent if the demodulator does not use information about the initial phases of demodulated signals. Obviously, it is possible, at least, at the BASK and BFSK signal demodulation, in which information is built into the amplitude and frequency of pulses, respectively, but not in the initial phases of pulses.

The purpose of incoherent demodulation usage is simplification of the demodulator scheme due to absence of carrier recovery of reference oscillations. Thus, in a case of BASK signal demodulator scheme has the form in Fig. 31.

The principal difference between incoherent demodulator scheme (Fig. 31) and coherent is the absence of a synchronous detector, which contains a carrier recovery of reference oscillations. Envelope detector (ED) is used instead of the synchronous detector. At the analog circuit implementation the ED was carried out very simply (Fig. 32).

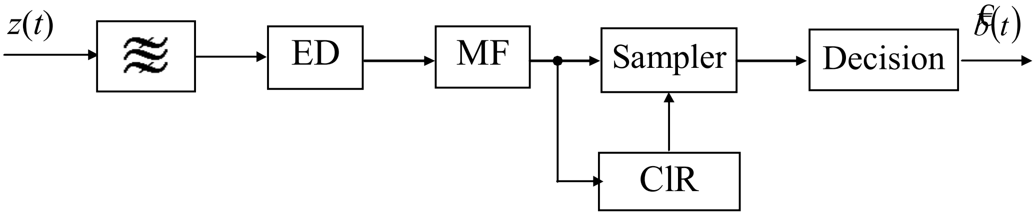



**Figure 31 –** Scheme of incoherent BASK signal demodulator

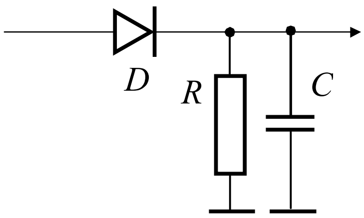



**Figure 32** – Envelope detector

At the processor implementation the envelope is calculated as the square root of the sum of the quadrature components squares of the detected signal. Bandpass filter should pass to the detector signal and reduce noise at frequencies that are outside the frequency band of signal. Other elements of the scheme are the same as in the scheme of the coherent demodulator.

Scheme shown in Fig. 31, can be used to the $M$ASK signals demodulation, when $M > 2$, but with a boundary conditions on parameters: information coefficients $a_i$ must be nonnegative, since the output voltage ED does not depend on the phase of the radio pulse. QASK signal constellation, for which the incoherent demodulation is possible, is shown in Fig. 33.

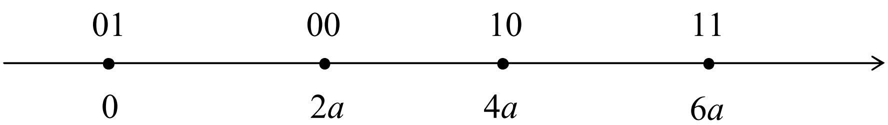



**Figure 33** – QASK signal constellation

Analysis of noise immunity transmission systems with incoherent demodulation is difficult, because the noise distribution on the ED output is significantly different from normal. Definitely they can say that the signal/noise ratio at the output ED is in 2 times less than on the synchronous detector output. The final formula for the bit error probability on the demodulator output

$$p_{\text{BASK inc}} = \frac{1}{2}\exp\left(-h_b^2/2\right). \tag{93}$$

Channel symbols (radio pulses) at BFSK are different in frequencies:

$$\begin{aligned} s_1(t) &= a\sqrt{2}A(t)\cos 2\pi f_1 t, \\ s_0(t) &= a\sqrt{2}A(t)\cos 2\pi f_0 t. \end{aligned} \tag{94}$$

In the scheme of coherent BFSK signal demodulator a synchronous detector separates the radio pulses. In the scheme of incoherent demodulator bandpass filters are used for the separation of radio pulses (Fig. 34).

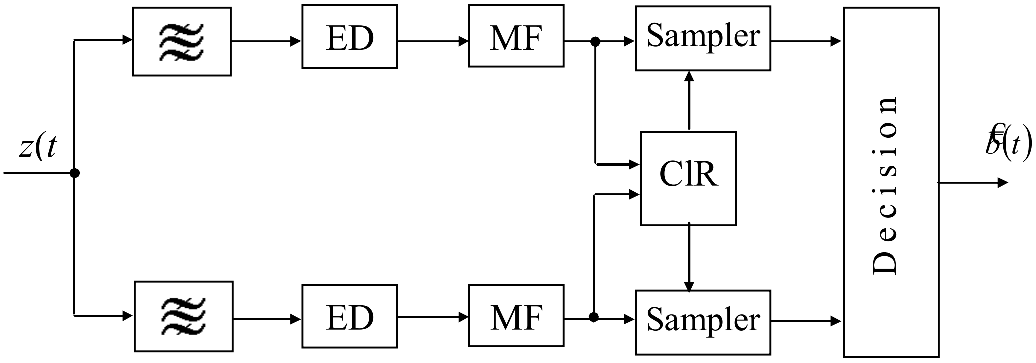



**Figure 34 –** Scheme of incoherent BFSK signal demodulator

Bandpass filters have average bandwidths $f_1$ and $f_0$. Scheme to the right of the envelope detector is the same as in the scheme of the coherent demodulator. The formula for the bit error probability on the BFSK demodulator output is the same as at BASK:

$$p_{\text{BFSK, inc}} = \frac{1}{2}\exp(-h_b^2/2). \tag{95}$$

Incoherent demodulation of BDPSK signal is possible in the sense as defined above the concept of "incoherent demodulation" at the beginning of the subsection. At the BDPSK information lies not in the initial phases of radio pulse, but in the phase difference of adjacent in time radio pulses. So we can, without knowing the initial phases of radio pulses, perform demodulation by comparing the phases of adjacent radio pulses. BDPSK signal incoherent demodulator scheme is shown in Fig. 35.

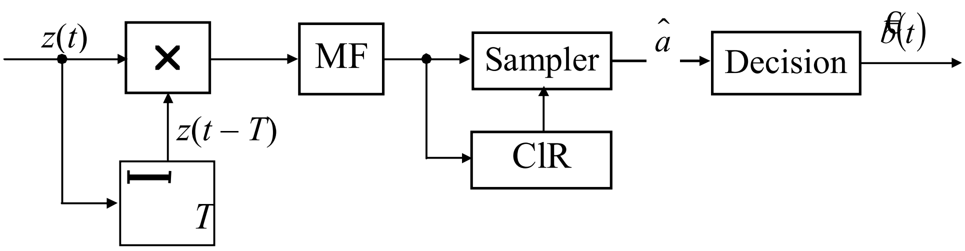



**Figure 35 –** Scheme of incoherent BDPSK signal demodulator

Delay circuit at the time of clock interval is used in the demodulator scheme. We can assume that the synchronous detector is made on multiplier, where the reference oscillation is a previous radio pulse. The polarity of the voltage on the multiplier output (and on the outputs of next blocks) depends on the phase difference oscillations, which are multiplied:

case 1 – the same phases of next pulses:

$$a\sqrt{2}A(t)\cos 2\pi f_0 t \cdot a\sqrt{2}A(t-T)\cos 2\pi f_0 t = a^2 A(t)A(t-T) + a^2 A(t)A(t-T)\cos 2\pi 2 f_0 t;$$

case 2 – phases of next pulses differs on $\pi$:

$$-a\sqrt{2}A(t)\cos 2\pi f_0 t \cdot a\sqrt{2}A(t-T)\cos 2\pi f_0 t = -a^2 A(t)A(t-T) - a^2 A(t)A(t-T)\cos 2\pi 2 f_0 t.$$

The first terms provide on the MF output voltage, the polarity of which depends on the same or different phase of the multiplied oscillations. From the above it is evident that:

1) rule of decision calculating:

$$\text{if } \hat{a} > 0, \text{ then } \hat{b}_i = 0,$$
$$\text{if } \hat{a} < 0, \text{ then } \hat{b}_i = 1;$$

2) differential decoder is not required.

The distribution of instantaneous values of noise on the multiplier output is no Gaussian because of presence of a component generated by the product of the noise $n(t)$ and $n(t-T)$. Analysis of error probability is difficult. The final formula for the bit error probability on the demodulator output has the form:

$$p_{\text{BDPSK, inc}} = \frac{1}{2}\exp(-h_b^2). \tag{96}$$

Note that the following terminology is widely used: incoherent BDPSK signal demodulation is called the method of phase comparison reception, coherent BDPSK signal demodulation is called the method of polarities comparison reception.

Let's compare the noise immunity of the coherent and incoherent binary signals demodulators. We use dependence $p = f(h_b^2)$ for comparison (Fig. 36).

The figure shows that passing from incoherent to coherent demodulation at keeping the error probability it is necessary to increase the signal/noise ratio by about 1 dB.

Incoherent demodulation is widely used for many decades, when the hardware was carried out on the vacuum tube and transistor. Now the incoherent demodulation is used, if the communication channel is rapidly changing the initial phase of the signal, for example, radio communication between objects that are moving relatively quickly.

## 13 CLOCK RECOVERY SYSTEMS

In the digital signal demodulator ClR system is designed to build sampling clock pulses, which provide the capture of signal samples at the output of matched filter at the time of maximum signal/noise ratio.

In demodulators of digital signals matched filtering of channel symbol $A(t-\tau)$ takes place (Fig. 37). Delay of channel symbols $\tau$ depends on the length of the transmission line. The delay is a random (unknown) value. Under such conditions, there is a delayed on time τ pulse $P(t - \tau)$ on the MF output, maximum of which $P(0)$ is observed at unknown moment of time $t = \tau$.

ClR system contains a clock generator, which generates a short pulse $д(t-\tau_G)$ at every clock interval duration $T$, closing the key at the moment $t = \tau$. But the impulse is formed in an unknown arbitrary time $t = \tau_G$. As a result, the sample of pulse $P(t-\tau)$ not be maximal, since $\tau_G \neq \tau$, therefore $P(\tau - \tau_G) < P(0)$. In addition, there is intersymbol interference, as the samples are taken not at the times when $P(t) = 0$.

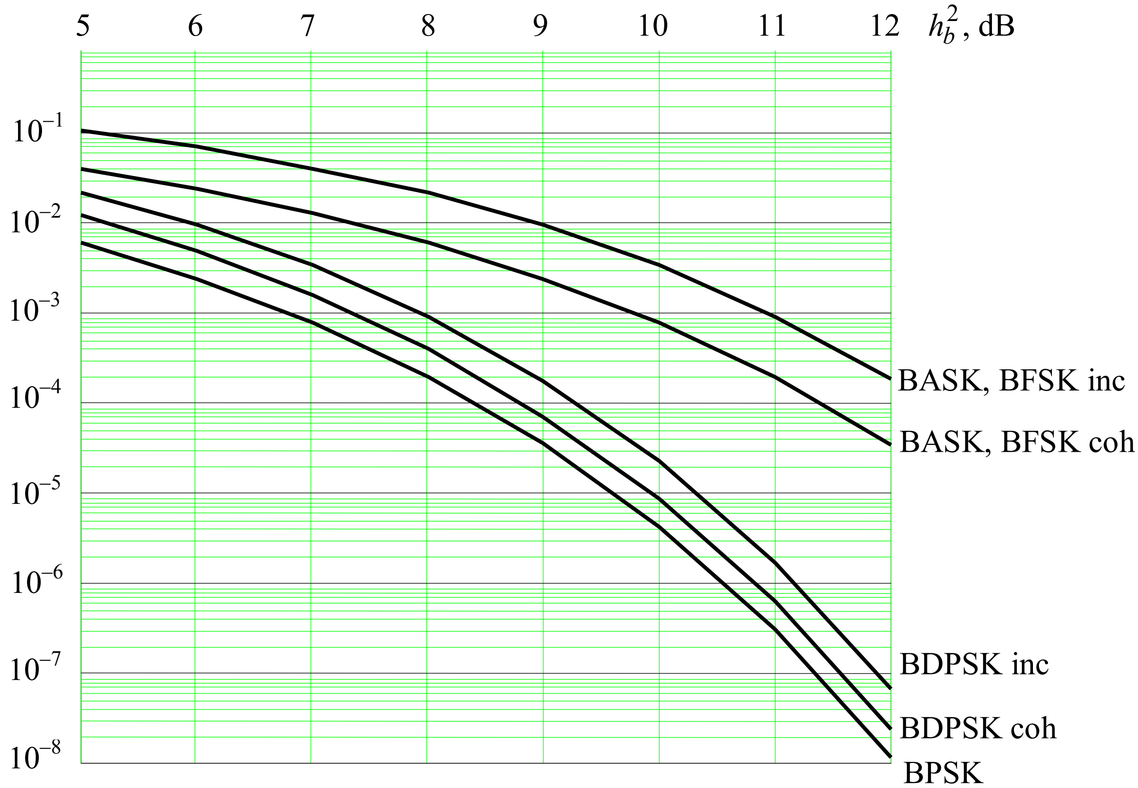



**Figure 36 –** Noise immunity of binary signals demodulators

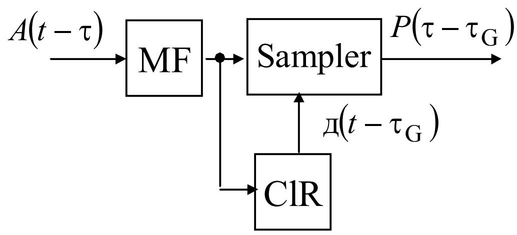



**Figure 37 –** Signal processing by matched filter

System ClR must ensure the correct choice of the sample moments. Modern demodulator implemented on the processors, this problem is solved by adjusting of the clock generator by error signal, formed by a special detector. That is, the basis for constructing a ClR system is a PLL system (Fig. 26).

All principles of ClR systems construction are based on the properties of the digital signal to change sign. A fragment of the signal on the MF output in the symbols 1 and 0 transmission is shown in Fig. 38, which implies that the change in the sign of the signal occurs approximately midway between the moments in which it is necessary to take samples.

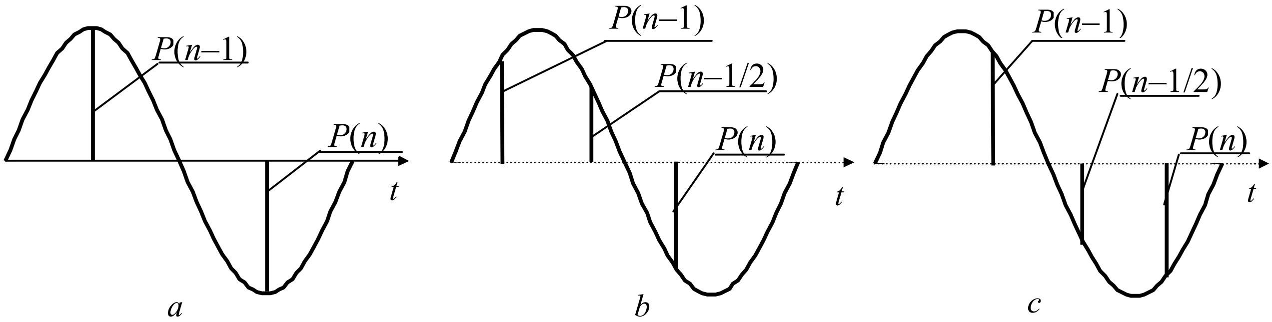



**Figure 38** – Signal sampling after MF at ClR system working

Fig. 38, *a* shows the case where sample pulses are installed correctly. Samples are taken at the moments of maximum signal. This figure clearly shows that the digital signal is zero midway between the moments of taking samples. Detector of ClR system errors can be constructed based on the following principle. At each clock interval pulse is taken in the middle of the clock interval $P(n-1/2)$. If $P(n-1/2)=0$, then samples, which will be made to the decision taken in the best moments, otherwise an ClR system error signal is formed:

$$\varepsilon(n) = P(n-1/2)\{\mathrm{sign}[P(n)] - \mathrm{sign}[P(n-1)]\}, \tag{97}$$

where $P(n)$ is sample, which decision is made on this the clock interval;

$P(n-1)$ is sample, which decision is made on previous the clock interval;

$\mathrm{sign}[x] = \begin{cases} +1, & x > 0, \\ 0 & x = 0, \\ -1 & x < 0 \end{cases}$ is sign determining function.

