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Methodical guidelines for fulfilling laboratory works and individual tasks in Communication Theory

Module №2 – Communication Signals

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Odessa 2008

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The laboratory work 1.2
RANDOM PROCESSES PROBABILITY DISTRIBUTIONS RESEARCHING

1. Objectives

Studying and experimental determination of one-dimensional probability distribution functions and the probability density functions of random processes.

2. Main positions

2.1. It is assumed that studying processes are stationary and ergodic. In such processes one-dimensional probability distribution function and one-dimensional probability density function do not depend on time.

2.2. By definition the value of one-dimensional probability distribution function $F(x)$ is equal to the probability of that in the arbitrary time moment process. $X(t)$ will take on the value that does not exceed x:

$$F(x) = P\{X(t) \le x\}. \tag{1}$$

The value of one-dimensional probability density function $p(x)$ is equal to the limit of ratio of probability of that in the arbitrary time moment process $X(t)$ will take on the value from interval $(x - \Delta x/2, x + \Delta x/2)$ to the interval length Δx when $\Delta x \to 0$:

$$p(x) = \lim_{\Delta x \to 0} \frac{P\{x - \Delta x/2 < X(t) \le x + \Delta x/2\}}{\Delta x}. \tag{2}$$

The properties of $F(x)$ and $p(x)$ functions shown on the table below are easy to prove using their definitional formulas (1) and (2).

Table 1 – The properties of the functions $F(x)$ and $p(x)$

	$p(x)$	$F(x)$
1	$P\{x < X(t) \le x + dx\} = p(x)dx$	$F(x) = P\{X(t) \le x\}$
2	$P\{x_1 < X(t) \le x_2\} = \int_{x_1}^{x_2} p(x)dx$	$P\{x_1 < X(t) \le x_2\} = F(x_2) - F(x_1)$
3	$\int_{-\infty}^{\infty} p(x)dx = 1$	$F(\infty) = 1; \quad F(-\infty) = 0$
4	$p(x) \ge 0$	$F(x_2) \ge F(x_1)$ when $x_2 > x_1$
5	$p(x) = \frac{dF(x)}{dx}$	$F(x) = \int_{-\infty}^{x} p(x)dx$

The functions $F(x)$ and $p(x)$ are used to calculate the probabilities of getting values of the process in the given interval (line 2, table 1), to perform statistical averaging at determination of process characteristics or the result of certain operation with random process.

2.3. For processes which are often used, analytical expressions of functions $F(x)$ and $p(x)$ are known.

For a **Gaussian (a normal) process** (for example, fluctuation noise):

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-a)^2}{2\sigma^2}}, \tag{3}$$

where $a = \overline{X(t)}$ is the average value or the expectation of a random process

$$a = \int_{-\infty}^{\infty} x\, p(x) dx ; \quad (4)$$

σ – root-mean-square deviation of a random process, it is determined as $\sigma = \sqrt{D[X(t)]}$;

$D[X(t)]$ – variance of a random process (an average value of a squared deviation of a value of random process from his average value)

$$D[X(t)] = \int_{-\infty}^{\infty} (x - a)^2 p(x) dx . \quad (5)$$

The probability distribution function of normal process has the following expressions:

$$F(x) = 1 - Q\left(\frac{x - a}{\sigma}\right), \quad (6)$$

here

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_{z}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt$$

– *Q*-function or addition to Gaussian probability distribution function.

On a fig. 1*a*, the graphs of the probability distribution are given at $a = 1$ and $\sigma = 0{,}5$.

Probability distribution of a **harmonic oscillation** $X(t) = A \cdot \cos(2\pi f t + \varphi)$, where *A* and *f* are constants, and φ is random value, is described by the expressions:

$$p(x) = \begin{cases} \dfrac{1}{\pi\sqrt{A^2 - x^2}}, & |x| \le A, \\ 0, & |x| > A; \end{cases} \qquad F(x) = \begin{cases} 0{,}5 + \dfrac{1}{\pi}\arcsin\dfrac{x}{A}, & |x| \le A, \\ 0, & |x| > A. \end{cases} \quad (7)$$

The average value of harmonic oscillation is equal to zero, and root-mean-square deviation is equal to $A/\sqrt{2}$. In fig. 1 *b*, the graphs of probability distribution of harmonic oscillation are given at $A = 2$. If $x = A$, then the value of probability density tends towards infinity.

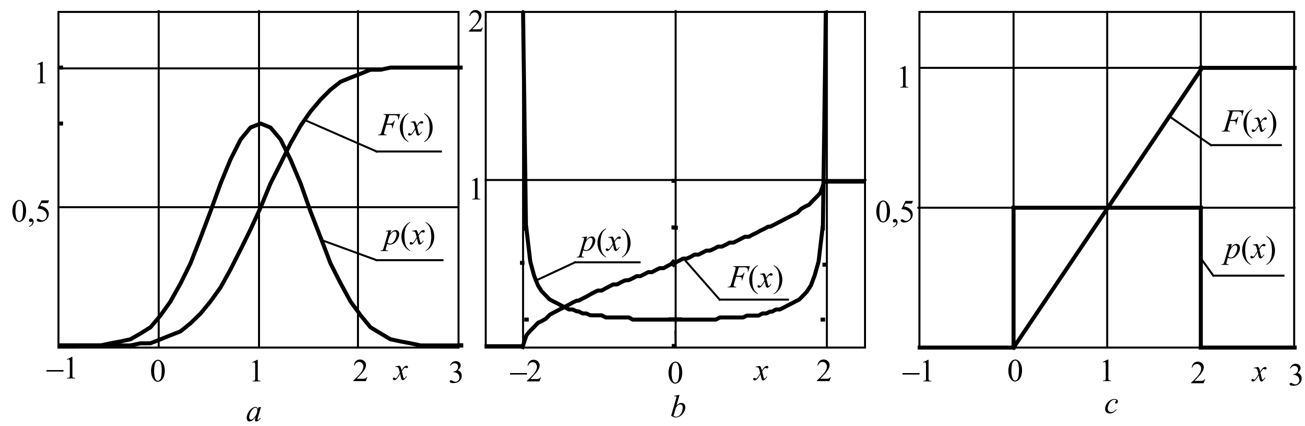

Figure 1 – The probability distributions: *a* –of the Gaussian random process; *b*– of the harmonic oscillation; *c* – random process with a uniform distribution

Functions *F*(*x*) and *p*(*x*) for the random process with a **uniform distribution** on the interval (x_{min}, x_{max}) are written down as:

$$p(x)=\begin{cases}\dfrac{1}{x_{max}-x_{min}}, & x_{min}<x\le x_{max},\\ 0, \quad x\le x_{min},\ x>x_{max};\end{cases} \qquad F(x)=\begin{cases}0 & \text{при} \quad x\prec x_{min};\\ \dfrac{x-x_{min}}{x_{max}-x_{min}} & \text{при} \quad x_{min}\le x\le x_{max};\\ 1 & \text{при} \quad x\succ x_{max}.\end{cases} \qquad (8)$$

The average value of the random process with a uniform distribution is equal to $(x_{min}+x_{max})/2$ and root-mean-square deviation is equal to $(x_{max}-x_{min})/\sqrt{12}$. The graphs of a uniform probability distribution for $x_{min}=0$ and $x_{max}=2$ are given on a fig. 1, *c*.

3. Questions

3.1. Which processes are called stationary and ergodic?

3.2. Give the definition of the one-dimensional probability distribution function of random process and prove its properties.

3.3. Give the definition of the one-dimensional probability density function of the random process and prove its properties.

3.4. How can you find the probability of getting values of the process in the set interval, using the probability distribution function or the probability density function?

3.5. Write down the expressions for the expectation and variance of a random process. What is their physical meaning?

3.6. Write down expression for the normal probability distribution function and explain the content of values connected with it.

3.7. Explain the type of the graphs of probability distribution function of the harmonic oscillation with an accidental phase, fluctuation noise, and the random process with a uniform distribution.

3.8. Describe the principle of operation of devices to measure the probability distribution function and probability density function of random process.

4. Home task

4.1. Study "Probabilistic characteristics of the random processes" from the compendium of lectures and literature.

4.2. Execute calculations and build probability distribution function $F(x)$ and probability density function $p(x)$ graphs of the normal (Gaussian) random process, $a=0$ and root-mean-square deviation $\sigma=1+0{,}1N$(where N is number of workplace) for the values $-3\sigma<x<3\sigma$. In the absence of the integral table of probability it is possible to take advantage of the approximation formula:

$$Q(z)\cong 0{,}65\exp[-0{,}44(z+0{,}75)^2] \text{ under } z>0;$$

$$Q(z)=1-Q(|z|) \text{ under } z<0,\ Q(0)=0{,}5,\ Q(\infty)=0.$$

Results of calculations should be presented in the form of tables and graphs.