Need of multiplier $\{\mathrm{sign}[P(n)] - \mathrm{sign}[P(n-1)]\}$ in terms of formation of the error signal (97) due to the following: if a digital signal does not change sign, then $P(n-1/2) \neq 0$, even if the samples $P(n-1)$ and $P(n)$ taken in the best moments of samples. Therefore, in such situation it is necessary that the signal $P(n-1/2)$ is not used for error calculation. Indeed, if the signs of samples $P(n-1)$ and $P(n)$ identical, then their difference is equal to zero and sample $P(n-1/2)$ is not used.

Algorithm for error calculating (97) was obtained by Gardner. It corresponds to the ClR system, represented by Fig. 39. On the diagram ED is error detector. The scheme takes into account that the frequency of taking samples twice the clock frequency.

An examination of the ClR scheme shows that the adjustment of the generator will be implemented only if the digital signal transitions from 1 to 0 and from 0 to 1. Therefore, the formation of the transmitted digital signal it is necessary to eliminate long sequences of identical symbols. To achieve this, the transmitted digital signal is converted into a special device – a signal scrambler with the characteristics of a random digital signal or use of a linear AMI code.

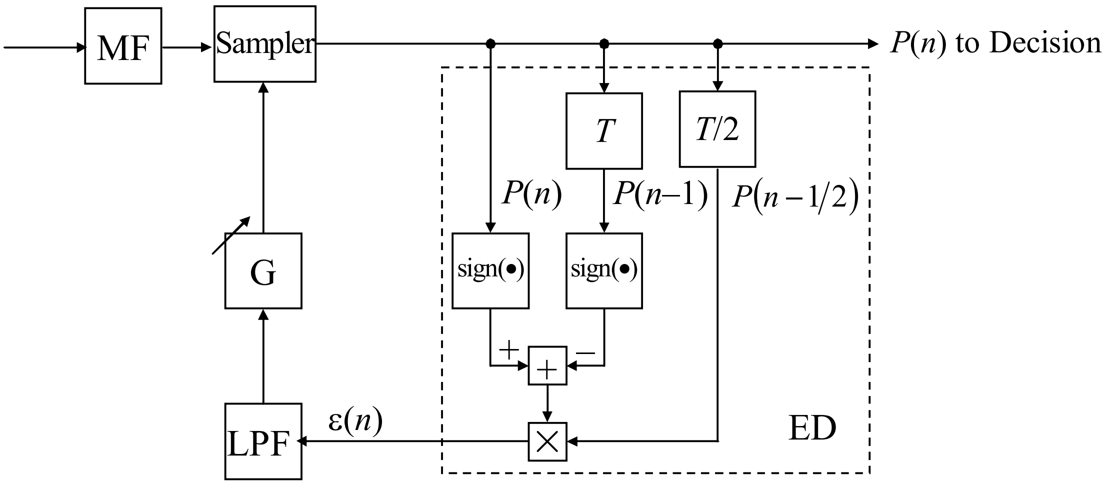



**Figure 39 –**ClR system with Gardner's error detector

## 14 NOISE IMMUNITY IN CHANNELS WITH VARIABLE PARAMETERS

A typical model of the radio channel in mobile communication systems and wireless access is a channel with variable parameters

$$z(t) = k(t)\cdot s(t) + n(t), \tag{98}$$

where $n(t)$ is additive noise, we believe, as before, that AWGN;

$k(t)$ is multiplicative noise, it plays the role of transfer coefficient of communication channel; mathematical function $k(t)$ is random function which varies slowly compared with changes in the signal $s(t)$ parameters.

They say that the relation (98) is a model of the fading channel (fading is the random variation of the signal level at the output of a communication channel). Multipath propagation of radio waves is a reason of fading. In the radio antenna receives two or more copies of the signal $s(t)$, past different ways; copies have different phases, as they were different distances. Adding, we can give a copy of an increase or attenuation of signal $s(t)$ depending on the difference of their phases.

Statistical characteristics is used to describe the function $k(t)$. It is important for us to know the probability distribution of values $k$. Most believe that $k(t)$ has Rayleigh distribution

$$p(k) = \begin{cases} \dfrac{2k}{\overline{k^2}} \exp\left(-\dfrac{k^2}{\overline{k^2}}\right), & k \ge 0, \\ 0, & k < 0, \end{cases} \tag{99}$$

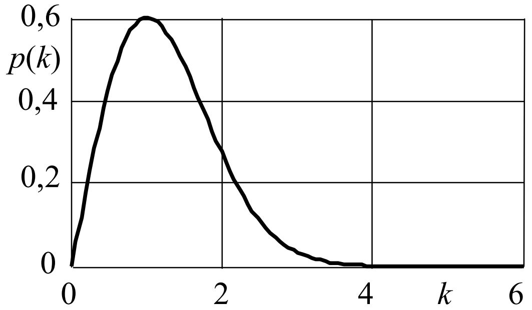



**Figure 40 –** Rayleigh distribution ( $\overline{k^2} = 1$)

where $\overline{k^2}$ means a square of the transmission coefficient $k$.

Dependence (99) with $\overline{k^2} = 1$ shown in Fig. 40. The graph shows that there may be situations when the signal is close to zero. Because of these situations, the problem is the recovery of the carrier signal, so demodulators are realized incoherent reception typically in such communication channels.

Signal/noise ratio depends on the transmission coefficient of communication channel, so the value of $h_b$ is random with the same distribution as the $k$ value:

$$p(h_b)=\begin{cases}\dfrac{2h_b}{\overline{h_b^2}}\exp\left(-\dfrac{h_b^2}{\overline{h_b^2}}\right), & h_b \ge 0,\\ 0, & h_b < 0.\end{cases} \tag{100}$$

Since $h_b$ changes, then we can say about the conditional error probability. Thus, in the case of BFSK

$$P_{\text{er}}(h_b)=\frac{1}{2}\exp(-h_b^2/2). \tag{101}$$

Define the unconditional bit error probability

$$\begin{aligned}\overline{P_{\text{er}}} &= \int_0^\infty p(h_b)P_{\text{er}}(h_b)dh_b = \int_0^\infty \frac{2h_b}{\overline{h_b^2}}\exp\left(-\frac{2h_b}{\overline{h_b^2}}\right)\frac{1}{2}\exp\left(-\frac{h_b^2}{2}\right)dh_b = \\ &= \frac{1}{\overline{h_b^2}}\int_0^\infty h_b\exp\left(-h_b{}^2\cdot\frac{2+\overline{h_b^2}}{2\overline{h_b^2}}\right)dh_b = \frac{1}{\overline{h_b^2}}\left(-\frac{1}{2}\frac{2\overline{h_b^2}}{2+\overline{h_b^2}}\exp\left(-h_b^2\cdot\frac{2+\overline{h_b^2}}{\overline{h_b^2}}\right)\right)\Bigg|_0^\infty = \frac{1}{2+\overline{h_b^2}}.\end{aligned} \tag{102}$$

Compare the noise immunity of the reception in a channel with constant parameters – the formula (101) – and in a channel with variable parameters – the formula (102).

To achieve the error probability $10^{-6}$ in a channel with constant parameters required $h_b^2 = 26{,}2$ or 14,2 dB, and in the channel with variable parameters $\overline{h_b^2} = 10^6$ or 60 dB, i.e. signal/noise ratio must be increased in 40000 times.

Similarly analyze the noise immunity of BDPSK signal reception in a channel with constant parameters

$$P_{\text{er}}(h_b)=\frac{1}{2}\exp\left(-h_b^2\right); \tag{103}$$

in a channel with variable parameters

$$\overline{P_{\text{er}}}=\frac{1}{2+\overline{h_b^2}}. \tag{104}$$

## 15 NON-OPTIMAL DEMODULATORS

We have considered the scheme of optimal demodulators and their noise immunity in all the previous sections. Recall that the criterion of demodulation optimality is the minimum error probability of signal and bit. Scheme of optimal *M*PSK ($M \ge 4$), *M*APSK, *M*QAM demodulators are shown in Fig. 20. To achieve minimum error probability in this scheme the following conditions are satisfied.

1. References oscillations, which are fed to the multiplier, is in phase with the cosine and sine components of the channel symbol, because this is a complete separa-

tion of cosine and sine components in order to follow their separate treatment, and the output voltage multipliers maximum.

2. Matched filter is matched with the pulse-carrier that provides maximum signal/noise ratio at the sample moment.

3. The ClR system provides taking samples at times when the instantaneous values of pulse-carrier are maximal.

4. Due to the fact that the shaping filters of the modulator and matched filter of the demodulator have the same AR, described by the dependence of the "root of the Nyquist spectrum", and transmission line does not distort the signal at the samplers' inputs we have impulses without intersymbol interference.

5. Decision rule at the decision circuit is based on the optimal partition of the space of signals on the signal areas.

Disturbance of any of these conditions leads to deterioration of noise immunity (increasing of the error probability). As a rule, all conditions in real demodulators in one way or another are not satisfied. So they say that the real demodulator is non-optimal.

1. References oscillations are generated from the noisy signal and do not correspond exactly in phase with the cosine and sine components of the channel symbol, so there is no complete separation of the cosine and sine components, there are transient noise, and the output voltage of multipliers is not maximal.

2. Because of the finite precision implementation of shaping and matched filters, signal distortion in a transmission line the matched filters are not exactly matched with the pulse-carrier and does not provide the maximal signal/noise ratio at the sample moment.

3. Sample pulses in the ClR system are formed from the noisy signal and don't provide taking samples at the instants when the instantaneous value of the pulse-carrier is maximum.

4. Because of the finite precision implementation of shaping and matched filter, signal distortion in a transmission line the intersymbol interference takes place at the samplers' inputs.

5. In the multilevel signal's demodulator there is not optimal partition of signals space to the signals areas as demodulated signal has levels that differ from the levels for which partitioning of the signals areas was calculated. To reduce this effect, the demodulator should have an automatic control of signal gain.

It is not possible to calculate the noise immunity of non-optimal demodulator. Noise immunity of non-optimal demodulator is determined experimentally or by simulation on a computer. In Fig. 41 shows the calculated dependence of the error probability on the signal/noise ratio for optimum demodulator and obtained experimentally.

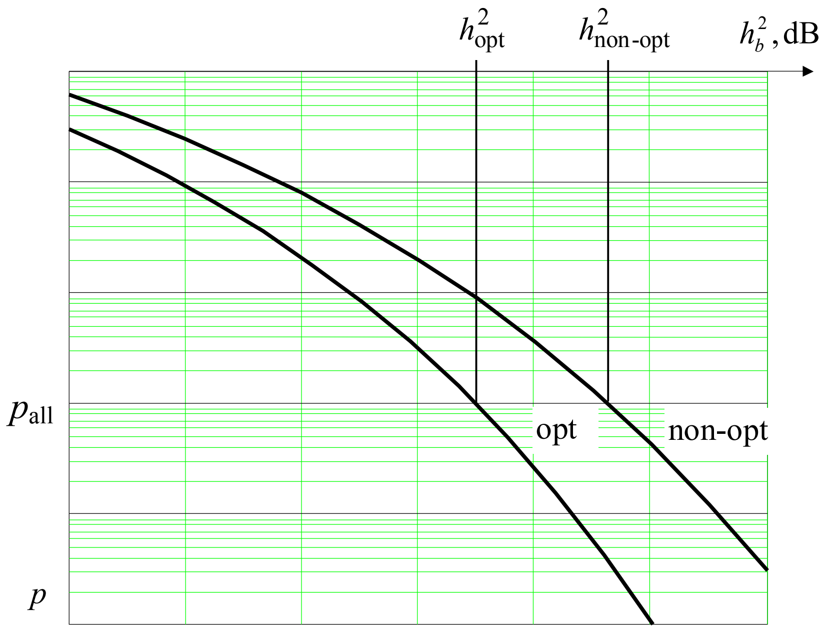



**Figure 43 –** Comparison of noise immunity of optimal and non-optimal demodulation

From the curves the value of the signal/noise ratio at the optimum reception $h^2_{\mathrm{opt}}$ and non-optimal reception $h^2_{\mathrm{non-opt}}$ can be determined. The difference between these values is called the energy losses of demodulation (EL):

$$\mathrm{EL} = h^2_{\mathrm{non-opt}} - h^2_{\mathrm{opt}}. \tag{105}$$

In modern well-implemented modems in the channel with AWGN for QPSK EL = 0,8...1,0 dB. With the increasing of signal levels number losses increases and for 64QAM, 256QAM they can reach 2 dB.

## 16 GENERAL INFORMATION ABOUT THE ANALOG DEMODULATION SIGNAL MODULATION TYPES

An analog signal is a baseband signal $b(t)$, which can be continuously variable in time and accept any form. Such signals take place, for example, in telephony, broadcast, television and telemetry. The signal $b(t)$ in these cases is beforehand unknown: it is known only, that it belongs to some great number or is realization of some random process $B(t)$. Analog signals can be transmitted *directly* or with the use of *modulation.* In first case the transmitted signal is proportional the analog signal $s(t) = kb(t)$, where $k$ is a constant factor. In second case a signal $s(t)$ is some function of analog signal $b(t)$ (see module 1).

Block-diagram of the analog transmission system is similarly as well as block-diagram of the digital transmission system (Fig. 1). A signal on the demodulator input is total waveform of the transmitted signal $s(t)$ and noise $n(t)$

$$z(t) = s(t) + n(t). \tag{106}$$

Noise $n(t)$ is realization of stationary Gaussian process with the power spectral density $N_0$ in the signal frequencies band. Out of signal frequencies band noise PSD equal to the zero.

A task consists of that, on the input signal $z(t)$ to get (to recover) a baseband signal $\hat{b}(t)$, the least different in sense of some criterion, from the transmitted signal $b(t)$. The signal $\hat{b}(t)$ reproduced with some error is named the signal $b(t)$ estimation. Thus, the task of signal $s(t)$ demodulation can be examined as a task of receipt of signal estimation $\hat{b}(t)$. In special case, when a signal $s(t)$ is the function of some time-constant parameter l, a task is taken to the signal parameter l estimation. At the direct transmission of signal $s(t)= kb(t)$ calculation of estimation $\hat{b}(t)$ is taken to signal linear filtration. At a transmission the modulated signal $s(t)$ the signal estimation $\hat{b}(t)$ in demodulator is carried out by detection and signal processing. In many cases signals processing is taken to one or another methods of filtration and can be carried out both before the detector and after it.

As measure of noise immunity at analog signals transmission can be a degree of "deviation" of the got estimation $\hat{b}(t)$ from the transmitted signal $b(t)$. An average square deviation or an average square error is usually accepted

$$\overline{\varepsilon^2(t)} = \overline{\left[\hat{b}(t) - b(t)\right]^2}, \tag{107}$$

where averaging is executed on all possible realization of signals $\hat{b}(t)$ and $b(t)$.

Difference $\varepsilon(t) = \hat{b}(t) - b(t)$ is a noise on the demodulator output. Its average square $\overline{\varepsilon^2(t)} = P_\varepsilon$ is noise power on the demodulator output. Power of the transmitted signal $P_b = \overline{b^2(t)}$ is considered set. Then it is possible to define the powers of signal and noise ratio on the demodulator output $\rho_{out} = P_b/P_\varepsilon$. Powers of signal and noise ratio on the demodulator input $\rho_{in}$ it is usually known. In many cases as a criterion of noise immunity accept not error average square $\overline{\varepsilon^2(t)}$, but signal/noise ratio $\rho_{out}$. Demodulator can improve signal/noise ratio $\rho_{in}$. This improvement depends not only on the method of the signal processing but also on the modulation type. Therefore comfortably to estimate noise immunity of the analog transmission systems by the signal/noise ratio gain

$$g = \frac{\rho_{out}}{\rho_{in}} = \frac{P_b/P_\varepsilon}{P_s/P_n}. \tag{108}$$

At $g > 1$ signal/noise ratio is improved in demodulator. On occasion $g < 1$, that means, that not gain, but losses take place at demodulation.

Other correctness criteria of continuous messages transmitting are used in practice. For example, criterion of speech intelligibility at transmitting of vocal messages, criterion of maximal error in a telemetry and other are used.

## 17 OPTIMUM DEMODULATOR OF ANALOG MODULATION SIGNALS

The set forth below going near the construction of optimum demodulator is applicable to the case of weak noise (high signal/noise ratio on demodulator input). Such situation is incident to the analog transmission systems, which is used in communication and broadcasting networks.

The criterion of optimality is a minimum of error means a square of baseband signal recovery. Demodulator of analog modulation signals necessarily contains a detector, corresponding to the used modulation type – its output is proportional the values of informative parameter of the modulated signal. For error minimization of baseband signal recovery it is necessary to execute optimum processing:

- to recover the modulated signal by a pre-detector filter;
- to weaken unwanted components, taking place on the detector output by a post-detector filter.

The analog modulation signals optimum demodulator scheme follows from said (Fig. 44)

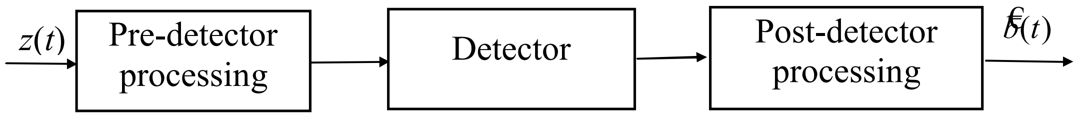



**Figure 44** – Analog modulation signals optimum demodulator scheme

As devices of pre-detector and post-detector processing, as a rule, linear filters are used, although nonlinear processing is required on occasion. It is explained, foremost, by simplicity of realization of linear filters, which comparatively synthesized easily, and there is the developed theory of their construction, what cannot be said about nonlinear filters.

## 18 OPTIMUM LINEAR FILTRATION OF CONTINUOUS SIGNALS

We will consider the theory of optimum linear filtration – Kolmogorov-Wiener filter or OLF.

Let on the linear filter input the sum of useful signal $s(t)$ and noise $n(t)$ operates (106) with the transfer function $H(j\omega)$. It is required to find such function $H(j\omega)$, which provides a minimum of error mean square of recovered signal

$$\overline{\varepsilon^2(t)} = \overline{\left[\hat{s}(t) - s(t)\right]^2}, \tag{109}$$

where $\hat{s}(t)$ is an estimation of signal on the filter output.

We will suppose that $s(t)$ and $n(t)$ – stationary uncorrelated processes with the known power spectral density (PSD) $G_s(f)$ and $G_n(f)$. In such raising a task was decided A.N. Kolmogorov (1939) for discrete random sequences and N. Wiener (1941) for continuous processes. Therefore an optimum (in the indicated sense) linear filter is called the Kolmogorov-Wiener filter.

Operating on the filter input the signal $s(t)$ and noise $n(t)$ is passed through a filter independently and create on the filter output accordingly filtered signal $s^f(t)$ and noise $n^f(t)$. Taking into account it an error on the filter output will be written down

$$\varepsilon(t) = \hat{s}(t) - s(t) = \left(s^f(t) - s(t)\right) + n^f(t) = \Delta s(t) + n^f(t). \quad (110)$$

Element $\Delta s(t)$ reflects the error component, conditioned linear distortions of useful signal by a filter. Error mean square $\overline{\varepsilon^2(t)}$ equals

$$\overline{\varepsilon^2(t)} = \overline{\Delta s^2(t)} + \overline{\left(n^f(t)\right)^2}. \quad (111)$$

The size of linear distortions of useful signal by a filter depends on the degree of difference of filter AR from a constant value and degree of PR difference from linear dependence. The noise mean square on the filter output depends only of filter AR. In order that linear distortions of useful signal were minimum will accept filter PR linear

$$\varphi(\omega) = -\omega t_0, \quad (112)$$

where $t_0$ is a delay of signal in a filter.

We will pass to determination of filter AR. For this purpose we will define the power spectral density of the formula left and right parts (110)

$$G_\varepsilon(\omega) = G_{\Delta s}(\omega) + G_{nf}(\omega). \quad (113)$$

We will give components in right part correlations (113) through signal $s(t)$ and noise $n(t)$ PSD and necessary filter AR:

$$G_{nf}(\omega) = G_n(\omega) H^2(\omega); \quad (114)$$

$$G_{\Delta s}(\omega) = S_{\Delta s}^2(\omega), \quad (115)$$

where $S_{\Delta s}(\omega)$ is an error amplitude spectrum $\Delta s(t)$;

$$S_{\Delta s}(\omega) = S(\omega)[H(\omega) - 1], \quad (116)$$

where $S(\omega)$ is a signal amplitude spectrum $s(t)$;

$$G_{\Delta s}(\omega) = G_s(\omega)\left[H(\omega) - 1\right]^2. \quad (117)$$

After the substitution of relations (114) and (117) in (113) will get

$$G_\varepsilon(\omega) = G_s(\omega)\left[H(\omega) - 1\right]^2 + G_n(\omega) H^2(\omega). \quad (118)$$

The recovery error mean square (average power) is calculated as

$$\overline{\varepsilon^2(t)} = P_\varepsilon = \frac{1}{\pi}\int_0^\infty G_\varepsilon(\omega) d\omega. \quad (119)$$

As function $G_\varepsilon(\omega) \ge 0$ on all frequencies, then, providing min $G_\varepsilon(\omega)$ on all frequencies, we will attain a minimum of size $\overline{\varepsilon^2(t)}$. Necessary AR $H(\omega)$ we will define from a condition

$$\frac{dG_\varepsilon(\omega)}{dH(\omega)} = 0: \quad (120)$$

$$\frac{dG_\varepsilon(\omega)}{dH(\omega)} = 2G_s(\omega)[H(\omega)-1] + 2G_n(\omega)H(\omega) = 0\,. \qquad (121)$$

After the decision of equalization (121) will get expression for filter AR

$$H(\omega) = \frac{G_s(\omega)}{G_s(\omega) + G_n(\omega)}\,. \qquad (122)$$

On a Fig. 45 OLF AR, determined relation (122) is illustrated.

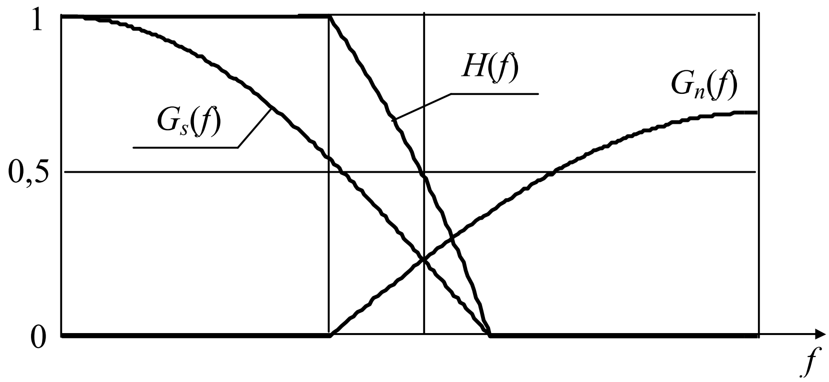



**Figure 45** – Illustration of OLF AR

From a Fig. 45 the features of OLF AR are visible:

– in frequencies band, where $G_n(f) \equiv 0$, AR value $H(f) = 1$ – in this bandwidth a filter must not bring in distortions;

– in frequencies band, where $G_s(f) \equiv 0$, AR value $H(f) = 0$ – in this bandwidth a filter must fully weaken noise;

– on frequency on which $G_s(f) = G_n(f)$, AR $H(f) = 0{,}5$;

– on other frequencies AR value determined by calculations on a formula (122).

If to put expression (122) in relation (118), will get expression for error PSD

$$G_\varepsilon(\omega) = \frac{G_s(\omega)G_n(\omega)}{G_s(\omega) + G_n(\omega)}\,. \qquad (123)$$

At the substitution of relation (123) in expression (119) it is possible to calculate the recovery error mean square of signal $\overline{\varepsilon^2(t)}$.

Easily to notice that error $\overline{\varepsilon^2(t)} = 0$ only in that case, when $G_s(f)\cdot G_n(f) = 0$, i.e. when the signal and noise spectrums of are not overlapped. In all other cases an optimum filter transmits the oscillations of different frequencies with that less weight, than relation $G_n(f)/G_s(f)$ more on this frequency.

It is necessary to notice that optimum linear filters, determined equation (122), substantially differ from the matched filters, considered above. If the basic setting of the filters examined here consists of the best reproducing of signal form, the task of the matched filters consists in forming of maximal signal/noise ratio in the sampling moment.

At the use of OLF as a device of pre-detector processing such feature comes to demodulators of the analog transmission systems. The high ratio of signal and noise spectral densities takes place $G_s(f)/G_n(f) >> 1$. Expression for OLF AR (122) passes to the following

$$H(f) = \begin{cases} 1, & f_{\min} \le f \le f_{\max}, \\ 0, & f < f_{\min},\ f > f_{\max}, \end{cases} \tag{124}$$

where $f_{\min}$ and $f_{\max}$ are a scope frequencies of signal spectrum.

Thus, the device of pre-detector processing is a bandpass filter with rectangular AR.

## 19 COMPARISON OF NOISE IMMUNITY OF OPTIMUM DEMODULATORS ANALOG SIGNAL MODULATION TYPES

We found that the demodulator should contain:

– pre-detector processing filter;

– detector;

– post-detector processing filter.

Filters should be optimal for optimal demodulation. In the weak noise filters AR must be rectangular:

– pre-detector processing filter is a bandpass filter, passband edge frequency of which coincide with the boundary frequencies of the spectrum of the modulated signal;

– post-detector processing filter is a LPF, cutoff frequency of which coincides with the maximum frequency spectrum of the baseband signal $F_{\max}$.

Noise immunity defines in terms of the AWGN. Analysis of noise immunity is to determine the gain demodulator in the signal/noise

$$g = \frac{P_b / P_\varepsilon}{P_s / P_n}. \tag{125}$$

To determine the gain required to determine the 4 values: $P_b$, $P_\varepsilon$, $P_s$, $P_n$. In view of the cumbersome calculations (they can be found in textbooks on the theory of telecommunication) we will give the final expressions and compare the analog transmission system. Demodulator gain in signal/noise ratio $g$ and the coefficient of expansion of the bandwidth at modulation $\alpha = \Delta F_s/F_{\max}$ are the main parameters on which transmitting systems are compared. For the considered modulation techniques, these parameters are summarized in table 8.

Let's make a comparison of numerical values of the parameters in some initial data: $K_A = 5$; $m_{FM} = m_{PM} = 6$; $m_{AM} = 1$.

Calculations give: $g_{AM} = 0{,}038$; $g_{DSB\text{-}SC} = 2$; $g_{SSB} = 1$; $g_{FM} = 60{,}5$; $g_{PM} = 20{,}2$.

Comparison of the numerical values of the gain shows that the lowest noise immunity has transmission system with AM: gain $g_{AM} << 1$, logically be called a losses. However, AM is used in the broadcasting system, where this deficiency is compensated by the simplicity of the demodulator on the basis of the envelope detector (a huge number of more simple radios and a radio transmitter with more power than at DSB-SC or SSB using).

**Table 8** – The main parameters of analog transmission systems

| **Modulation method** | ***g*** | **α** | **Notes** |
|---|---|---|---|
| AM | $\frac{2m_{AM}^2}{m_{AM}^2 + K_A^2}$ | 2 | Synchronous detector |
| | $\frac{m_{AM}^2}{m_{AM}^2 + K_A^2}$ | 2 | Envelope detector |
| DSB-SC | 2 | 2 | |
| SSB | 1 | 1 | |
| FM | $\frac{3m_{FM}^2}{K_A^2} \cdot \alpha$ | $2(m_{FM} + 1)$ | $c_{in} \geq c_{thr}$ |
| PM | $\frac{m_{PM}^2}{K_A^2} \cdot \alpha$ | $2(m_{PM} + 1)$ | $c_{in} \geq c_{thr}$ |

The highest noise immunity has a transmission system with FM. "Payment" for high noise immunity is a wide bandwidth signal. Thus, when $F_{max}$ = 3,4 kHz $\Delta F_{FM}$ = = 47,6 kHz, while the bandwidth of the signal SSB $\Delta F_{SSB}$ = 3,4 kHz.

**The threshold of noise immunity of the FM signals demodulator.** From the relation of FM demodulator gain (table 8) follows that the larger index $m_{FM}$, the larger gain (though at the price of signal bandwidth increasing). It makes you think that this gives the opportunity to work demodulators with a weak signal (low signal/noise ratio). One should take into account the phenomenon of the threshold of noise immunity. It is as following: if the signal/noise ratio at the input of the demodulator $\rho_{in}$ is higher than threshold signal/noise ratio $\rho_{thr}$, demodulator that provides high gain, defined by the relation given in table 8, but if $\rho_{in} < \rho_{thr}$, then gain decreases sharply, and this range of values $\rho_{in}$ is nonworking.

In the demodulator, performed on the basis of the standard frequency detector $\rho_{thr}$ =10 dB. So-called circuit threshold-extension FM demodulator has been proposed, which are called:

– demodulator with tracking filter;
– demodulator with frequency feedback;
– demodulator on the basis of synchronous phase detector.

In the demodulator threshold signal/noise ratio, depending on the initial data on the transmission system may be 5...7 dB. Reduction in the $\rho_{thr}$ allows:

1) to operate demodulators with lower signal/noise ratio;
2) increase the gain, as

$$c_{\text{in}} = \frac{P_s}{N_0 \Delta F_{\text{FM}}} = \frac{P_s}{N_0 2F_{\max}\left(m_{\text{FM}} + 1\right)}, \tag{126}$$

and assuming a decrease signal/noise ratio $\rho_{\text{in}}$, is possible to increase the index $m_{\text{FM}}$, and increase $m_{\text{FM}}$ leads to an increase in gain.

## ATTACHMENT A. METHODICAL MANUAL FOR THE COURSE WORK

Topic of the course work **«Calculation of a noise immunity of the digital signal demodulator»**

The digital signal is transmitted by the modulated signal on a communication channel with constant parameters and additive white gaussian noise.

**It is given:**

- the digital signal rate $R$;
- the type of modulation;
- the allowable probability of a bit error on an output of the demodulator $p_{\text{all}}$;
- the roll-off factor of a spectrum $\alpha$.