4.3 Be ready to discuss key questions.

5 Laboratory task

5.1 Acquaintance with a virtual model on a workplace

Start the program 1.2, using the icon TT(English) on the desktop. It is necessary to study the structure of a virtual model using its description in part 6 of this LW and to master introduction of parameters. Coordinate the plan of performance of the laboratory task the teachers one.

5.2 Research of the random process with a uniform distribution probability

Click in the menu “Choice of process“ item “With a uniform distribution”. Place in corresponding windows values $x_{min}=-1$ and $x_{max}=1$. Ultimate values of argument at the analysis of distributions are $x_{low}=-2$ and $x_{up}=2$. Write down measured average value, and root-mean-square deviation, graphs of probability distribution function and probability density function. On the instructions of the teacher repeat measurements for other values x_{min} and x_{max}.

5.3 Research of a Gaussian process

Click in the menu “Choice of process” item “Gaussian process ". Place in corresponding windows values a and σ, from the homework, and choose values x_{min} and x_{max} such that they cover a range of values x from $a - 3\sigma$ up to $a + 3\sigma$. Write down measured average value, and root-mean-square deviation, graphs of probability distribution function, and the probability density function. On the instructions of the teacher repeat measurements for other values of average value a and root-mean-square deviation σ.

5.4 Research of statistical characteristics of a harmonic oscillation

Click in the menu “Choice of process” item “Harmonic oscillation”. Place in corresponding windows value of amplitude $A = 1$, value of frequency f the order 10...20 kHz and value of an accidental phase φ. Establish ultimate values of argument at the analysis of distributions so that they cover a range of values x from $-A$ up to $+A$. Write down measured average value and root-mean-square deviation, graphs of probability distribution function, and the probability density function. On the instructions of the teacher repeat measurements for other values of A, frequency f, phase φ.

6 Description of laboratory model

Laboratory work is executed on a computer in the HP VEE environment using a virtual model. The block diagram of virtual model is given on the fig. 2. The model enables to investigate characteristics of random process with a uniform probability distribution, Gaussian random process, and harmonic oscillation.

This virtual model realizes two basic functions for each process:

1. Generation of the N samples of researched random process $X(t)$. Samples are displayed. This display is called "Realization of the process";

2. Calculations on basis of the generated samples of values and displaying them:
   a) probability distribution function;
   b) probability density function;
   c) average value of process;
   d) root-mean-square deviation of process.

For every researched random process different methods of generation of samples, different parameters of processes are used.

The generation of samples of process with a uniform distribution is executed with the built-in function “randomize”. The values of x_{min} and x_{max} are preset in the model.

The generation of samples of Gaussian process is executed by nonlinear transformation of two arrays of samples $u(i)$ and $v(i)$ of random process with a uniform distribution on an interval (0, 1).

Transformation is given by

$$X(i) = a + \sigma \cdot \sqrt{-2\ln(u(i))} \cdot \cos(2\pi v(i)), \; i = \overline{1, N}, \qquad (9)$$

here i is the number of the sample in an array; a and σ are the average value and root-mean-square deviation of researched random process, which a researcher sets on a model.

A built-in functional generator executes the generation of samples of harmonic oscillation. A researcher sets amplitude, frequency, and the initial phase of oscillation.

The calculation of values of probability distribution function and probability density function is executed in the range of argument values from lower-range value x_{low} and to upper-range value x_{up}. An interval (x_{low}, x_{up}) is divided on M of identical subintervals $\Delta x = (x_{up} - x_{low})/M$; the quantity of samples k_j, which get in a j-th subinterval is calculated (j takes on values from 1 to M). Frequency of getting of sample values in the j-th subinterval $q_j = k_j/N$. At sufficiently large values M and N (in the model $M = 200$, $N = 10000$) values of frequency q_j gives the probability of getting the sample values in the j-th subinterval. Probability of getting of sample values in the j-th subinterval is $q_j = p(x_j)\Delta x$, where $x_j = j\Delta x$ (according line 1 in table 1). Therefore

$$p(x_j) = \frac{k_j}{N\Delta x} = \frac{k_j M}{N(x_{\text{up}} - x_{\text{low}})}, \quad j = \overline{1, M}$$

Arrays of values $p(x_j)$ and x_j readout on the display “Probability density function”.

Using property of probability distribution function $F(x)$ (line 5 table 1), the array of values is calculated:

$$F(x_j) = \Delta x \sum_{k=1}^{j} p(x_k), \quad j = \overline{1, M}$$

Arrays of values $F(x_j)$ and x_j readout on the display "Probability distribution function".

The average value of the researched process is calculated on the formula

$$\overline{X(i)} = \frac{1}{N} \sum_{i=1}^{N} X(i),$$

here $X(i)$, $i = \overline{1, N}$ is i-th sample of the researched process. The value $\overline{X(i)}$ is displayed. This display is called "Measured average value".

Root-mean-square deviation of the researched process calculates as

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X(i) - \overline{X(i)})^2} .$$

The value σ is displayed. This display is called “Measured root-mean-square deviation”.

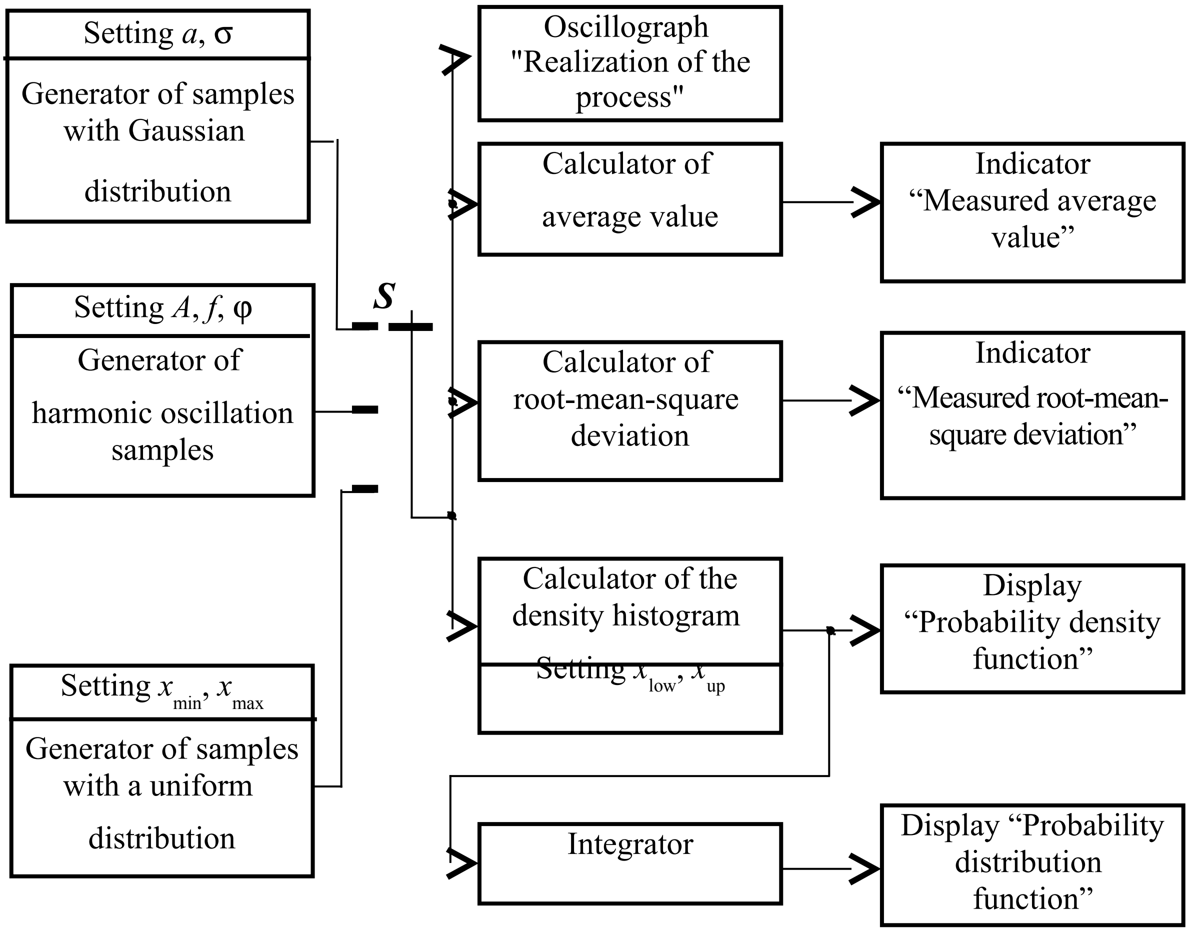

Figure 2 – Virtual model block diagram

7 Requirements to the report

7.1 Title of laboratory work.
7.2 Objectives of laboratory work.
7.3 Results of the homework execution.
7.4 Block diagram of researches.
7.5 Results of the execution of items 5.2–5.5 of laboratory task (graphs, oscillograms, numerical values, etc.).
7.6 Conclusions on every item of the laboratory task, with analysis of the got results:

- coincidence of form of functions $p(x)$ and $F(x)$ each of researched process to theoretical;
- implementation of properties $p(x)$ and $F(x)$,
- coincidence of measured average value and root-mean-square deviation with calculated, on the given parameters of the researched process (x_{min} and x_{max}, A);
- dependence of functions $p(x)$ and $F(x)$ from frequency and initial phase of harmonic oscillation.