**It is necessary:**

1. To paint the block diagram of the digital transmission system (DTS). To specify a place of the modulator and the demodulator inclusion. To explain assignment of separate blocks.
2. To draw signal constellation of the given method of modulation and to give a mapping code.
3. To paint the block diagram of the demodulator of a given modulation method, to explain assignment of separate blocks.
4. To calculate and graph of dependence of bit error probability from the signal/noise ratio on an input of the demodulator $p = f(h_b^2)$ for the given modulation method and for QPSK.
5. To define necessary signal/noise ratio $h_b^2$ on inputs of demodulators of the given modulation method and QPSK for maintenance of allowable bit error probability on an output of the demodulator $p_{\text{all}}$, to show them in figure $p = f(h_b^2)$, to find their difference.
6. To calculate necessary bandwidth of a communication channel $F_{\text{ch}}$, necessary for transmission of the given method modulation signal and a QPSK signal.
7. To draw conclusions concerning an exchange of a necessary bandwidth for the necessary signal/noise ratio at transition from QPSK to the given modulation method.
8. To define the necessary ratio of average powers of a signal and noise $P_s/P_n$ in a signal bandwidth on an input of the demodulator of the given modulation method for maintenance of allowable probability bit error on an output of the demodulator $p_{\text{all}}$.

**Additional assign**

9. Calculate and plot the pulse-carrier at the matched filter output of demodulator, to verify the sampling conditions.
10. Calculate and plot the normalized power spectral density of the signal at the modulator output (value of the frequency of the carrier signal is given). To show the width of the signal spectrum on the graph.

**Instructions to carrying out**

Item 1: see Section 1.

Item 2: to draw signal constellation of the given modulation method, it is necessary to remember, that in case of *M*PSK signals a points of signal constellation are placed in regular intervals on a circle, and in case of *M*QAM signals are placed in points of a square lattice (see examples in figure 21); for designing of mapping code see material Module 1 – the mapping code should be a Gray code.

Item 3: see Section 8.

Item 4 and 5: see Section 9. A noise immunity of the demodulator defines with a bit error probability $p$. Bit error probability $p$ depend on a method of modulation, characteristics of a communication channel and the ratio of average energy of the modulated signal, spent on transmission of one bit, to noise power density $h_b^2 = E_b/N_0$.

In case of signals *M*PSK ($M \geq 4$), transmitting by gaussian channel with constant parameters, a bit error probability is defined [1, p. 256]

$$p = \frac{2}{n} Q\left( \sqrt{2n} \sin \frac{\pi}{M} \cdot h_b \right), \tag{1}$$

and in case of signals *M*QAM – [1, p. 586]

$$p = \frac{4\left(\sqrt{M} - 1\right)}{n\sqrt{M}} Q\left( \sqrt{\frac{3n}{M - 1}} \cdot h_б \right), \tag{2}$$

where $n = \log_2 M$;

$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-t^2/2) dt$ – gaussian $Q$-function (probability integral), it is tabulated in mathematical directories. At absence of tables it is possible to use the formula of approximation

$$Q(x) = 0{,}65 \exp\left(-0{,}44\,(x + 0{,}75)^2\right). \tag{3}$$

For the given modulation method and QPSK it is necessary to calculate and plots dependence $p = f(h_b^2)$. At a diagram the scale for $p$ should be logarithmic, and the scale for values $h_b^2$, expressed in decibels ($h_b^2$ [dB] = 10 lg $h_b^2$), should be linear. At calculations increase $h_b^2$ with step equals 1 dB, starting from 2 dB, to such value, at which $p$ does not appear less than $p_{\text{all}}$. The example of such dependence is showed in Fig. 36.

Item 6: see material Module 1. The necessary bandwidth of a communication channel for transmission of two-dimensional signals *M*PSK and *M*QAM is defined

$$F_{\text{ch}} = \frac{R(1 + \alpha)}{\log_2 M}. \tag{4}$$

Item 7: compare the received numerical values.

Item 8: Transition from $h_b^2$ to $P_s/P_n$ is carried out in view of the following ratio

$$h_b^2 = \frac{E_b}{N_0} = \frac{P_s \cdot T_b}{P_n / F_{\text{ch}}} = \frac{P_s}{P_n} \cdot \frac{F_{\text{ch}}}{R}. \tag{5}$$

Item 9: the plot of pulse-carrier at the demodulator output of matched filter $P(t)$ should be constructed according to the formula (52). To facilitate the calculations appropriate to introduce a variable $x = f_N t$ and change it with step 1/8. The calculations may be uncertain. So,

$$\frac{\sin x}{x} = 1, \text{ when } x \to 0; \quad \frac{\cos\ 2\pi\alpha f_N t}{1 - (4\alpha f_N t)^2} = \frac{\pi}{4}, \text{ when } 4\alpha f_N |t| \to 1.$$

Plot of the pulse $P(t)$ must be built using a numerical scale on the axes. Show the moments of taking samples at the time axis. Make conclusions about the absence of intersymbol interference.

Item 10: power spectral density (PSD) of the signal at the modulator output is described by the dependence of the Nyquist spectrum (formula (49)). The expression for the normalized PSD is written as: $G_s(f) = N(f - f_0)/T$, where $f_0$ is frequency of the signal carrier.

**Conclusions to a work**

To state the list of the executed calculations shortly. To specify, whether the executed calculations answer the initial data and the task. And what calculations and why do not answer the task.

**The initial data to the task on the course work, a part 1**

| | Number of the variant | $R$, kbits/s | $\alpha$ | Type of modulation | $p_{\text{all}}$ |
|---|---|---|---|---|---|
| Number of a variant for carrying out of the course work should correspond to number of a student surname in the group register | 01 | 120 | 0,25 | 8PSK | 2E–6 |
| | 02 | 320 | 0,3 | 16QAM | 1E–6 |
| | 03 | 80 | 0,35 | 16PSK | 5E–7 |
| | 04 | 120 | 0,4 | 64QAM | 2E–7 |
| | 05 | 120 | 0,2 | 8QAM | 1E–7 |
| | 06 | 240 | 0,25 | 8PSK | 5E–8 |
| | 07 | 400 | 0,3 | 16QAM | 2E–8 |
| | 08 | 800 | 0,35 | 16PSK | 1E–8 |
| | 09 | 60 | 0,4 | 64QAM | 5E–6 |
| | 10 | 120 | 0,2 | 8QAM | 2E–6 |
| | 11 | 60 | 0,25 | 8PSK | 1E–6 |
| | 12 | 240 | 0,3 | 16QAM | 5E–7 |
| | 13 | 320 | 0,35 | 16PSK | 2E–7 |
| | 14 | 180 | 0,4 | 64QAM | 1E–7 |
| | 15 | 150 | 0,2 | 8QAM | 5E–8 |
| | 16 | 180 | 0,25 | 8PSK | 2E–8 |
| | 17 | 40 | 0,3 | 16QAM | 1E–8 |
| | 18 | 320 | 0,35 | 16PSK | 5E–6 |
| | 19 | 240 | 0,4 | 64QAM | 2E–6 |
| | 20 | 240 | 0,2 | 8QAM | 1E–6 |
| | 21 | 120 | 0,25 | 8PSK | 5E–7 |
| | 22 | 60 | 0,3 | 16QAM | 2E–7 |
| | 23 | 1000 | 0,35 | 16PSK | 1E–7 |
| | 24 | 120 | 0,4 | 64QAM | 5E–8 |
| | 25 | 300 | 0,2 | 8QAM | 2E–8 |

***Explanation:*** 4E–5 means $4 \cdot 10^{-5}$.

## Attachment B. Education manual for laboratory works

### LW 3.1 Researching of matched filters

**1 Objectives**

Study and experimental verification of matched filters' properties.

**2 Main positions**

See section 4:

– definition of matched filter (MF);
– calculation signal to noise ratio at output of MF;
– definition of MF AR and MF PR;
– matched filter with rectangular videopulse;
– form of signal on the output of MF at action of an any signal on its input;
– form of signal on the output of MF at action on its input the signal which a filter is matched.

Except for the listed properties the MF has the following property. For some complex signals in which product of spectrum width on their duration $FT_s >> 1$, the correlation function $K_s(t)$ has only the "narrow" troop landing in an area around $\tau = 0$ by duration $2\tau_c \approx 1/F$ ($\tau_c$ is a correlation interval of signal). During filtration of such signals the matched filters there is a compression of signals at times ($2\tau_c << T_s$). The example of such complex signals are two-level signals, built on the basis of Barker`s sequences (codes). If such signal in a sum with a noise to skip through MF, will take a place not only maximization of signal/noise ratio but also compression of signal in time.

**3 Questions**

3.1 What filter is called matched?

3.2 What parameters of signal must be known for a MF synthesis?

3.3 Write down expressions for AR and PR of MF. Give physical interpretation for them.

3.4 How is the impulse response of MF determined?

3.5 What is the condition of physical realization of MF?

3.6 What form does have response of MF at a serve on its input of signal, which it is matched with?

3.7 How is the signal/noise ratio determined on the MF output?

3.8 Represent the chart of filter, matched with a rectangular pulse.

3.9 Explain settings of Barker`s sequences (codes) and their properties.

**4 Home task**

4.1 To study the main positions of section 4 "Matched filter".

4.2 To calculate and build the graph normalized correlation function of the set signal (tab. 1).

**Table 1 –** Basic data to the home task

| Number of brigade | Signal |
|---|---|
| 1, 7 | Videopulse, $T_s = 2$ ms |
| 2, 8 | Radiopulse with rectangular envelope, $T_s = 2$ ms, $f_0 = 1000$ Hz |
| 3, 9 | Barker`s sequence $\{a_i\}$ = +1, +1, +1, –1, –1, +1, –1, $T_s = 2$ ms |
| 4, 10 | Videopulse, $T_s = 4$ ms |
| 5, 11 | Radiopulse with rectangular envelope, $T_s = 4$ ms, $f_0 = 1000$ Hz |
| 6, 12 | Barker`s sequence $\{a_i\}$ = +1, +1, +1, –1, –1, +1, –1, $T_s = 4$ ms |

*Note:* Information about the correlation functions of videopulse and radiopulse it is possible to find in sections 4 and 5. The example of calculation of correlation function of complex signal is resulted in Appendix of this LW.

4.3 Represent charts for:

a) research of time and spectral characteristics of the used signals;

b) research of impulse responses and AR of filters;

c) research of responses of filters on arbitrary signals;

d) research of gain in signal/noise ratio, which is provided a filter.

4.4 Prepare to the discussion on questions.

**5 Laboratory task**

5.1 **Acquaintance with a virtual model.** For this purpose start the program 3.1, using an icon “Laboratory works” on a desktop, and then folder of “TT-2”. Learn a model chart, using description in item 6 this LW. Specify the plan of performance of laboratory objective with a teacher.

5.2 **Research of time and spectral characteristics of the signals used in work.** Draw the time diagrams $s(t)$ and amplitude spectrum $S(f)$ of signals to the report: videopulse, radiopulse and complex signal at duration whether 4 or 2 ms, and radiopulse at frequencies whether 1000 or 1500 Hz. Write down the values of average powers of signals on the MF input $P_{\mathrm{s\,in}}$.

5.3 **Research of impulse responses and AR of filters.** Feed delta-function on the input of the matched filter. Bring the MF impulse responses $g(t)$ and AR $H(f)$ to the report (by the way, a amplitude spectrum on the output of filter repeats its AR). Execute a task for two signals: from a home task and set by a teacher.

In conclusions, compare time and spectral diagrams, got at the performance of this objective and task 5.2, and set connection between time and spectral characteristics of signals and filters, matched with signals.

5.4 **Research of responses of filters on signals which they are matched with.** Execute a task for signals: from a home task and set by a teacher. Bring to the report the responses of filters on signals which they are matched with. Write down the sampling values of signals on the MF output $y_{\mathrm{s}}(t_0)$. For this purpose in turn set the set signals and filters matched with them.

*Note*. On the MF output time diagrams must repeat the correlation functions of signals displaced at times.

In conclusions, compare the results of execution of it and home tasks and analyze compliance of MF properties.

5.5. **Define gain in signal/noise ratio, which is provided MF.** For this purpose:

– feed on the input of filter the sum of signal $s(t)$ (for example, videopulse) and noise $n(t)$ with power 0,1 V$^2$ or 1,0 V$^2$; to set a filter, matched with an input signal from a home task; after execution of the program compare temporal diagrams on an input and output of MF, make sure in the considerable weakening of noise by a filter;

– start the program on execution at the disconnected signal; write down the values of average powers of noise on the MF input $P_{n\ \mathrm{in}}$ and on the output of MF

$P_{n\,\text{out}}$; power of initial signal in sampling moment $P_\text{s}$ out is determined as a square of sampling of maximal value of initial signal (task 5.4) $P_{\text{s out}} = y_\text{s}^2(t_0)$; value $P_{n\,\text{in}}$ is certain during implementation task of 5.2: for videopulse and complex signal $P_{n\,\text{in}}$ equals the square of amplitude, and for radiopulse $P_n$ in half as much square of amplitude;

– expect gain in signal/noise ratio, provided by MF,

$$g_{\text{MF}} = \frac{P_{\text{s out}}/P_{\text{n out}}}{P_{\text{s in}}/P_{\text{n in}}}.$$

In conclusions, compare the got values with the expected value of gain $2F_\text{n}T_\text{s}$ ($F_\text{n}$ is a bandwidth of noise from a generator).

Divergence calculated and experimentally found values of gain can make tens percents is explained that realization of noise at a computer design relatively short, and average power of noise changes substantial appearance from realization to realization (it is herein possible to make sure, starting repeatedly the program and registering value $P_{\text{n in}}$ and $P_{\text{n out}}$).

**6 Description of laboratory model**

Laboratory work is executed on a computer in the environment of HP VEE with the use of virtual model the flow diagram of which is resulted in Fig. 1.

The generator of a signal is intended for formation of the signals $s(t)$ shown in Fig. 2:

a) single videopulse
b) radiopulse with rectangular envelope;
c) complex signal on the basis of 7-elements Barker`s sequence.

By handing down menus a management is conducted the generator of signal:

– setting any from the signals listed above;
– setting of duration of signal $T_s$ 4 or 2 ms;
– setting of frequency of radiopulse $f_0$ is 1000 Hz or 1500 Hz.

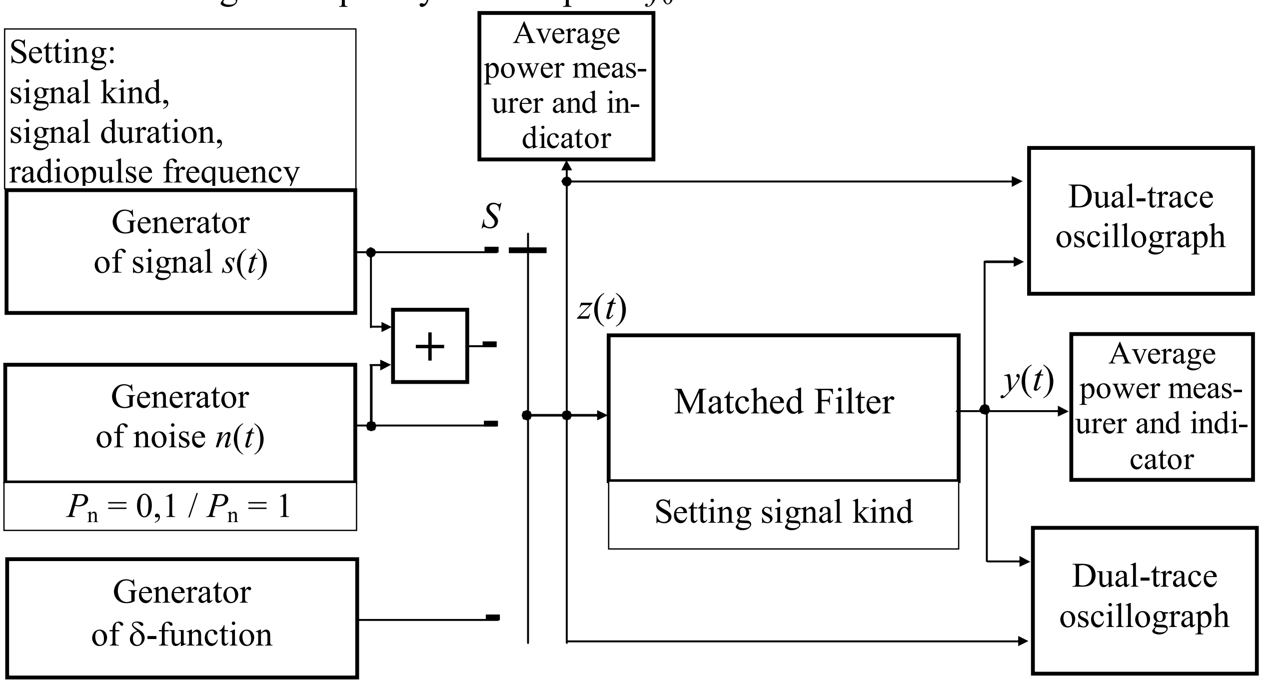



**Figure 1** – The laboratory breadboard model block diagram

The noise generator produces realization of noise $n(t)$ with average power $P_{\text{n}}$ in 0,1 or 1 $\text{V}^2$, quasi-white noise in the bandwidth of frequencies $f_{\text{n}}$ = 35 kHz.

A model contains the δ-function generator also.

The receipt of sum of signal and noise provides summarizing.

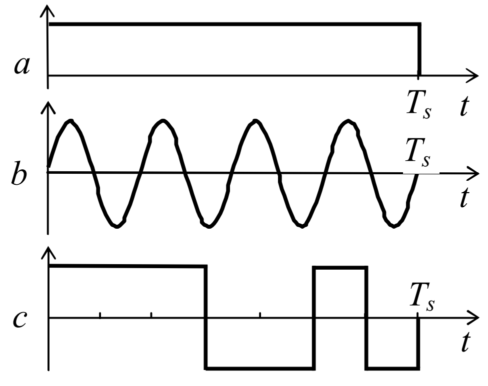



**Figure 2** – Researching signals

The switch $S$ lets to give on the input of the matched filter: $s(t)$, $n(t)$, $s(t)+ n(t)$ or $\delta(t)$.

A handing down menu for a management the matched filter lets to set MF for signals: videopulse, radiopulse and complex signal. If in the generator of signal to change duration of signal whether 2 or 4 ms or frequency of radiopulse whether 1000 or 1500 Hz, descriptions of MF change properly.

As descriptions of MF are determined within an arbitrary coefficient $a$, that in a model coefficient $a$ got out so that it was comfortably to look after temporal and frequency descriptions.

A model contains measuring devices and indicators of average powers of processes $z(t)$ (on the MF input) and $y(t)$ (on the MF output), oscillographs and spectrum analyzers.

**7 Requirements to the report**

7.1 The name of laboratory work.

7.2 The purpose of laboratory work.

7.3 The results of the home task.

7.4 Block diagrams of researches and results of implementation 5.2....5.5 laboratory task (oscillograms, spectrograms, and numerical values).

7.5 The conclusions on every item of laboratory task, in which it is necessary to give the analysis of the got results, such as the coincidence of theoretical and experimental data, etc.

7.6 The date, the signature of student, the visa of teacher with estimation on a 100-mark scale.

**Appendix. Calculation of complex signal correlation function**

The correlation function of nonperiodic signal is determined duration $T_{\text{s}}$ (for $0 \le \tau \le T_{\text{s}}$)

$$K_{\text{s}}(\tau) = \int_{0}^{T_s-\tau} s(t)\,s(t+\tau)\,dt\,. \tag{1}$$

We will consider a complex signal, which presents by itself the sequence of videopulses

$$s(t) = \sum_{i=1}^{n} a_i\, 1(t-(i-1)\,\tau_0)\,, \tag{2}$$

where $1(t)$ is a rectangular impulse of amplitude equals 1 and by duration $\tau_0$;

$a_i$ are coefficients which acquire a value +1 or -1;

$n$ is a number of impulses in a sequence.

At signals which are with videopulse, a correlation function is the broken line and for the construction of its graph it is enough to expect the values of function for τ, that multiple $\tau_0$. After the substitution of expression (2) in (1) will get correlation which determines the values of correlation function of signal (A.2) for the values τ, that multiple $\tau_0$

$$K_s(k\tau_0) = \sum_{i=1}^{n} a_i\, a_{i+k} \,. \tag{3}$$

Duration of signal $T_s = n\tau_0$, it is therefore necessary to expect value $K_s(k\tau_0)$ for $k = 0, 1, 2, ..., n - 1$, and $K_s(n\tau_0) = 0$.

For an example will conduct the calculation of correlation function of complex signal, built on the basis of 7-elements sequence ($n = 7$): $\{a_i\}$ = +1, +1, +1, –1, +1, +1, –1. Calculation on a formula (3) is taken to the following:

$k = 0$, $K_s(0) = 7$;

$k = 1$, $K_s(\tau_0) = 1{\cdot}1 + 1{\cdot}1 + 1{\cdot}(-1) + (-1){\cdot}(+1) + (+1){\cdot}1 + 1{\cdot}(-1) = 0$.

Like conducted calculation for $k$ = 2, 3, 4, 5, 6. The results of calculation are taken in table 2.

**Table 2 –** Results of calculation of correlation function

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $K_s(k\tau_0)$ | 7 | 0 | –1 | +2 | +1 | 0 | –1 | 0 |

The graph of the normalized correlation function of the considered signal is resulted in the Fig. 3.

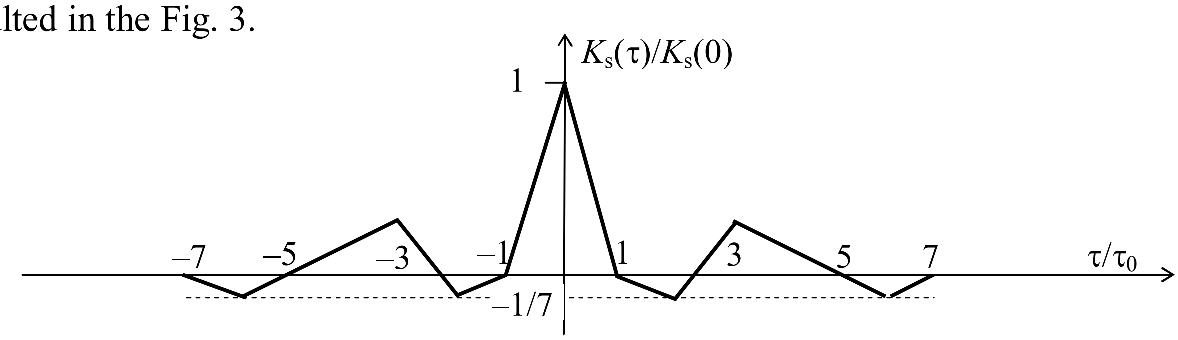



**Figure 3** – Calculated normalized correlation function of complex signal

**LW 3.2 Optimal demodulators of binary modulation signals schemes researching**

**1 Objectives**

1.1 Study of functional diagrams of optimal coherent demodulators of BASK, BFSK and BPSK signals.

1.2 Signals transformations in the separate demodulators' blocks researching.

**2 Main positions**

See section 8:

– criterion of an optimality of the demodulator;
– the mathematical description of the modulated signal of digital modulation;

– the mathematical description of channel symbols of BASK, BFSK and BPSK;
– the scheme of optimal demodulator of signals BASK and BPSK;
– the scheme of optimal demodulator of signal BFSK;
– inversion work of demodulator of signal BPSK;

**3 Questions**

3.1 Explain setting of modulator and demodulator during the transmission of digital signals.

3.2 Represent the time diagrams of BASK, BPSK and BFSK signals for a digital signal 101100.

3.3 Formulate the criterion of optimal demodulators of digital modulation.

3.4 Represent the functional diagrams of BASK, BPSK and BFSK signals demodulators and explain, what transformations are executed by every block.

3.5 What coherent detector, and what purpose of recovering carrier scheme?

3.6 What role is played by the matched filters in the schemes of demodulator?

3.7 What setting of time synchronization scheme?

3.8 Formulate the process of finding decision by the deciding schemes of BASK, BPSK and BFSK signals demodulator.

**4. Home task**

4.1 Learn a section 8.

4.2 Represent the functional diagrams of BASK, BPSK and BFSK signals demodulators.

4.3 Write down a number 48 + $N$ in the binary scale of notation ($N$ is a number of Your brigade). Represent the got digital signal, considering that rate $R$ = 1000 bit/s. Represent the time diagrams of BASK, BPSK and BFSK signals for this purpose digital signal, if $A(t)$ – videopulse, $f_0$ = 4 kHz, $f_1$ = 5 kHz.

4.4 On the input of BASK signal demodulator the BASK signal is given, got in item 4.3. Represent the time diagrams of signals in all points functional diagram of demodulator: on each of entrances and output of multiplier, on the output of matched filter, on the output of sampler, there are schemes on the output of decision.

4.5 Prepare to the discussion by questions.

**5 Laboratory task**

5.1 **Familiarize with a virtual model.** For this purpose run the program of 3.2a, using an icon “Laboratory works” on a desktop, and then folder of “TT”. Learn a model scheme, using description in item 6 this LW. Specify the plan of the laboratory work processing with a teacher.

5.2 **Signals transformations in BASK signal demodulator researching.** For this purpose it is necessary to set a digital signal, used for the home work processing, to turn off noise. Start the program on implementation. Draw time diagrams from oscillographs on the model panel. Compare results with home task processing.

5.3 **Signals transformations in BPSK signal demodulator researching.** Conditions of the work processing are such as in item 5.2. The phase of recovering carrier is equal 0 and 180°. Draw time diagrams from oscillographs on the model panel. Describe the differences of oscillogramms at the different phases of recovered carrier.

5.4 **Signals with noise transformations in BASK signal demodulator researching.** Conditions of the work processing are such as in item 5.2, but the noise generator is on and the noise level is two conditional unit. Run the program on execute and check, whether there was an error in the accepted digital signal. If it did not arise up, repeatedly start the program to appearance even of one error. Draw time diagrams from oscillographs on the model panel. Explain reason of errors origin.

5.5 **Signals with noise transformations in BPSK signal demodulator researching.** Conditions of the work processing are such as in item 5.4. The phase of recovered carrier is equal 0. Draw time diagrams from oscillographs on the model panel. To explain reason of errors origin.

5.6 **Signals transformations in BFSK signal demodulator researching**. For this purpose it is necessary to run the program 2.3b, using an icon "Laboratory works" on a desktop, and then folder of "TT", and to learn a model scheme; set a digital signal, used for the home work processing. Draw time diagrams from oscillographs on the model panel at first in upper subchannel, and then in lower. Explain their symbol.

**6 Description of laboratory model**

Laboratory work is executed on a computer in the HP VEE environment with the use of two virtual models.

A model 3.2a (Fig. 1) is intended for research of demodulators of signals of BASK and BPSK. He contains:

– generator of digital signal with speed of 1000 bit/s, a signal consists of 6 binary symbols (bit), symbols are set on the panel of model;

– modulator of signals – BASK or BPSK is set, envelope radiopulses – rectangular, frequency of carrier 4 kHz;

– a model of communication channel is summarizing of signal and noise;

– noise generator, which gives possibility to include and turn off him, and also set the level of noise;

– demodulator of signals of BASK and BPSK is after a scheme, resulted in Fig. 1; at a change in the model of type of modulation there is the proper change of rule of decision in a deciding scheme; the schemes of proceeding in carrier and time synchronization create "ideal" oscillations; at BPSK it is possible to change the phase of picked up a thread carrier on 180° for the imitation of inversion work;

– 5 oscillographs for the supervision of sentinel diagrams in all points functional diagram of demodulator.

A model of 3.2b (Fig. 2) is intended for research of demodulator of BFSK signal. It contains:

– generator of digital signal with speed of 1000 bit/s, a signal consists of 6 binary symbols (bit), symbols are set on the panel of model;

– modulator of signal of BFSK, envelope radiopulses – rectangular, frequencies of radiopulses $f_0 - \Delta f/2$ =4 kHz and $f_0 + \Delta f/2$ = 5 kHz;

– a model of communication channel is undistorted quadripole;

– demodulator of signal of BFSK is after a scheme, resulted in Fig. 3.3;

– 5 oscillographs for the supervision of sentinel diagrams in all points functional diagram of demodulator; oscillographs are commuted for the supervision of sentinel diagrams in upper or lower subchannel.

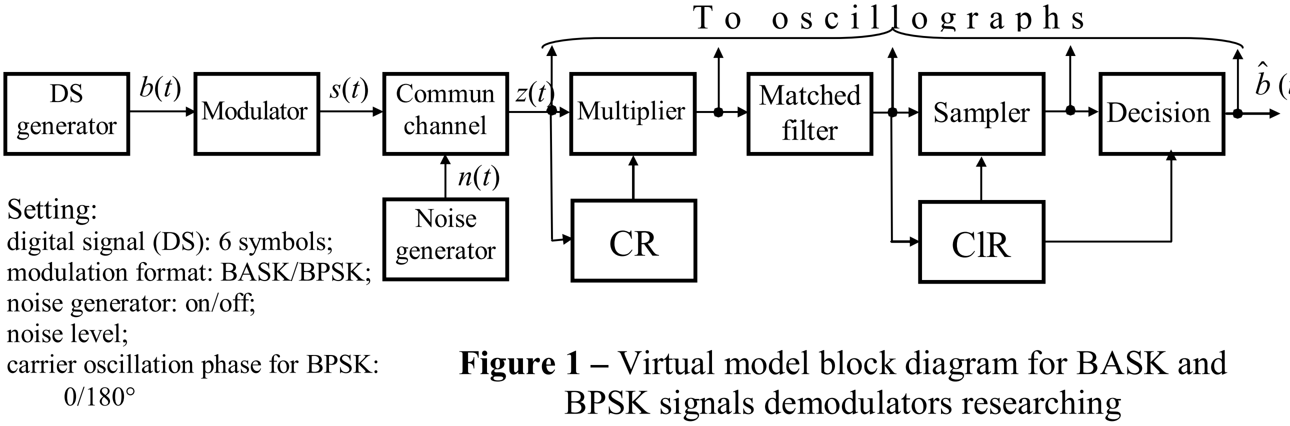



**Figure 1 –** Virtual model block diagram for BASK and BPSK signals demodulators researching

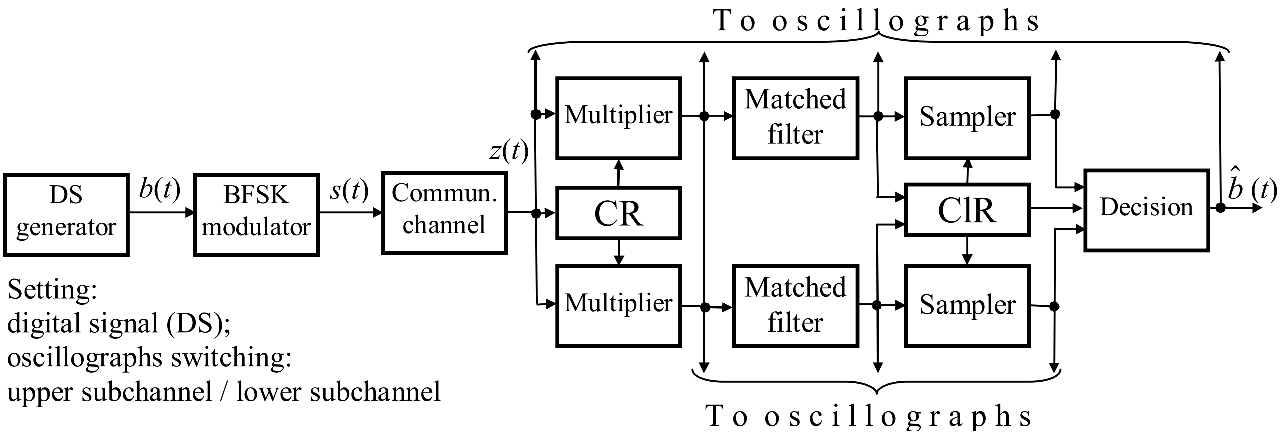



**Figure 2 –** Virtual model block diagram for BFSK signal demodulators researching

**7 Requirements to the report**

7.1 Name of laboratory work.

7.2 Purpose of laboratory work.

7.3 Results of the home task processing.

7.4 Block diagrams of researches and results of implementation of items 5.2…5.7 laboratory task (diagrams and explanations to them).