7.7 Signature of student about the laboratory work execution, teachers signature for the laboratory work defence with estimation and date.

Literature

1. **Баскаков С.И.** Радиотехнические цепы и сигналы: Учебник для вузов.– М.: Радио и связь, 1988 (1983).

2 **Теория** передачи сигналов: Учебник для вузов / А.Г. Зюко и др. – М.: Радио и связь, 1986.

The laboratory work 1.3
CORRELATION CHARACTERISTICS OF RANDOM PROCESSES AND DETERMINISTIC SIGNALS

1. Objectives

1.1 Studying the method of experimental determination of correlation characteristics of random processes and deterministic signals.

1.2 Research the connection between correlation functions and spectrums of random processes and deterministic signals.

2. Main positions

2.1 The correlation function (CF) of the random process $X(t)$ is the expectation of the process values product, which it takes on in the time moments t_1 and t_2:

$$K_X(t_1, t_2) = \overline{X(t_1) \cdot X(t_2)}. \tag{1}$$

CF values $K_X(t_1, t_2)$ determine the quantity of statistical dependence between the values of process in the time moments t_1 and t_2. For the stationary processes, the values of the CF do not depend on choice and t_2. They depend on the distance between them $\tau = t_2 - t_1$. CF is denoted as $K_X(\tau)$. Further we will consider only stationary processes and suppose that they are ergodic. For the ergodic processes CF is determined as:

$$K_X(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t+\tau)\, dt, \tag{2}$$

here $x(t)$ is realization of the process $X(t)$.

2.2 Independent of the form of the CF of different processes, correlation function has such properties as:

– $K_X(0) = P_X$, here P_X is average power of process;

– $K_X(0) \geq K_X(\tau)$ – if $\tau = 0$ the value of the function $K_X(\tau)$is maximal;

– $K_X(\tau) = K_X(-\tau)$ – $K_X(\tau)$ is an even function;

– $K_X(\infty) \to \overline{X(t)}^2$, here $\overline{X(t)}$ is the average value of the process.

2.3 Than less value of $K_X(\tau)$ in comparison with $K_X(0)$, the less statistical dependence between the values of process, distant on τ one after another. If $K_X(\tau) = 0$, then values of process $X(t)$, distant on such time interval as τ, are uncorrelated. It is easier to compare the values $K_X(\tau)$ and $K_X(0)$, if to pass to the normalized correlation function

$$R_X(\tau) = \frac{K_X(\tau)}{K_X(0)} \tag{3}$$

$R_X(0) = 1$ and $-1 \leq R_X(\tau) \leq 1$.

2.4 Often, for a description of correlation properties of random processes instead of the CF a correlation time τ_c is used. The correlation time is used for "rough" description of correlation properties of process. Values of process, distant one after another on $\tau > \tau_c$, are uncorrelated. Values of process, distant one after another on $\tau \leq \tau_c$, are correlated. Different methods of determination of correlation time are used:

1) Correlation time τ_c is the base of rectangle in high $K_X(0)$, the area of this rectangle is equal to the area under the curve of the CF module (fig. 1, *a*):

$$\tau_c = \frac{1}{K_X(0)} \int_0^{\infty} |K_X(\tau)|\, d\tau \tag{4}$$

2) Such values of τ_c, that $\tau > \tau_c$ values of CF do not exceed some given level (fig. 1, *b*).

3) If the CF has an oscillating character, it is possible to take a value of τ at which CF first time takes on a zero value, as the correlation time τ_c, (fig. 1, *c*).

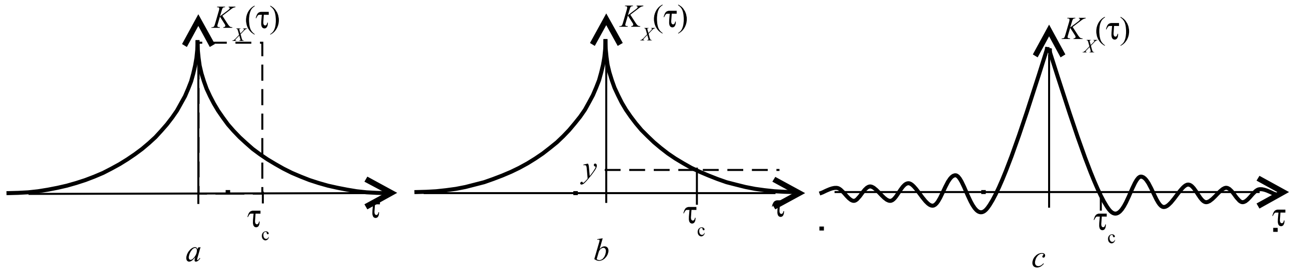

Figure 1 – Determination of correlation time

2.5 In accordance with (2) it is impossible to measure CF precisely, because for this purpose realization of process of infinite duration is needed. It is possible to measure CF in case of realization of the random process of finite duration. It is obvious that the longer the realization of the process T_{real}, the more precisely measured CF of realization represents CF of process. The device for measuring CF of realization is named a correlation meter (fig. 2). Here delay time τ determines the argument of the measured value of the CF. If correlation meter, shown on a fig. 2, execute on a processor or on a computer, it is possible to get the array of the $K_X(kT_s)$ values, where T_s is sampling interval of the process realization *x(t)*; values of argument taken from the interval $-T_{real} \le kT_s \le T_{real}$. The got arrays of values kT_s and $K_X(kT_s)$ are displayed.

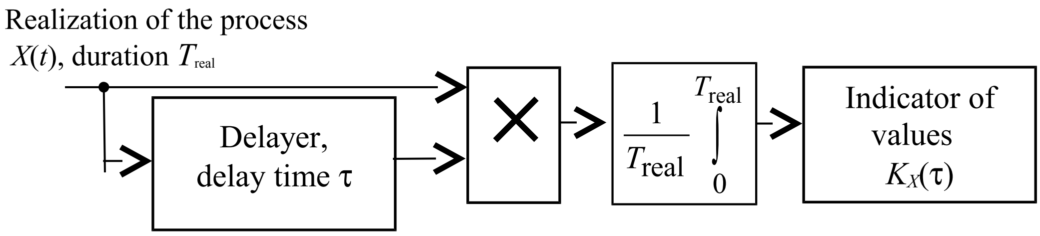

Figure 2 – Block diagram of correlation meter

2.6 The power spectral density $G_X(f)$, which determines the distribution of power of the process on frequencies, is a main spectral description of random processes. Quantitatively function $G_X(f)$ determines power of process in band extension 1 Hz near frequency *f*. Khinchin-Wiener theorem asserts that the functions $K_X(\tau)$ and $G_X(\omega)$ are bound by the Fourier transform

$$\left. \begin{aligned} G_X(\omega) &= 2\int_0^{\infty} K_X(\tau)\cdot\cos(\omega\tau)d\tau; \\ K_X(\tau) &= \frac{1}{\pi}\int_0^{\infty} G_X(\omega)\cdot\cos(\omega\tau)d\omega\ . \end{aligned} \right\} \quad (5)$$

If the $G_X(f)$ function is known, it is possible to define average power of a process

$$P_X = \int_0^{\infty} G_X(f)\,df \quad (6)$$

In particular, if a process is a quasi-white noise with the power spectral density N_0 in band $(0, F_{max})$,

$$P_X = N_0 \cdot F_{max}. \quad (7)$$

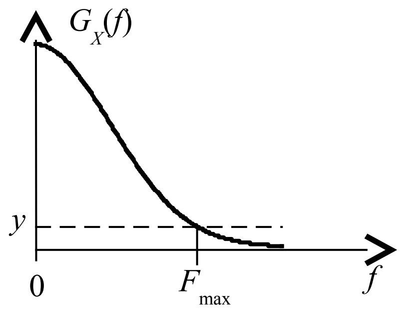

Figure 3 - Determination of bandwidth

2.7 It is often enough to know the bandwidth of the process F_{max}. The bandwidth of random process is determined by the function $G_X(f)$ in such a way as the bandwidth of the deterministic signal. In fig. 3 it is shown, how to determine a bandwidth at set level y, i.e. F_{max} is band extension, outside which the power spectral density of process does not exceed the value y.