7.5 Conclusions after every point tasks in which you give the analysis of the got results (coincidence of experimental and theoretical data).

7.6 A date, signature of student, visa of teacher, with estimation on a 100-point scale.

**LW 3.3 Researching of optimal demodulator noise immunity of digital modulated signals**

**1. Objectives**

1.1 Studying the method of experimental researching of the digital modulated signals noise immunity.

1.2 Experimental research of noise immunity of next signals: BASK, BFSK, BPSK, BDPSK, QPSK.

**2. Main positions**

2.1 See section 9.

2.2 Optimal demodulator will realize noise immunity of the used channel symbol. Noise immunity of arbitrary binary equiprobable signals, if $n(t)$ is white Gaussian noise, is expressed by the formula for the signal error probability

$$P_{\mathrm{err}}(2) = Q\left(\frac{d}{\sqrt{2N_0}}\right), \tag{2}$$

where $d$ is distance between channel symbols;

$N_0$ is power spectral density of noise;

$$Q(x) = \frac{1}{\sqrt{2\text{p}}}\int_x^{\infty} e^{-t^2/2}dt \tag{3}$$

– Gaussian $Q$-function (integral of probability).

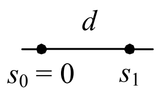
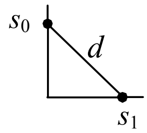
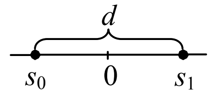
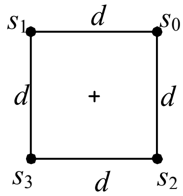
At the binary transmission systems bit error probability $p = P_{\mathrm{err}}(2)$.

Function $Q(x)$ is monotone decreasing. Therefore, when more function argument, then error probability is less. Obvious, than less power spectral density of the noise $N_0$, then error probability $p$ is less. Growth of distance between the signals $d$ results in decrease of error probability. The value $d$ is determined from signal constellation of the modulated signal and is expressed through energy of signal which is expended on the transmission of one bit $E_{\mathrm{b}}$. Energy per bit can be expressed through average signal power $P_{\mathrm{s}}$, duration of the bit interval $T_{\mathrm{b}}$ or the transmitted digital signal rate $R$:

$$E_{\mathrm{b}} = P_{\mathrm{s}} \cdot T_{\mathrm{b}} = P_{\mathrm{s}}/R. \tag{4}$$

2.3 In the case of multilevel signals ($M > 2$) error probability of signal $s_i(t)$ is expressed by the sum of error probabilities in the binary systems. These binary systems are formed by the elementary signal $s_i(t)$ and signals, transition in which is most probable. So, in case of multilevel signals the signal error probability depends on $N_0$ and $d$. Conversion of signal error probability in bit error probability takes into account the mapping code. For the researched modulation types signal constellations, distances between signals and expressions for bit error probability on table 1 are resulted. For space-saving notation of bit error probability formulas substitution $h_{\mathrm{b}}^2 = E_{\mathrm{b}}/N_0$ (briefly – the signal/noise ratio) is used. The formula of bit error probability for BDPSK is written down taking into account that at the relative decoding the number of errors is doubled: $p_{\mathrm{BDPSK}} = 2p_{\mathrm{BPSK}}$, that is true at $p_{\mathrm{BPSK}} \ll 1$.

**Table 1** – Descriptions that determine signals noise immunity

| Modulation type | BASK | BFSK | BPSK | QPSK |
|---|---|---|---|---|
| Signal constellation |  |  |  |  |
| Distance between signals $d$ | $\sqrt{2E_b}$ | $\sqrt{2E_b}$ | $2\sqrt{E_b}$ | $2\sqrt{E_b}$ |
| Bit error probability $p$ | $Q(h_b)$ | $Q(h_b)$ | $Q(\sqrt{2}h_b)$ | $Q(\sqrt{2}h_b)$ |

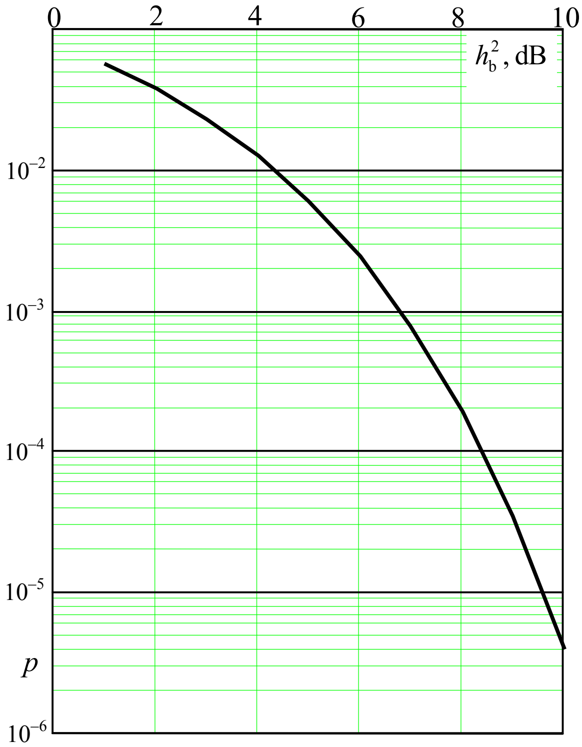



**Figure 1 –** Example of bit error probability dependence $p = f(h_b^2)$

2.4 Dependence $p = f(h_b^2)$ is built for convenient determination of bit error probability at the given SNR or determination of necessary SNR at the given bit error probability (example is resulted in Fig. 1). The SNR value is accepted to express in decibels and use a linear scale for it. It should be remembered that in formulas for error probability the size of $h_b$ is expressed in times. Transition can be executed on formulas

$$\left.\begin{aligned} h_b^2[\text{dB}] &= 10\lg h_b^2[\text{times}], \\ h_b^2[\text{times}] &= 10^{0,1h_b^2[\text{dB}]}. \end{aligned}\right\} \qquad (5)$$

Graphs of bit error probability dependence from SNR $p = f(h_b^2)$ build with using of logarithmic scale for error probability $p$, as shown in Fig. 1.

2.5 Error ratio is determined experimentally

$$K_{\text{err}} = N_{\text{err}} / N_{\text{all}}, \qquad (6)$$

where $N_{\text{all}}$ – number of the transmitted bits during observation time $T_{\text{ob}}$;

$N_{\text{err}}$ – number of the error bits received during $T_{\text{ob}}$.

Error probability and error ratio coincide at the large number of transmitted bits

$$p = \lim_{N_{\text{all}} \to \infty} K_{\text{err}}. \qquad (7)$$

Observation time (or $N_{\text{err}}$) takes large enough, that the error ratio practically gives the values of error probability. Consider, what such approximation takes place at $N_{\text{err}} \geq 20$.

2.6 Signal/noise ratio (SNR) with use (4) can be represented as

$$h_b^2 = \frac{E_b}{N_0} = \frac{P_s T_b}{P_n / F_{ch}} = \frac{P_s}{P_n} \cdot \frac{F_{ch}}{R}, \tag{8}$$

where $P_s$ and $P_n$ are average powers of signal and noise on the demodulator input;

$F_{ch}$ – noise bandwidth, which is equal the communication channel bandwidth.

So, for SNR measuring it is necessary to measure signal and noise powers by quadratic voltmeter and execute calculations using expression (8) at known values $F_{ch}$ (Hz) and $R$ (bits/s).

2.7 The communication channel bandwidth $F_{ch}$ must be matched with the signal bandwidth $F_s$: $F_{ch} \geq F_s$. The digital modulated signals bandwidth (Hz) is determined in the case of $M$-PSK, $M$-DPSK and $M$-AM

$$F_s = \frac{R \cdot (1+\alpha)}{\log_2 M}, \tag{9}$$

and in the case of BFSK

$$F_s = 2R \cdot (1+\alpha), \tag{10}$$

where $\alpha$ is roll-off factor of spectrum, $0 \leq \alpha \leq 1$, usually values $\alpha$= 0,15...0,35.

**3 Questions**

3.1 What is the error rate, error ratio, error probability?

3.2 What is the signal noise immunity?

3.3 Write down and explain expressions for the calculation of bit error probability at the optimal demodulation BASK, BFSK, BPSK, BDPSK and QPSK signals.

3.4 Explain, how does bit error probability depend from digital signal rate when $P_s$ and $N_0$ are constants.

3.5 Compare BASK, BFSK, BPSK, BDPSK and QPSK signals noise immunity of optimal demodulation at the fixed value $h_b^2$.

3.6 What is the quantitative measure of difference between of signals? Compare difference between BASK, BFSK, BPSK and QPSK signals.

3.7 How to calculate the necessary communication channel bandwidth for the BASK, BFSK, BPSK, BDPSK and QPSK signals?

3.8 How can we experimentally measure the SNR?

**4 Home task**

4.1 Study section 9 and item 2 "Main positions".

4.2 Calculate the table of bit error probability values for the BASK, BFSK, BPSK, BDPSK and QPSK signals, increasing value $h_b^2$ from 2 to 10 dB with a step 1 dB.

Calculations of error probability it is possible to execute in the mathematical packages MathCAD and MatLab, using the error function erf(•):

$$Q(x) = 0,5(1 - \text{erf}(x/\sqrt{2})) \tag{11}$$

High accuracy of calculations for values $Q(x) > 10^{-10}$ the such formula provides

$$Q(x) = 0{,}65\ \exp(-0{,}44(x + 0{,}75)^2). \qquad (12)$$

Using calculation results build graphs of $p$ from SNR $h_b^2$ dependence. Build graphs according to the sample in Fig. 1. For this graph take one page in copybook.

4.3 Represent a block diagram for researching of digital modulated signals noise immunity.

4.4 Be ready to discuss key questions.

**5 Laboratory task**

5.1 **Acquaintance with a virtual model.** Run the program 3.3, using the icon TT (English) on a desktop. It is necessary to study the structure of a virtual model using its description in item 6 of this LW. Specify with the teacher the laboratory task performance plan.

5.2 **Calibration of SNR:**

1) Measure average power of signal. For this purpose it is necessary to set:

– digital signal rate $R$ from the interval 1000–10000 bits/s;
– the modulation type is arbitrary;
– a signal is "On", a noise is "Off";
– communication channel bandwidth must satisfy the condition $F_{ch} \geq F_s$, the signal bandwidth $F_s$ is determined from expressions (9) and (10);
– measuring of "Average power".

Run the program and write down measured value of signal average power $P_s$.

2) Calculate and set the power spectral density of noise $N_0$, at which SNR $h_b^2 = 1$, i.e. 0 dB. For this purpose, using definition $h_b^2 = \dfrac{E_b}{N_0} = \dfrac{P_s T_b}{N_0} = \dfrac{P_s}{N_0 \cdot R} = 1$, calculate $N_0 = P_s/R$. Use symbol $m$ for $10^{-3}$ and $u$ for $10^{-6}$ at setting the value $N_0$.

For example, BPSK is set, $R = 10$ kbit/s, $F_{ch}$·= 12 kHz; measured $P_s = 1$ B$^2$; calculated $N_0 = 10^{-4}$ V$^2$/Hz. It is necessary to set $N_0 = 0.1m$.

Turn off a signal, set attenuation in the noise path 0 dB, run the program and write down the measured value of average noise power $P_n$. Make sure that $P_n = F_{ch} \cdot N_0$. In the further measurements of error probability do not change $R$ and $N_0$ setting, and the SNR change by setting of the proper attenuation in the noise path: attenuation of noise, represented in decibels, sets the same SNR value $h_b^2$ in decibels.

5.3 **Measuring of error probability** (coefficient of error). For this purpose set:

- measuring of "Coefficient of errors";
- a signal is "On";
- modulation type set on the task of teacher (BASK, BFSK, BPSK, BDPSK or QPSK);
- communication channel bandwidth is set from condition $F_{ch} \geq F_s$, and the signal bandwidth $F_s$ is determined from relation (9).

Make a table according to the sample table 2. Setting attenuation of noise from 3 to 9 dB (for BASK, BFSK) and from 2 to 6 dB (for BPSK, BDPSK and QPSK) with a step 1 dB and run the program, fill the columns of table: the modulation type, the SNR $h_b^2$, bit error number $N_{err}$, number of the transmitted bits $N_{all}$. Complete execution of the program provides $N_{all}$ = 10000. Execution of the program can be stopped, if the number of error bits attained several tens.

**Table 2** – Results of error probability measuring

| Modulation type | Communication channel bandwidth $F_{ch}$, Hz | SNR $h_b^2$, dB | Number of error bit, $N_{err}$ | Number of the transmitted bits, $N_{all}$ | Error probability (error ratio) $p$ |
|---|---|---|---|---|---|
| BASK | | 3 | | | |
| | | 4 | | | |
| | | ⋮ | | | |
| BFSK | | 3 | | | |
| | | ⋮ | | | |

On measuring results calculate the error ratio using (6); considering that number of the transmitted bits $N_{all}$ large enough, take calculated value of error ratio as error probability.

Build the graphs of dependences $p = f(h_b^2)$ for measuring results and calculations of all modulation types. Build the graphs on the picture, where the home task is resulted.

5.4 **Measuring of probability of error at the changed communication channel bandwidth**. To set the communication channel bandwidth, multiply it in 1,5...2 times, and repeat measurements of error probability for one of modulation types. Make sure, that error probability does not depend from the communication channel bandwidth and from the value of noise average power at the demodulator input; it depends on noise power spectral density $N_0$.