The $K_X(\tau)$ and $G_X(f)$ functions are bound by the Fourier transform, there is connection between the bandwidth F_{max} and correlation time $\tau_к$ of the process:

$$\tau_к \cdot F_{max} = 0{,}5. \qquad (8)$$

In expression (8), sign "=" means that the product of correlation time and bandwidth of process is a value order of magnitude 0,5.

2.8 A correlation function is a description for the deterministic signal, but it does not have such interpretation, as for a random process. CF of a nonperiodic deterministic signal is determined

$$K_s(\tau) = \int_0^{T_s} s(t)s(t+\tau)dt, \qquad (9)$$

here T_s is duration of signal $s(t)$.

To measure the CF of the deterministic signal it is possible with the correlation meter, the block diagram of which is resulted on a fig. 2. According to this diagram integration is executed on an interval $(0, T_s)$ and a factor before this integral is missed.

Let $s(t)$ is rectangular video pulse of amplitude A and duration T_p

$$s(t) = \begin{cases} A, & 0 \le t < T_p, \\ 0, & t < 0, \ t \ge T_p. \end{cases} \qquad (10)$$

After substitution of expression (10) in expression (9) we will get

$$K_s(\tau) = \begin{cases} A^2 T_p (1 - |\tau|/T_p), & |\tau| \le T_p, \\ 0, & |\tau| > T_p. \end{cases} \qquad (11)$$

The CF of a rectangular video pulse is shown on fig. 4, *a*.

As shown in expression (9) $K_s(0) = E_s$ – energy of a signal $s(t)$. The Fourier transform from $K_s(t)$ gives the square of amplitude spectrum (energy spectral density) of signal $s(t)$. The Fourier transform from expression (11) gives the square of the known expression for the amplitude spectrum of rectangular video pulse

$$S^2(f) = \left(A T_p \frac{\sin(\pi f T_p)}{\pi f T_p} \right)^2, \quad -\infty < f < \infty. \qquad (12)$$

2.9 We will consider rectangular radio pulse, duration T_p

$$s(t) = \begin{cases} A\sin(2\pi f_0 t + \varphi_0), & 0 \le t < T_p, \\ 0, & t < 0, \ t \ge T_p, \end{cases} \qquad (13)$$

where A, f_0 and φ_0 is amplitude, frequency and initial phase of oscillation.

After substitution (13) in (9) we will get

$$K_s(\tau) = \begin{cases} 0{,}5 A^2 T_p (1 - |\tau|/T_p)\cos 2\pi f_0, & |\tau| \le T_p, \\ 0, & |\tau| > T_p. \end{cases} \qquad (14)$$

From expression (14) follows, that CF of rectangular radio pulse is cosine curve with a zero initial phase and does not depend on the phase of rectangular radio pulse. Therefore, if the initial phase of rectangular radio pulse φ_0 is a variate, CF of rectangular radio pulse is determined by a formula (14). Envelope of CF of rectangular radio pulse coincides with CF of signal which represents envelope of rectangular radio pulse. The graph of CF of rectangular radio pulse, built on a formula (14) for $f_0 = 4/T_p$, on a fig. 4, *b* is given.

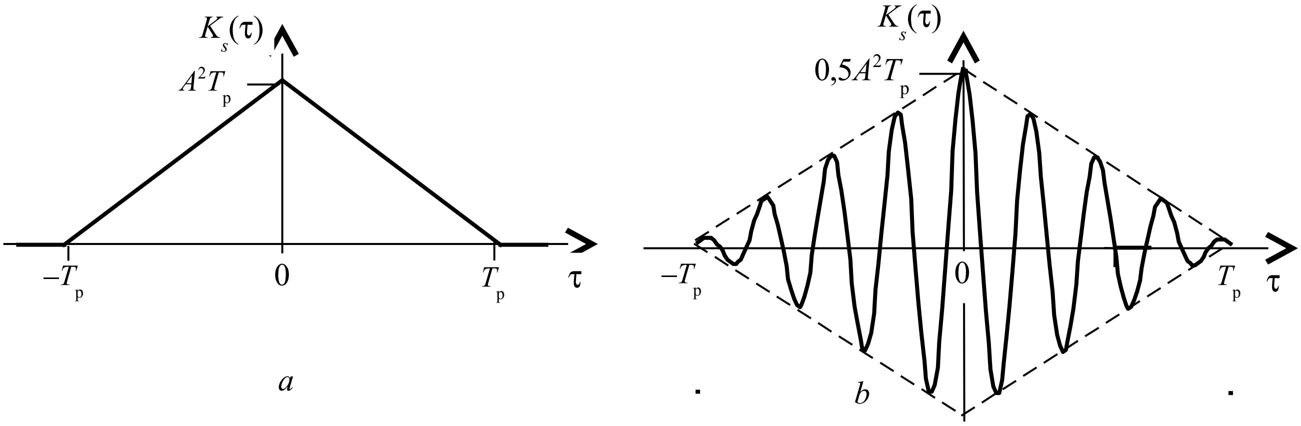

Figure 4 – Correlation functions of: *a* – rectangular video pulse, *b* – rectangular radio pulse

The Fourier transform from expression (14) gives the square of amplitude spectrum of signal (12)

$$S^2(f) = 0{,}25\left(AT_p \frac{\sin\left(\pi(f - f_0)T_p\right)}{\pi(f - f_0)T_p} \right)^2, \quad -\infty < f < \infty . \tag{15}$$

3. Questions

3.1 Give the definition of the CF of the random process.
3.2 How to determine CF of the process?
3.3 Enumerate main properties of the CF of random process.
3.4 What parameters of random process is it possible to define on its CF?
3.5 What is asserted by the Wiener-Khinchin theorem?
3.6 Enumerate methods of determination of correlation time.
3.7 How is a bandwidth and correlation time of random process connected?
3.8 What form has the CF of rectangular video pulse?
3.9 What form has the CF of rectangular radio pulse?
3.10 Why does the initial phase of rectangular radio pulse not influence on its CF?

4. Home task

4.1 Study the "Correlation theory of random processes" from the compendium of lectures and literature [1, p. 73...79, 149...164; 2, p. 67...72, 109...118].

4.2 Build a block diagram of the correlation meter for the research of correlation functions of random processes and deterministic signals.

4.3 Execute calculations and build graphs for the CF of rectangular video pulse and rectangular radio pulse for such input data: T_p = 2 ms, frequency of oscillation of radio pulse signal $f_0 = 500(N + 1)$ Hz, here N is the number of workplace. Execute calculations and build the graphs of spectrums for the given pulses using expressions (12) and (15).

4.4 Prepare for discussion on key questions.

5 Laboratory task

5.1 Acquaintance with a virtual model on a workplace

Start the program **1.3**, using the icon **TT(English)** on the desktop. It is necessary to study the structure of a virtual model using its description in part 6 of this LW and master set of parameters. Coordinate the plan of fulfilling of the laboratory task with the teacher.

5.2 Research of correlation and spectral characteristics of realization of noise

Set in the generator of the quasi-white noise F_{max} = 1000 Hz. After run the program, analyse the experimental data and write down it. Check up implementation of properties of correlation function, determine maximal frequency on a spectrum, and determine correlation time on a correlation function, find their product, compare him with the theoretical value (8). Provide the visual estimate of the average value of the power spectral density N_0 on an interval (0, F_{max}). Multiply the value of power spectral density N_0 on F_{max} and compare the product with the value of the measured average power of realization – expression (7).

On the instructions of the teacher repeat measurements for other values $F_{max.}$.

5.3 Research of correlation and spectral characteristics of rectangular video pulse

Set in the generator of rectangular video pulse A = 2 V, T_p = 0,5 ms. After run the program, sketch the $K_s(\tau)$ and $S^2(f)$ graphs. Analyse the experimental data and write down it. Compare the experimental dependence $S^2(f)$ with the theoretical (12); compare the experimental dependence $K_s(\tau)$ with the theoretical one(11); compare measured value of pulse energy with the value of $K_s(0)$

On the instructions of the teacher repeat research for other values A and T_p

5.4 Research of correlation and spectral characteristics of rectangular radio pulse

Set in the generator of rectangular radio pulse A = 2 V, f_0 = 1000 Hz. After run the program, sketch the $K_s(\tau)$ and $S^2(f)$ graphs. Analyse the experimental data and write down it. Compare the experimental dependence $S^2(f)$ with theoretical (15), compare the experimental dependence $K_s(\tau)$ with theoretical one(14), and compare the measured value of pulse energy with the value of $K_s(0)$. Write down the value of the initial phase. Launch the program and make sure, that a correlation function does not depend on an initial phase. On the instructions of the teacher repeat research for other values A and T_p

6 Description of laboratory model

Laboratory work is executed on a computer in the HP VEE environment using of virtual model. The block diagram of virtual model is given on the fig. 5. A model contains the following generators:

– generator of noise, which produce the realization of quasi-white noise with the band (0, F_{max}), duration 20 ms, as a 5000 samples; it is possible to set the F_{max} value 1000, 2000 and 3000 Hz;

– generator of single rectangular video pulse allows to set pulse duration 0,5, 1 and 1,5 ms and arbitrary amplitude of pulse;

– generator of rectangular radio pulse, duration 2 ms, allows to set arbitrary amplitude of pulse and frequency of oscillation f_0 1000, 2000 and 3000 Hz. The initial phase of oscillation is a random value, this value is readout on the indicator φ.

The switch S allows to choose the researched process.

If for research noise is chosen, on displays is represented:

– realization of noise;

– value of the measured average power of realization;

– correlation function of realization, calculated using the algorithm which is given on the fig. 2;

– power spectral density of realization of noise, got as the Fourier transform from the correlation function of realization. The program generates samples of quasi-white noise. However, because of a few of samples, the spectrum is far from white in the band (0, F_{max}).

If for research a rectangular video pulse or rectangular radio pulse is chosen, on displays is represented:

– oscillogram of pulse;

– value of measured pulse energy;

– correlation function of pulse, calculated on a formula (9);

– square of amplitude spectrum of pulse, got as the Fourier transform from the correlation function of pulse.

In all cases for the calculation of CF the built-in function “Xcorrelate” is used.

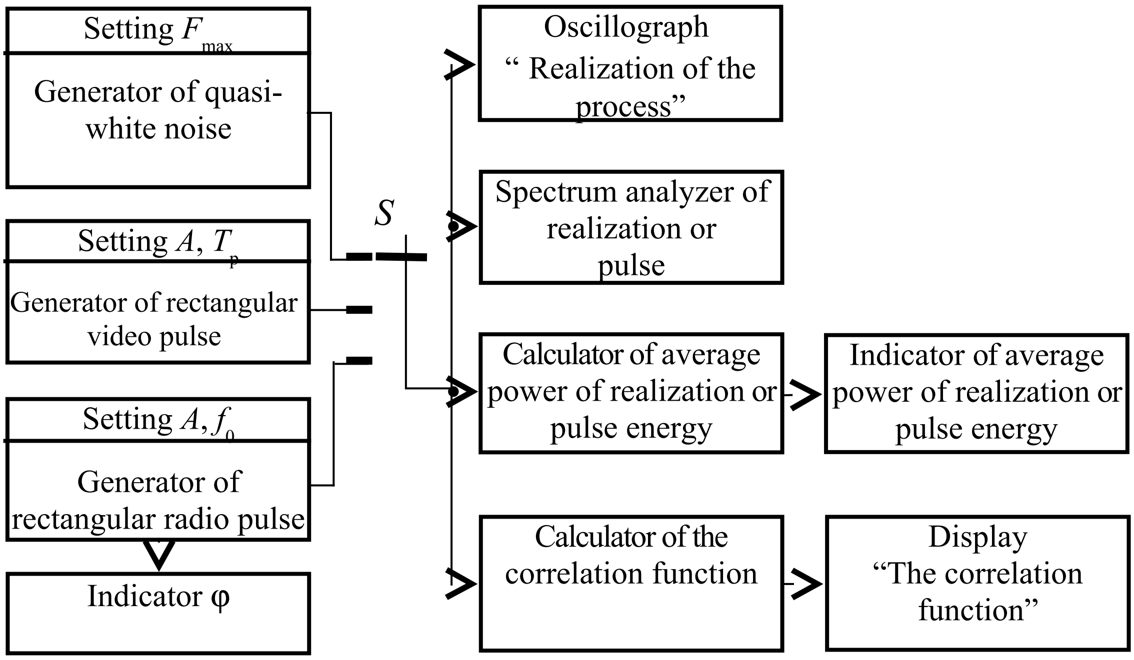

Fig. 5 - The block diagram of virtual model

7 Requirements to the report

7.1 Title of laboratory work.

7.2 Objectives of laboratory work.

7.3 Results of the homework execution.

7.4 Block diagram of researches, list of devices which are used in LW.

7.5 Results of implementation of items 5.2-5.4 of laboratory task (graphs, oscillograms, numerical values, etc.).

7.6 Conclusions on every item of laboratory task, with analysis of the got results (verification of implementation of properties of correlation functions, coincidence of experimental and theoretical data, etc.).

7.7 Signature of student about the laboratory work implementation, signature of teacher about the laboratory work defense with estimation, date.

Literature

1. **Баскаков С.И.** Радиотехнические цепы и сигналы: Учебник для вузов.– М.: Радио и связь, 1988 (1983).

2. **Гоноровский И.С.** Радиотехнические цепы и сигналы: Учебник для вузов. – М.: Радио и связь, 1986 (1977).

Laboratory work 1.4
AM, DSB-SC, SSB MODULATED SIGNALS RESEARCHING

1. Objectives

1.1 The modulated signals time and spectral characteristics research.
1.2 Relation between characteristics of the modulated and modulating signals research.

2 Main positions

2.1 Carrier of amplitude (AM), double-sideband-suppressed-carrier (DSB-SC) and single-side-band (SSB) modulations is harmonic oscillation $u_{car}(t) = A_0 \cos(2\pi f_0 t + \varphi_0)$. The modulating signal is a telecommunication baseband continuous signal $b(t)$ with such characteristics:
- the signal spectrum maximum frequency is F_{max};
- signal is normalized such as module maximum values $|b(t)|_{max} = 1$;
- signal average value $\overline{b(t)} = 0$.

2.2 In case of AM the carrier amplitude changes are proportional to instant values of a modulating signal, i.e. amplitude of the modulated signal $A(t) = A_0 + \Delta A b(t)$, where ΔA – factor of proportionality which choose such as amplitude $A(t)$ did not accept negative values. As $|b(t)|_{max} = 1$, that ΔA defines the greatest on the module a carrier amplitude change. In order that amplitude $A(t)$ did not accept negative values, it is necessary to provide $\Delta A \le A_0$. Frequency and initial phase of a carrier remain invariable. It is convenient to pass to a relative maximum change of amplitude – the amplitude modulation factor $m_{AM} = \Delta A/A_0$. Clearly, that $0 < m_{AM} \le 1$.

Analytical expression AM signal in case of any modulating signal looks like

$$s_{AM}(t) = A_0[1 + m_{AM} b(t)]\cos(2\pi f_0 t + \varphi_0). \quad (1)$$

We see, that parameters of AM signal are m_{AM}, A_0, f_0 and φ_0. Time diagram AM signal is shown on fig. 1. Attracts attention that envelope the modulated signal repeats the form of a modulating signal – amplitude AM signal $A(t)$ is envelope high-frequency oscillations $\cos(2\pi f_0 t + \varphi_0)$ (on fig. 1 envelope is represented by a dashed-line curve).

2.3 On fig. 2 any amplitude spectrum of a modulating signal and amplitude spectrum AM signal corresponding to it are shown. Amplitude spectrum AM signal consists of carrier frequency harmonic oscillation, upper sideband of frequencies (USB) and the lower sideband of frequencies (LSB). Thus USB is a copy of a spectrum of the modulating signal, which shifted on frequency on f_0. LSB is mirror reflection USB relative to carrier frequency f_0.

From fig. 2 the important result followers: the AM signal spectrum width F_{AM} is equaled to the doubled value of the modulating signal spectrum maximum frequency, i.e. $F_{AM} = 2F_{max}$.

2.4 Calculations show, if modulating signals are telecommunication baseband signals than sidebands power makes some percent from modulated signal power. Therefore it is expedient to generate a signal with a spectrum which consists only of two frequencies sidebands (carrier frequency oscillation is absent), – such signal is the signal of double-sideband-suppressed-carrier modulation.

Such kind of modulation when the modulated signal is product of a modulating signal and a carrier is called double-sideband-suppressed-carrier. Analytical signal DSB-SC expression looks like

$$s_{DSB\text{-}SC}(t) = A_0 b(t)\cos(2\pi f_0 t + \varphi_0). \quad (2)$$

Time diagrams of the modulating and modulated signals are resulted on fig. 3. As the modulating signal acts on amplitude of a carrier, DSB-SC considered as version AM. From fig. 3 is visible, that envelope signal DSB-SC $A(t) = A_0|b(t)|$ (shown by a dashed line) does not repeat modulating signal.