**6 Description of laboratory model**

Laboratory work is executed on a computer in the HP VEE with using a virtual model, the block diagram of which is resulted on the Fig. 2. Virtual model contains:

- digital signal source which produces equiprobable symbols 1 and 0, digital signal rate $R$, bit/s, is setting on the panel of model;
- modulator which forms the BASK, BFSK, BPSK, BDPSK and QPSK signals, average power of signal, for all modulation types $P_s$ = 1 B$^2$, the modulation type is setting on the panel of model;
- key in the path of modulated signal allows to connect and disconnect modulator output from the input of communication channel;
- noise generator, producing realization of white noise with the Gaussian probability distribution, the value of the power spectral density $N_0$ is set on the panel of model;
- attenuator for noise attenuation; on the panel of model it is possible to set attenuation from 0 to 10 dB with a step 1 dB or to turn off noise;
- communication channel which forms the sum of signal and noise, on the panel of virtual model the communication channel bandwidth $F_{ch}$, Hz is set;
- meter of average power, connected with communication channel output;
- demodulator for demodulation of the BASK, BFSK, BPSK, BDPSK and QPSK signals provides optimal demodulation of signal which acting in communica-

tion channel. At the change of set modulation type in modulator the algorithm of demodulation changes properly;

- comparator of bit on modulator input and bit on demodulator output, if these bits differ, then a signal about the error at demodulation is formed;
- counter of error bits $N_{\text{err}}$ ;
- counter of transmitted bits $N_{\text{all}}$;
- indicators of measured average power on the communication channel output, number of the of error bits $N_{\text{err}}$ and number of the transmitted bits $N_{\text{all}}$.

Program is stopped when achievement value of the transmitted bit number $N_{\text{all}}$ = 10000 is completed. If the number of error bits is insufficient for the calculation of error probability, the restart of the program (once again or anymore) executes, and for the calculation of error probability the corresponding values $N_{\text{err}}$ and $N_{\text{all}}$ are added.

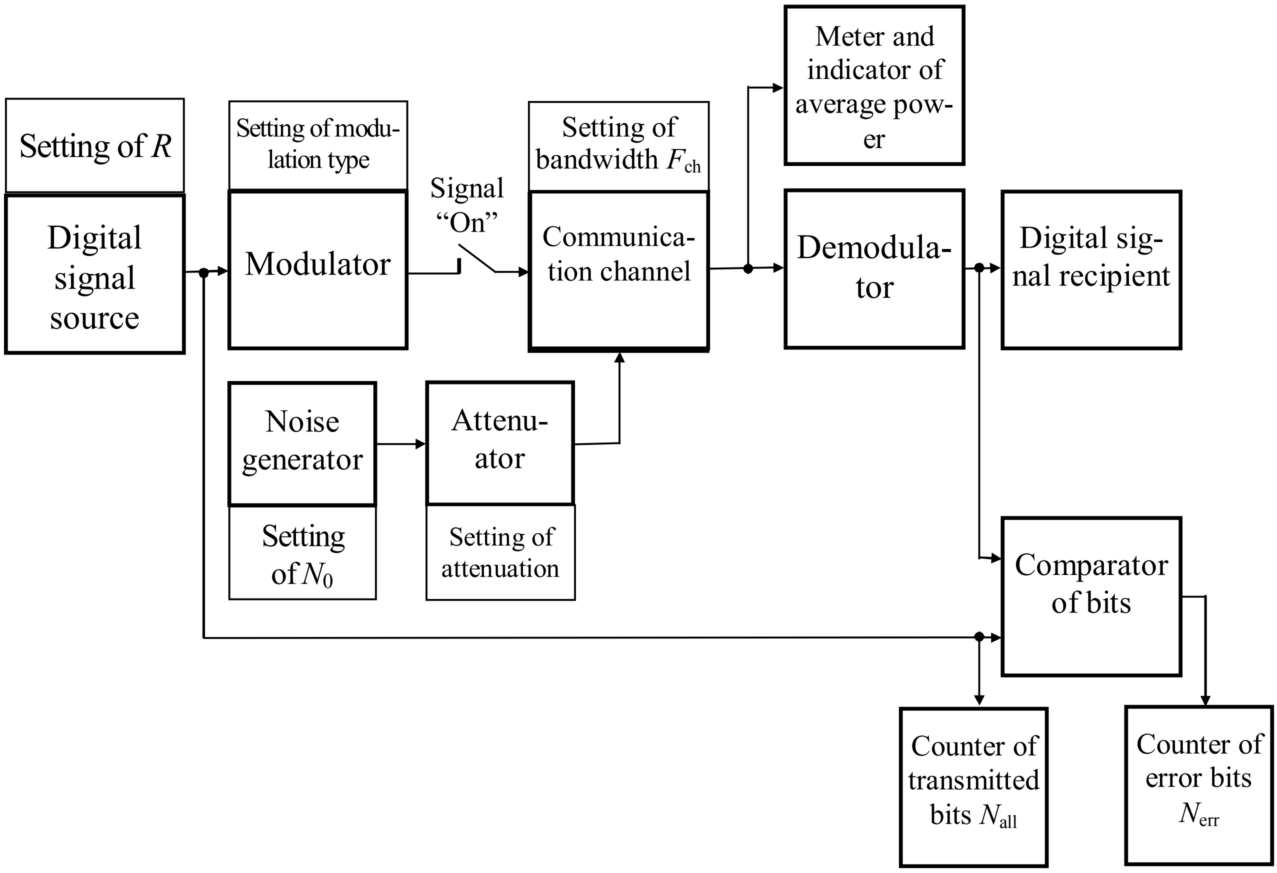



**Figure 2** – Virtual model block diagram

**7 Requirements to the report**

7.1 Title of laboratory work.

7.2 Objectives of laboratory work.

7.3 Results of the homework execution.

7.4 Block diagram of researches and results of the execution of items 5.2...5.4 of laboratory task (tables and graphs).

7.5 Conclusions on every item of the laboratory task, with analysis of the got results (coincidence of experimental and theoretical states).

7.6 Signature of student about the laboratory work execution, teacher's signature for the laboratory work defense with estimation and date.

## LW 3.4 Research of signals and noise passing through synchronous and frequency detectors

### 1. Objectives

Researching of noise characteristics on the synchronous and frequency detectors output, determination of gain in signal/noise ratio at AM, DSB SC, SSB and FM signals detection.

### 2. Main positions

2.1 A synchronous detector (SD) consists of multiplier and LPF (Fig. 1). On one multiplier input the detecting modulated signal, and on second reference waveform $u_{\text{ref}}(t) = 2\cos 2\pi f_0 t$ operates. There are low-frequency components in a band less then frequency $F_{\max}$ and components in a band from $2f_0 - F_{\max}$ to $2f_0 + F_{\max}$ ($F_{\max}$ – maximal frequency in the modulating signal $b(t)$ spectrum) appears at modulated signal and reference waveform multiplying. LPF cut frequency must be equal to $F_{\max}$, filter passes only low-frequency components, it is possible to execute if $f_0 > F_{\max}$.

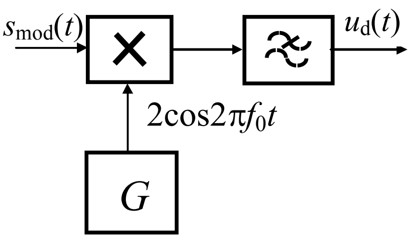


**Figure 1** – Synchronous detector block diagram

2.2 SD is used for detection of the AM, DSB SC and SSB signals

$$s_{\text{AM}}(t) = A_0(1 + m_{\text{AM}} b(t)) \cos 2\pi f_0 t, \tag{1}$$

$$s_{\text{DSB–SC}}(t) = A_0 b(t) \cos 2\pi f_0 t, \tag{2}$$

$$s_{\text{SSB}}(t) = A_0 b(t) \cos 2\pi f_0 t \pm A_0 \tilde{b}(t) \sin 2\pi f_0 t, \tag{3}$$

where $\tilde{b}(t)$ – Gilbert transformation from $b(t)$.

On the SD output we will get:

in the case of the AM signal $\quad u_{\text{d}}(t) = A_0(1+m_{\text{AM}} b(t))$; (4)

in the case of the DSB SC and SSB signals $\quad u_{\text{d}}(t) = A_0 b(t)$. (5)

2.3 Average power $P_{\text{s}}$ of the modulated signals described by expressions (1), (2) and (3):

$$P_{\text{sAM}} = 0{,}5 A_0^2 (1 + m_{\text{AM}}^2 P_{\text{b}}), \quad P_{s\text{DSB–SC}} = 0{,}5 A_0^2 P_{\text{b}}, \quad P_{\text{sSSB}} = A_0^2 P_{\text{b}}, \tag{6}$$

where $P_{\text{b}}$ – average power of signal $b(t)$.

Average power $P_{\text{s out}}$ of the signals on the synchronous detector output, described by expressions (4) and (5), (without constant component at AM):

$$P_{\text{s outAM}} = A_0^2 m_{\text{AM}}^2 P_{\text{b}}, \quad P_{\text{s outDSB}} = P_{\text{s outSSB}} = A_0^2 P_{\text{b}}. \tag{7}$$

2.4 If on the input of SD quasi-white noise in the band of the modulated signal frequencies acts, its spectrum is shifted in band near zero frequency and in a band near $2f_0$ frequency by a detector. LPF passes the noise components near zero frequency. As only the shifting of noise spectrum takes place, output noise is quasi-white in the LPF pass band.

Band noise can be considered as a sum of two quadrature components

$$n(t) = N_{\text{in-p}}(t)\cos(2\pi f_0 t) + N_{\text{q}}(t)\sin(2\pi f_0 t), \tag{8}$$

where $N_{\text{in-p}}(t)$ – amplitude of in-phase (in relation to reference waveform), cosine component;

$N_{\text{q}}(t)$ – amplitude of quadrature (in relation to reference waveform), sine component.

A synchronous detector reacts to only in-phase component, and on its output low-frequency noise takes place

$$\text{LF}\{n(t)2\cos(2\pi f_0 t)\} = N_{\text{in-p}}(t).$$

At presentation of band noise $n(t)$ by expression (8) its power $P_{\text{n}}$ is distributed equally between quadrature components, power of each processes $N_{\text{in-p}}(t)$ and $N_{\text{q}}(t)$ is also equal $P_{\text{n}}$. So, noise power on the SD output

$$P_{\text{n out}} = P_{\text{n}}. \tag{9}$$

2.5 Value which shows, in how many times the SNR is decreased at detection, is named detector gain in SNR.

$$g = \frac{P_{\text{s out}}/P_{\text{n out}}}{P_{\text{s}}/P_{\text{n}}}. \tag{10}$$

Using (6), (7) and (9) it is possible to get expressions determining gain for synchronous detection of the AM, DSB-SC and SSB signals:

$$g_{\text{AM}} = \frac{2m_{\text{AM}}^2}{K_A^2 + m_{\text{AM}}^2}, \qquad g_{\text{DSB-SC}} = 2, \qquad g_{\text{SSB}} = 1, \tag{11}$$

where $K_{\text{A}} = 1/\sqrt{P_b}$ – amplitude coefficient of modulating signal.

2.6 The FM signal is written down

$$s_{\text{FM}}(t) = A_0 \cos(2\pi f_0 t + 2\pi\Delta f_{\text{d}} \int_{-\infty}^{t} b(t)dt + \varphi_0), \tag{12}$$

where $\Delta f_{\text{d}}$ – frequency deviation.

Average power of signal (12) is determined

$$P_{\text{s FM}} = 0{,}5\, A_0^2. \tag{13}$$

At processor realization a frequency detector (FD) is built on the scheme resulted in Fig. 2. Filters LPF1 and LPF2 – components of quadrature splitter, have cut frequency $F_{\max}(m_{\text{FM}} + 1)$, where $m_{\text{FM}} = \Delta f_{\text{d}}/F_{\max}$. Filter LPF3 has cut frequency $F_{\max}$.

The analysis shows that $u_{\text{d}}(t) = b(t)$, so

$$P_{\text{s outFM}} = P_{\text{b}}. \tag{14}$$

It is possible to show that when on the FD input sum of reference waveform and quasi-white noise in the FM signal band with power $P_{\text{n}}$ acts, and SNR considerably more, then one the noise power spectral density on the FD output is described by quadrature dependence

$$G_{\rm N}(f)=\frac{2P_{\rm n}(2\pi f)^2}{A_0^2(2\pi\Delta f_{\rm d})^2(m_{\rm FM}+1)F_{\max}},\quad 0\le f\le F_{\max}. \tag{15}$$

Noise power on the detector output is determined

$$P_{\rm n\,out}=\int_0^{F_{\max}}G_{\rm N}(f)df=\frac{2P_{\rm n}}{3A_0^2m_{\rm FM}^2(m_{\rm FM}+1)}. \tag{16}$$

2.5 Using (13), (14) and (16) it is possible to get expressions for determining gain of FD

$$g_{\rm FM}=\frac{3m_{\rm FM}^2}{K_A^2}2(m_{\rm FM}+1). \tag{17}$$

Usually FM signal index equals to a few units up to 10. Therefore, as follows from a formula, gain can achieve the values $g_{\rm FM}$ >> 1. This assertion is correct when the SNR on the FD input is considerably more then one.

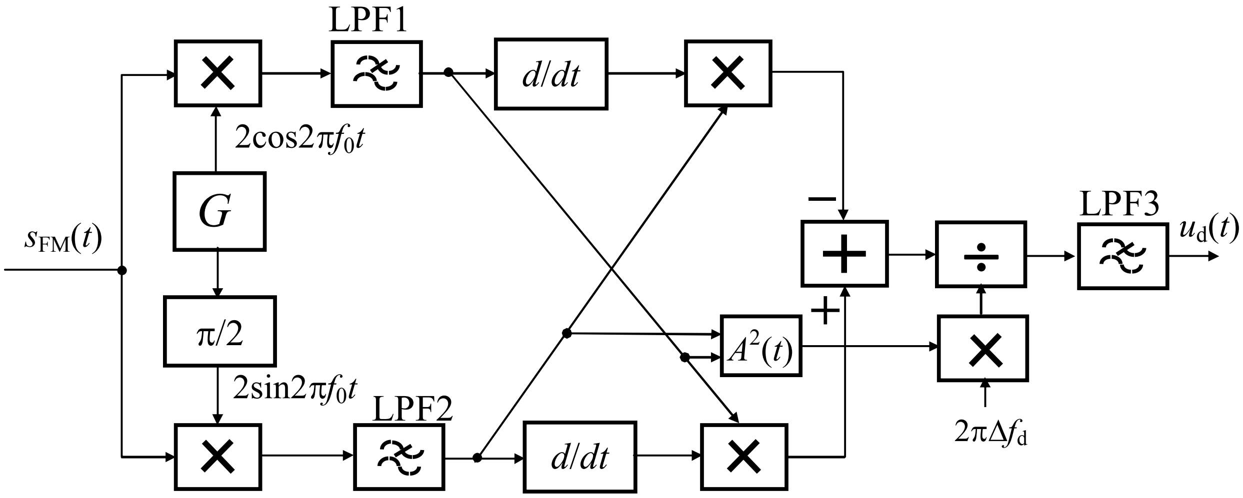



**Figure 2 –** Frequency detector block diagram

**3 Questions**

3.1 Give determination of amplitude (AM), double-sideband-suppressed-carrier (DSB-SC) and single-sideband (SSB) modulations.

3.2 How to calculate the spectrums of the AM, DSB-SC and SSB signals at the set spectrum of modulating signal?

3.3 What is synchronous detector (SD)? Draw its block diagram.

3.4 What form has the noise spectrum on the SD output?

3.5 What gain in the SNR does synchronous detector provide?

3.6 Give determination of frequency modulation (FM).

3.7 What is frequency deviation of the FM signal?

3.8 How to calculate and build the FM signal spectrum at modulation by harmonious waveform?

3.9 What form has the noise spectrum on the FD output when on its input the sum of reference waveform and weak noise operates?