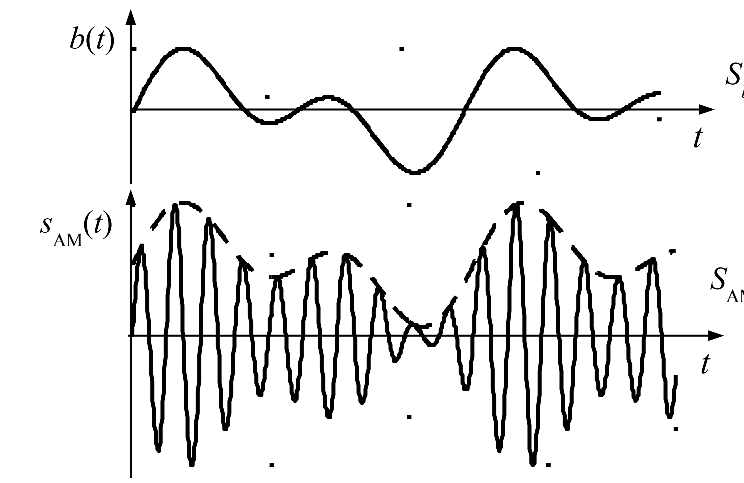

Figure 1 – Modulating $b(t)$ and modulated $s_{AM}(t)$ signals

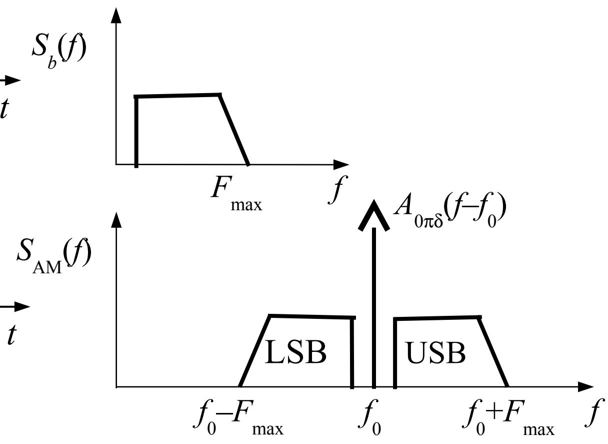

Figure 2 – Modulating and AM signals spectrum

From comparison of the mathematical expressions describing AM signal (1) and DSB-SC signal (2) we see, that spectrum DSB-SC signal differs from spectrum AM signal absence of carrier frequency oscillation. On fig. 4 any amplitude spectrum of a modulating signal and DSB-SC signal amplitude spectrum corresponding to it, which consists of USB and LSB, are shown. From fig. 4 follows, that DSB-SC signal spectrum width $F_{DSB\text{-}SC}$ is the same, as well as AM signal spectrum width: $F_{DSB\text{-}SC} = 2F_{max}$.

2.5 Such kind of modulation at which the modulated signal spectrum and modulating signal spectrum are the same shifted on carrier frequency or the first is an inversion of the shifted spectrum concerning carrier frequency is called single-sideband modulation. The SSB signal spectrum contains one sideband – upper or lower. The SSB signal can be written as

$$s_{SSB}(t) = A_0 b(t)\cos(\omega_0 t + \varphi_0) \mp A_0 \tilde{b}(t)\sin(\omega_0 t + \varphi_0), \quad (3)$$

where the sign "–" concerns the description of a signal with the upper sideband of frequencies, and a sign "+" – with the lower sideband; $\tilde{b}(t)$ – conjugated on Hilbert signal with a signal $b(t)$. The physical sense of Hilbert transform is enough simple: signal $\tilde{b}(t)$ differs from a signal $b(t)$ that phases of all its components are shifted on a π/2 angle.

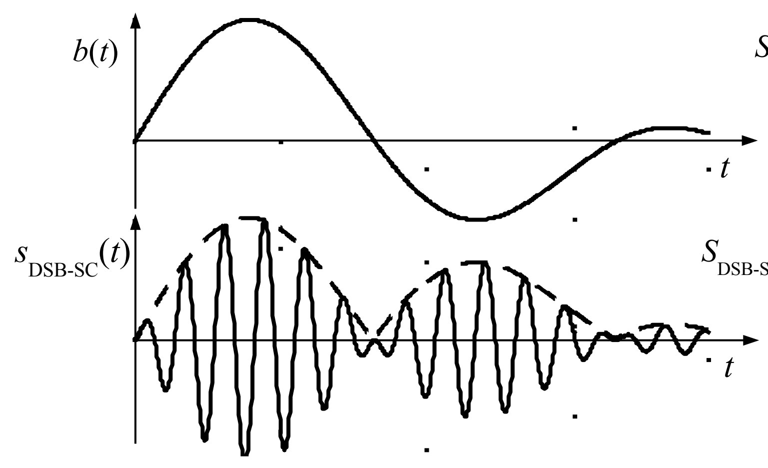

Figure 3 – Modulating $b(t)$ and modulated $s_{DSB\text{-}SC}(t)$ signals

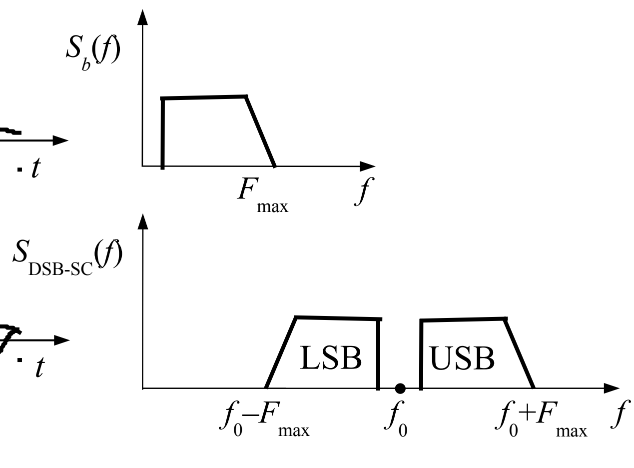

Figure 4 – Modulating and DSB-SC signals spectrum

Time modulating signal diagrams $b(t)$, conjugated on Hilbert $\tilde{b}(t)$ and SSB signal are situated on fig. 5. From fig. 5 is visible, that envelope SSB signal $A(t) = A_0\sqrt{b^2 + \tilde{b}^2}$ (it is shown by a dashed line) does not repeat modulating signal.

On fig. 6 any amplitude spectrum of a modulating signal and an amplitude spectrum corresponding to it SSB USB signal are shown. From fig. 6 follows, that the width of SSB signal spectrum F_{SSB} is twice less than width of AM and DSB-SC signals spectrum: $F_{SSB} = F_{max}$.

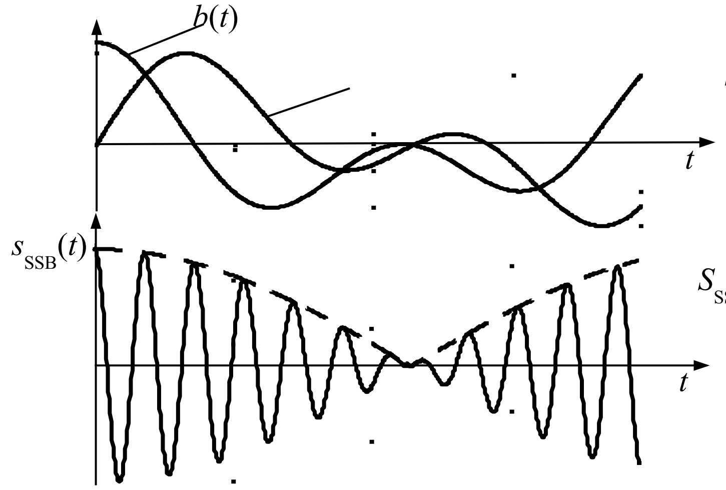

Figure 5 – Modulating $b(t)$ and modulated $s_{SSB}(t)$ signals

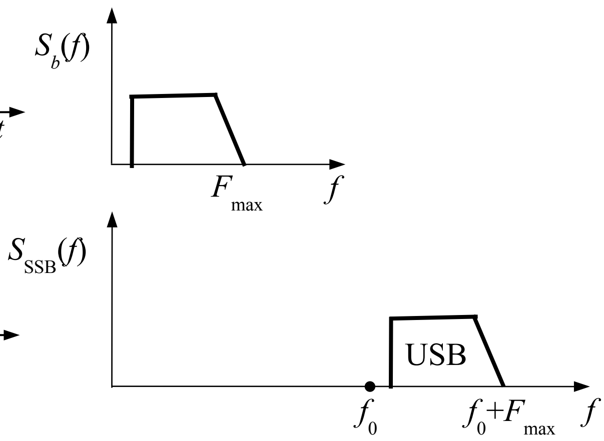

Figure 6 – Modulating and the SSB signals spectrum

2.6 Mathematical models of AM, DSB-SC and SSB signals as (1...3) are used for these signals formation and detection schemes construction.

3 Questions

3.1 What purpose is modulation in telecommunication systems used for?

3.2 Define amplitude, double-sideband-suppressed-carrier and single-sideband modulations.

3.3 What is amplitude modulation factor? What values it can accept?

3.4 What is Hilbert transform? What its physical essence?

3.5 Draw time diagrams AM, DSB-SC and SSB signals if a modulating signal is harmonic oscillation.

3.6 Represent AM, DSB-SC and SSB signals spectrum if a modulating signal is harmonic oscillation .

3.7 Represent AM, DSB-SC and SSB signals spectrum at set any modulating signal spectrum.

3.8 Explain, why SSB signal envelope on fig. 5 has such kind?

4 Home task

4.1 Study section “Amplitude modulation and its versions” under the compendium of lectures and the literature [1, pp. 53-60; 2, pp. 88-96] and the description of a laboratory model on section 6 of these instructions.

4.2 Carrier oscillation of frequency f_0 are modulated by a baseband signal $b(t) = A_1\sin(2\pi F_1 t) + A_2\sin(2\pi F_2 t) + A_3\sin(2\pi F_3 t)$. Represent baseband signal spectrum and AM, DSB-SC and SSB signals spectrum (put $m_{AM} = 1$). Initial data to the task according to your laboratory place number are given in table 1.

4.3 Be prepared for discussion on questions.

Table 1 – Initial data to a home task

Workplace number	A_1, V	F_1, Hz	A_2, V	F_2, Hz	A_3, V	F_3, Hz	f_0, Hz
1	0,3	50	0,4	100	0,3	250	800
2	0,3	100	0,3	200	0,4	300	900
3	0,4	50	0,3	200	0,3	250	1000
4	0,3	100	0,4	150	0,3	250	1100
5	0,3	50	0,3	250	0,4	300	1200
6	0,4	100	0,3	250	0,3	300	1000
7	0,3	50	0,4	100	0,3	150	800
8	0,3	100	0,3	200	0,4	300	900

5 Laboratory task

5.1 Familiarize with a virtual model on a workplace.
Start the program **1.4**, using the icon **TT(English)** on the desktop. Study scheme model, using the description in section 6 of this LW. Specify with the teacher the laboratory task performance plan.

5.2 Carry out researches of the modulated signals in time and frequency domain. For this purpose:

– set values A_1, F_1, A_2, F_2, A_3, F_3, factor m_{AM} and frequency f_0 the same, as well as in a homework;

– set an AM modulation kind and run program;

– sketch in the report the signals oscillogram and the spectrogram at the modulator input and output;

– set sequentially DSB-SC, USB SSB, LSB SSB modulation kinds, run program and sketch in the report the signals at the modulator output spectrogram;

– compare calculated in a homework and the obtained on model spectrograms, comparison results bring to report conclusions;

– conclude about concerning conformity of modulating signal forms and envelope the modulated signal for modulation different kinds.

5.3 Carry out modulated signals spectrum researches in case of changing carrier frequency. For this purpose at first increase by 200 Hz, and then reduce by 200 Hz carrier frequency, sketch in the report obtained signals spectrogram at the modulator output. Changes in spectrograms in comparison with received in item 5.2 bring in report conclusions.

5.4 Carry out research of AM signal spectrum dependence from modulation factor. For this purpose:

– set parameters A_1, F_1, A_2, F_2, A_3, F_3 and frequency f_0 the same, as well as in home task;

– set a kind of AM modulation and factor m_{AM} =0,7;

– compare the obtained oscillograms and spectrum at the output of the modulator with obtained in item 5.2, results of comparison to bring in report conclusions.

5.5 Carry out research the SSB signal in case of a single-tone modulating signal. For this purpose:

– set values A_1 = 1V, F_1 = 100 Hz, $A_2 = A_3 = 0$, frequency f_0 the same, as well as in a home task;

– set a kind of SSB USB, and then SSB LSB modulation;

– sketch in the report $b(t)$, $\tilde{b}(t)$ and $s_{SSB}(t)$ signals oscillogram and spectrogram;

– conclude about concerning conformity of $b(t)$, $\tilde{b}(t)$, $s_{SSB}(t)$ signals and envelope the modulated signal $A(t) = A_0\sqrt{b^2 + \tilde{b}^2}$.

6 Laboratory model description

Laboratory work is carried out on the computer in the environment of HP VEE with use of the virtual model which block diagram is situated on fig. 7.

Virtual model consists of the modulating continuous signal generator $b(t) = A_1\sin(2\pi F_1 t) + A2\sin(2\pi F_2 t) + A3\sin(2\pi F_3 t)$ and the modulator (the carrier generator is a part of the modulator). Harmonic oscillation frequencies and amplitudes values A_1, F_1, A_2, F_2, A_3, F_3, f_0, factor m_{AM} and carrier frequency is possible to change.

The virtual model scheme gives the chance to set modulation kinds: AM, DSB-SC, SSB USB and SSB LSB. Time and spectral diagrams of signals can be observed in two points of the virtual model scheme: at the modulator input and output. In a case of SSB on modulator input of the except modulating signal $b(t)$ is displayed a signal $\tilde{b}(t)$ on oscillograph. Together with modulated signal oscillogram, deduces the schedule envelope a signal by dotted line.

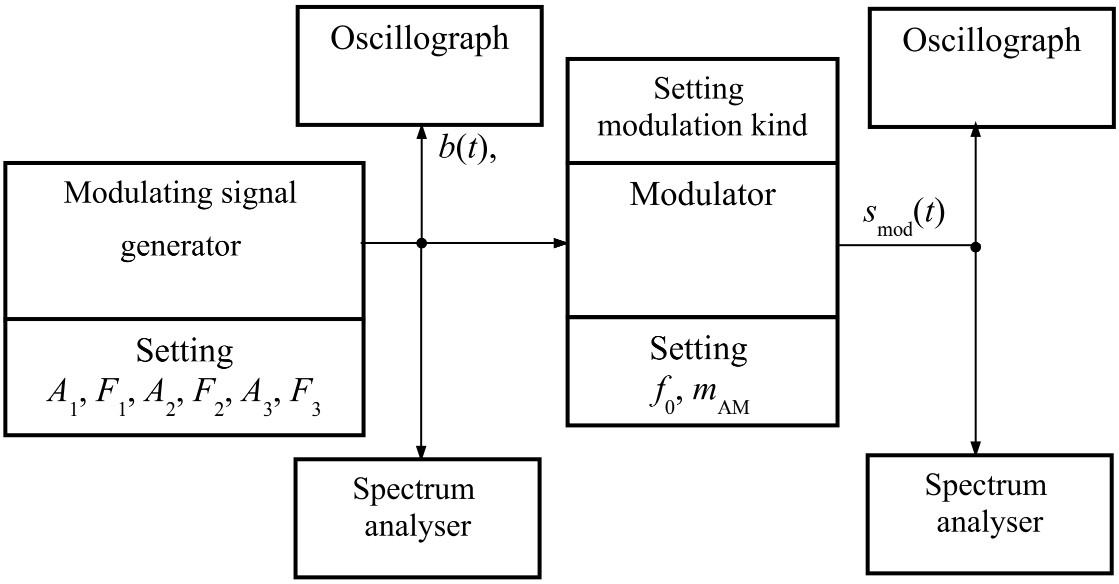

Figure 7 – Virtual model block diagram

7 Requirements to the report

7.1 Laboratory work title.

7.2 Work purpose.

7.3 Home task fulfillment results.

7.4 Block diagrams of researches and results of fulfillment of item 5.2... 5.5 laboratory tasks (oscillograms and spectrograms, each of them should have the subscription).

7.5 Conclusions on each item of the task in which you have to give the analysis of the obtained results (coincidence theoretical and experimental data, signals properties display, etc.).

7.6 Date, the student signature, the teacher visa with estimation in a 100-mark scale.

Literature

1. Панфілов І. П., Дирда В. Ю., Капацін А. В. Теорія електричного зв'язку: Підручник для студентів вузів 1-го та 2-го рівнів акредитації. – К.: Техніка, 1998.

2. **Баскаков С. И.** Радиотехнические цепы и сигналы. Учебник для вузов.– М.: Радио и связь, 1988 (1983).

Laboratory work 1.5
RESEARCH OF DIGITAL MODULATED SIGNALS

1. Objectives

1.1 Study of transmission methods of digital signals with modulated *M*-ASK, *M*-PSK and BFSK signals.

1.2 Research of time and spectral characteristics of *M*-ASK and *M*-PSK signals for $M = 2$ and 4 and BFSK signal.

2. Main positions

2.1 A baseband digital signal $b_u(t)$ is a sequence of binary symbols (bits) 1 and 0, that follow in an time interval T_b. In digital devices the rectangular pulse of high level corresponds to symbol 1, and the pulse of low level corresponds to symbol 0.

2.2 A digital modulation signal $s(t)$ is a sequence of radio pulses, that reflect a baseband signal and follow in the time interval T:

$$s(t) = \sum_{k=-\infty}^{\infty} s_i^{(k)}(t - kT), \tag{1}$$

where $s_i(t)$, $i = 0, \ldots, M-1$, are the elementary signals (radio pulses);

M is a number of elementary signals;

$s_i^{(k)}(t - kT)$ is the *i*-y radio pulse, that is translating in *k*-y time interval;

T is a clock period.