3.10 What gain in the SNR frequency detector provides at a weak noise on its input?

**4 Home task**

4.1 Study section 19.

4.2 Build the graphs of modulated signals spectrum, define the values of bandwidth, if reference waveform $u_{\text{ref}}(t) = \sin(2\pi 10000t)$, and modulating signal $b(t) = 0{,}3\sin(2\pi 120t) + 0{,}3\sin(2\pi 260t) + 0{,}4\sin(2\pi 460t)$:

- brigade № 1 and 5 – AM;
- brigade № 2 and 6 – DSB-SC;
- brigade № 3 – SSB LSB;
- brigades № 4 – SSB USB.

Calculate gain in the SNR, which synchronous detector provides (use, that $K_A^2 = 1/P_{\text{b}}$).

4.3 Build the graph of FM signal spectrum (for all brigades) and define a bandwidth if reference waveform $u_{\text{ref}}(t) = \sin(2\pi 10000t)$, modulating signal $b(t) = \sin(2\pi 220t)$, frequency deviation 800 Hz. Calculate gain in the SNR which is provided by a frequency detector at detection of signal with $K_A^2$=5,9, $\Delta f_{\text{d}}$ = 800 Hz, $F_{\max}$ = 220 Hz.

4.4 Be ready to answer questions.

**5 Laboratory task**

**5.1 Acquaintance with a virtual model.** Run the program 3.4, using the icon TT (English) on a desktop. It is necessary to study the structure of a virtual model using its description in part 6 of this LW. Specify with the teacher the laboratory task execution plan.

**5.2 Modulated signals detection research.** On the task of teacher AM, DSB-SC, SSB or FM signals are researched. For this purpose set the necessary modulation type in a model. Run the program, noise is “off” (factor of amplifying $K = 0$) and modulating and modulated signals are “on”. Compare time diagrams of signals on the modulator input and on the detector output and make sure, that they coincide. Add to the report the modulated signal spectrums and signal on the detector output diagram. Compare spectrums with the home task results. Write down power of signals on the detector input and output.

**5.3 Research of noise passing through synchronous detector.** AM, DSB-SC or SSB signals (on the task of teacher) are researched. For this purpose set the necessary modulation type (the corresponding detector switch on), set an amplification factor in the noise circuit $K$ = 3, turn off the modulated signal and run the program. Make sure, that the noise realization spectrums on the detector input and output can be considered uniformly distributed in the band of the modulated and baseband signals accordingly. Write down the values of maximal and minimal spectrum frequencies of noise realization on the detector input and output and compare them with maximal and minimal frequencies of modulated and modulating signals spectrums accordingly. Write down the noise power values on the detector input and output.

Calculate gain in the SNR which detector provides and compare experimental results with the home task results.

**5.4 Research of noise passing through frequency detector.** Set the modulation type "Frequency modulation", set an amplification factor in the noise circuit $K = 1...2$, turn off the modulated signal (reference waveform switch on) and run the program. Write down the noise power values on the detector input and output. Make sure, that SNR on the detector input considerably more then one (if it is not executed, it is necessary to decrease $K$). Write down the values of maximal and minimal spectrum frequencies of noise realization on the detector input and output and compare them with and minimal frequencies of modulated and modulating signals spectrums accordingly. Make sure, that the amplitude spectrum of noise realization on the detector output can be considered linearly increasing in the baseband signal band. Calculate gain in the SNR which detector provides and compare got results with the home task results.

**6 Description of laboratory model**

Laboratory work is executed on a computer in the HP VEE with using a virtual model, the block diagram of which is resulted in Fig. 3. Virtual model contains AM, DSB-SC and SSB signals modulators with the modulating signal $b(t) = 0{,}3\sin(2\pi 120t) + 0{,}3\sin(2\pi 260t) + 0{,}4\sin(2\pi 460t)$, and FM signal modulator with the modulating signal $b(t) = 0{,}3\sin(2\pi 100t) + 0{,}3\sin(2\pi 160t) + 0{,}4\sin(2\pi 220t)$. Reference waveform frequency is equal to 10 kHz. Coefficient $m_{\mathrm{AM}}=1$. SSB signal modulator forms the upper sideband. FM signal frequency deviation $\Delta f_{\mathrm{d}} = 800$Hz.

After setting of modulation type synchronous detector for detection of the AM, DSB-SC and SSB signals or frequency detector for FM signal detection uses. FD is built on the scheme resulted in Fig. 2.

The band noise generator produces realizations of quasi-white noise, the spectrum of which is concentrated in the modulated signals frequency band, namely:

- in the band 9500...10500 Hz at AM and DSB-SC signals detection research;
- in the band of 10000...10500 Hz at SSB signal detection research;
- in the band of 9000...11000 Hz at FM signal detection research.

An amplifier in the noise path is controlled – it is possible to change an amplification factor.

The sum of signal and noise is on the input of synchronous or frequency detector. For separate signal and noise passing research the modulated signal switch and amplification factor equal to zero in the noise circuit are used. Research of noise passing through FD is executed at the unmodulated reference waveform. For this purpose modulating signal is “off” (by switch “modulation on”), and the modulator output is “on” (by switch “signal on”).

There are three power measuring devices for bandpass noise, modulated signal and process on the detector output. Two amplitude spectrum analyzers – on the detectors input and output are used. Two oscillographs, one for the process on the detector input, second for the processes on the modulator input and detector output (by turns or simultaneously) are used.

In the program for the generation of signals and noises realization the number of samples is 4000, duration of realization 0,05 s, therefore sampling frequency is equal 80 kHz.

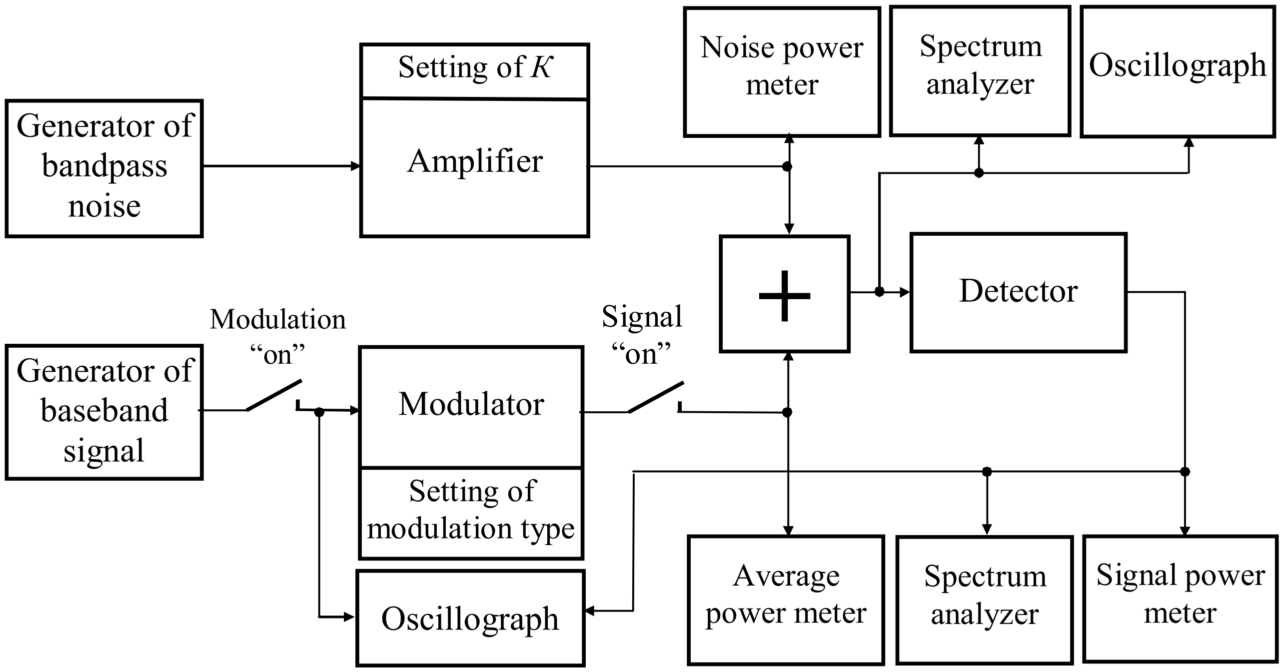



**Figure 3 –** Virtual model block diagram for research of signal and noise passing through detectors

**7 Requirements to the report**

7.1 Title of laboratory work.

7.2 Objectives of laboratory work.

7.3 Results of the homework execution.

7.4 Results of the execution of laboratory task (tables and graphs).

7.5 Conclusions on every item of the laboratory task, with analysis of the got results (coincidence of experimental and theoretical information).

7.6 Signature of student about the laboratory work execution, teachers signature for the laboratory work defense with estimation and date.

# Attachment C. Dictionaries

## C.1 English-Russian dictionary

| | |
|---|---|
| amplifier | усилитель |
| amplitude factor | коэффициент амплитуды |
| attenuation | ослабление |
| attenuator | аттенюатор |
| bandpass noise | полосовой шум |
| binary modulation types | двоичные виды модуляции |
| carrier recovery (CR) | восстановление несущей (ВН) |
| channel bandwidth | полоса пропускания канала |
| channel symbols | канальные символы, элементарные сигналы (импульсы) |
| clock period, timing period, timing interval | тактовый интервал |
| clock recovery (ClR) | восстановление тактовой синхронизация (ТС) |
| constant component | постоянная составляющая |
| decision | решение, решающая схема |
| detecting signal | детектируемый сигнал |
| detector gain | выигрыш детектора |
| differential decoder | относительный декодер |
| differential encoder | относительный кодер |
| differentially encoded phase modulation (DPSK) | фазоразностная модуляция, относительная фазовая модуляция |
| energy losses (EL) | энергетические потери |
| energy per bit | энергия на бит |
| envelope detector (ED) | детектор огибающей |
| equiprobable | равновероятный |
| error detector | детектор ошибки |
| error ratio | коэффициент ошибок |
| frequency detector | частотный детектор |
| frequency deviation | девиация частоты |
| frequency modulation index | индекс ЧМ |
| gain in signal-to-noise ratio | выигрыш в отношении сигнал/шум |
| Gaussian $Q$-function | гауссовская $Q$-функция |
| impulse response | импульсный отклик |
| incoherent demodulation | некогерентная демодуляция |

| | |
|---|---|
| in-phase, cosine component | синфазная, косинусная составляющая |
| intersymbol interference | межсимвольная интерференция |
| Kolmogorov-Wiener filter | фильтр Колмогорова-Винера |
| low-frequency components | низкочастотные составляющие |
| matched filter | согласованный фильтр |
| maximum a posteriori probability | максимум апостериорной вероятности |
| maximum likelihood rule | правило максимума правдоподобия |
| monotone decreasing | монотонно убывающая |
| noise immunity | помехоустойчивость |
| noise power spectral density | удельная мощность шума |
| optimal demodulator | оптимальный демодулятор |
| phase locked-loop (PLL) | фазовая автоподстройка частоты (ФАПЧ) |
| phase uncertainty | неопределенность фазы |
| power spectral density (PSD) | спектральная плотность мощности (СПМ) |
| pulse-carrier | импульс-переносчик |
| quadratic voltmeter | квадратичный вольтметр |
| quadrature, sine component | квадратурная, синусная составляющая |
| quadrature splitter | квадратурный расщепитель |
| raised cosine | поднятый косинус |
| reference waveform | опорное колебание |
| roll-off factor of spectrum | коэффициент ската спектра |
| root-mean square deviation (RMS) | среднеквадратическое отклонение (СКО) |
| sampler | дискретизатор |
| sampling condition | условие отсчетности |
| signal to noise ratio (SNR) | отношение сигнал/шум (ОСШ) |
| square root of the Nyquist spectrum | корень квадратный из спектра Найквиста |
| square root of the Nyquist spectrum | корень квадратный из спектра Найквиста |
| symbol-by-symbol demodulation | поэлементный прием |
| synchronous detector | синхронный детектор |
| threshold | порог |
| tracking filter | следящий фильтр |

| | |
|---|---|
| voltage-controlled oscillator (VCO) | генератор, управляемый напряжением (ГУН) |

## C.2 Russian-English dictionary

| | |
|---|---|
| аттенюатор | attenuator |
| восстановление несущей (ВН) | carrier recovery (CR) |
| восстановление тактовой синхронизации (ТС) | clock recovery (ClR) |
| выигрыш в отношении сигнал/шум | gain in signal-to-noise ratio |
| выигрыш детектора | detector gain |
| гауссовская $Q$-функция | Gaussian $Q$-function |
| генератор, управляемый напряжением (ГУН) | voltage-controlled oscillator (VCO) |
| двоичные виды модуляции | binary modulation types |
| девиация частоты | frequency deviation |
| детектируемый сигнал | detecting signal |
| детектор огибающей | envelope detector (ED) |
| детектор ошибки | error detector |
| дискретизатор | sampler |
| импульсный отклик | impulse response |
| импульс-переносчик | pulse-carrier |
| индекс ЧМ | frequency modulation index |
| канальные символы | channel symbols |
| квадратичный вольтметр | quadratic voltmeter |
| квадратурная, синусная составляющая | quadrature, sine component |
| квадратурный расщепитель | quadrature splitter |
| корень квадратный из спектра Найквиста | square root of the Nyquist spectrum |
| коэффициент амплитуды | amplitude factor |
| коэффициент ошибок | error ratio |
| коэффициент ската спектра | roll-off factor of spectrum |
| максимум апостериорной вероятности | maximum a posteriori probability |
| межсимвольная интерференция | intersymbol interference |
| монотонно убывающая | monotone decreasing |
| некогерентная демодуляция | incoherent demodulation |
| неопределенность фазы | phase uncertainty |
| низкочастотные составляющие | low-frequency components |

| | |
|---|---|
| опорное колебание | reference waveform |
| оптимальный демодулятор | optimal demodulator |
| ослабление | attenuation |
| относительная фазовая модуляция | differentially encoded phase-shift keying |
| относительный декодер | differential decoder |
| относительный кодер | differential encoder |
| отношение сигнал/шум (ОСШ) | signal to noise ratio (SNR) |
| поднятый косинус | raised cosine |
| полоса пропускания канала | channel bandwidth |
| полосовой шум | bandpass noise |
| помехоустойчивость | noise immunity |
| порог | threshold |
| постоянная составляющая | constant component |
| поэлементный прием | symbol-by-symbol demodulation |
| правило максимума правдоподобия | maximum likelihood rule |
| равновероятный | equiprobable |
| решение, решающая схема | decision |
| синфазная, косинусная составляющая | in-phase, cosine component |
| синхронный детектор | synchronous detector |
| следящий фильтр | tracking filter |
| согласованный фильтр | matched filter |
| спектральная плотность мощности (СПМ) | power spectral density |
| среднеквадратическое отклонение (СКО) | root-mean square deviation (RMS) |
| тактовый интервал | clock period, timing period, timing interval |
| удельная мощность шума | noise power spectral density |
| усилитель | amplifier |
| условие отсчетности | sampling condition |
| фазовая автоподстройка частоты (ФАПЧ) | phase locked-loop (PLL) |
| фазоразностная модуляция | differentially encoded phase-shift keying |
| фильтр Колмогорова-Винера | Kolmogorov-Wiener filter |
| частотный детектор | frequency detector |
| энергетические потери | energy losses (EL) |
| энергия на бит | energy per bit |

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**OF TELECOMMUNICATION SIGNALS RECEPTION**

**Education manual**

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