2.3 There is the general mathematical expression for radio pulse:

$$s_i(t) = a_i A(t)\cos(2\pi f_i t + \varphi_i), \quad i = 0, 1, \ldots M-1, \tag{2}$$

where a_i, f_i, φ_i – the parameters in which represent transmitted symbols;

$A(t)$ – a function, that determines the form of pulse.

Radio pulses can differ in amplitudes, phases or frequencies. There are different types of digital modulation, for example:

– *M*-ASK – *M*-ary amplitude modulation (pulses differ in parameter a_i);
– *M*-PSK – *M*-ary phase modulation;
– *M*-APSK – *M*-ary amplitude-phase modulation;
– *M*-QAM – *M*-ary quadrature-amplitude modulation;
– *M*-FSK – *M*-ary frequency modulation.

If $M = 2$, there is the binary $s(t)$ signal: radio pulse $s_0(t)$ is used for transmission 0, and radio pulse $s_1(t)$ – for transmission 1. If $M > 2$, the multi-level signal $s(t)$ takes place. As a rule, $M = 4, 8, \ldots, 2^n$, where n is an integer. Here every radio pulse $s_i(t)$ is used for transmission of $n = \log_2 M$ bits of baseband digital signal $b_u(t)$. The concrete bit sequence, that each radio pulse keeps, the mapping code sets. In the case of binary signals the clock period $T = T_b$, but in the case of multi-level signals, the clock period enlarges: $T = T_b \log_2 M$.

In the case of *M*-ASK and BPSK signals, elementary signals can be written as:

$$s_i(t) = a_i A(t)\cos(2\pi f_0 t), \quad i = 0, 1, \ldots, M-1, \tag{3}$$

where a_i is a number which represents n bits, that the $s_i(t)$ signal keeps;

f_0 is the carrier frequency.

In the case of *M*-PSK ($M \geq 4$) and *M*-APSK, it is convenient for to describe the elementary $s_i(t)$ signals with cosine and sinus components:

$$s_i(t) = a_i A(t)\cos 2\pi f_0 t + b_i A(t)\sin 2\pi f_0 t, \quad i = 0, 1, \ldots, M-1, \tag{4}$$

where a_i, b_i are coefficients, representing a sequence of *n* bits, that is transferring by the elementary signal $s_i(t)$.

The following record is equivalent to expression (4):

$$s_i(t) = A_i A(t)\cos\left(2\pi f_0 t - \varphi_i\right), \quad i = 0,\ 1, ...,\ M - 1, \tag{5}$$

$A_i = \sqrt{a_i^2 + b_i^2}$, $\varphi_i = \mathrm{arctg}\left(b_i / a_i\right)$, i.e. expression (4) maps radio pulse.

2.4 It is using conditionally to represent the elementary signals $s_i(t)$ as signal points in a certain space. Diagrams on which elementary signals are represented as signal points are named signal constellations. The purpose of such representation is to reflect the reciprocal difference of signals.

As it follows from the expression (3), elementary signals, in the case of *M*-ASK and BPSK signals, differ only in the coefficients a_i. Therefore, the signal points of *M*-ASK and BPSK signals are disposed on a numerical axis, and the *M*-ASK and BPSK signals are named one-dimensional (fig.1). On this figure the mapping codes are reflected also (the index *i* corresponds to a binary number, which is formed by transmitting bits):

– BASK signal: the transmission of 0 corresponds to $a_0 = 0$, and the transmission of 1 corresponds to $a_1 = a$.

– BPSK signal: 0 → $a_0 = -a$; 1 → $a_1 = a$.

– QASK signal: 00 → $a_0 = -a$; 01 → $a_1 = -3a$; 10 → $a_2 = a$; 11 → $a_3 = 3a$.

The number a determines the energies of elementary signals.

M-APSK and *M*-PSK ($M \geq 4$) signals are two-dimensional since elementary signals in expression (4) are described by two coefficients. Functions $A(t)\sin 2\pi f_0 t$ and $A(t)\cos 2\pi f_0 t$, that are presented in expression (4), are orthogonal, and they form two-dimensional space. Signal constellations of two-dimensional signals are reflected on a plane. For example, the signal constellation of QPSK signal is shown on the fig. 2. Here x symbolizes $\cos 2\pi f_0 t$ oscillation, and y symbolizes $\sin 2\pi f_0 t$ oscillation. It is taken into account that for the *M*-PSK signals expression (5) can be rewritten in that form:

$$s_i(t) = aA(t)\cos\left(2\pi f_0 t - \varphi_i\right), \quad i = 0,\ 1, ...,\ M - 1. \tag{6}$$

The mapping code of QPSK signal on a fig.2, is such:

00 → $\varphi_0 = 135°$ ($a_0 = -a; b_0 = a$);

01 → $\varphi_1 = 45°$ ($a_1 = a; b_1 = a$);

10 → $\varphi_2 = 225°$ ($a_2 = -a; b_2 = -a$);

11 → $\varphi_3 = 315°$ ($a_3 = a; b_3 = -a$).

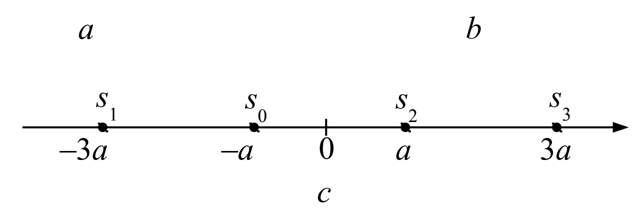

Figure 1 – Signal constellations of signals: *a* – BASK; *b* – BPSK; *c* – QASK

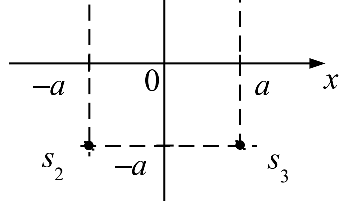

Figure 2 – Signal constellation of QPSK signal

2.5 As it follows from the expression (2), the elementary pulses are the signals of analog double-sideband-suppressed-carrier (DSB-SC) and, therefore, the spectrum of radio pulse $s_i(t)$ consists of two side bands, concentrated near the carrier frequency f_0. Spectral properties of $s_i(t)$ radio pulse are entirely determined by the $A(t)$ function.

If $A(t)$ function is a rectangular pulse of T duration, a radio pulse spectrum is wide. But it is important for the transmission of digital signals to form a compact spectrum. In order, that the spectrum of $s_i(t)$ radio pulse will be compact, and intersymbol interference would be absent, a function $A(t)$ must be the Nyquist pulse. Then side bands will be the copies of Nyquist spectrum (fig. 3), and the width of spectrum of *M*-ASK and BPSK signals is determined by expression:

$$F = 2f_N(1+\alpha) = \frac{1+\alpha}{T} = \frac{1+\alpha}{T_b \log_2 M}, \qquad (7)$$

where $f_N = 0{,}5/T$ is the Nyquist frequency;

α is a roll-off factor ($0 \le \alpha \le 1$).

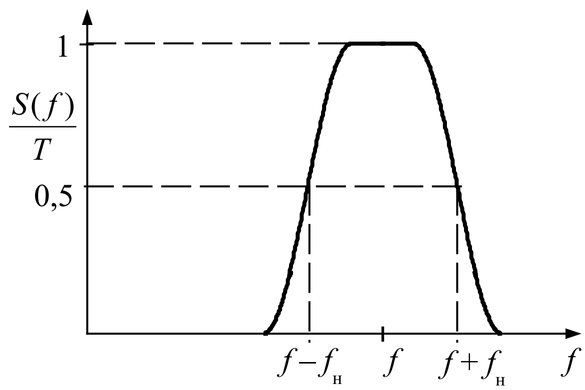

Figure 3 – Spectrum of elementary signals *M*-ASK and PM-*M* ($\alpha = 0{,}6$)

Signals, that are introduced by expression (4), are the sum of two DSB-SC signals with the identical amplitude spectrums, that are determined by the spectrum of $A(t)$ signal. If $A(t)$ is the Nyquist pulse, the amplitude spectrum of every summand, and also the spectrum of their sum, has the shape, shown on a fig. 3. Therefore, the bandwidth of elementary signals by *M*-PSK and *M*-APSK is described by expression (7).

An important conclusion follows from expression (7) – increasing the number of signal positions courses decreasing the bandwidth of elementary signals (2).

2.6 Process of forming one-dimensional and two-dimensional signals on the basis of expressions (3) and (4) is following: the mapper puts in accordance $n = \log_2 M$ input bits to the two rectangular pulses with amplitudes a_i and b_i (in the case of one-dimensional signals only one pulse with amplitude a_i takes place; $b_i = 0$); rectangular pulses are filtered in the shaping low-pass filters (LPF) for getting the Nyquist pulses; the pulses $a_i A(t)$ and $b_i A(t)$ enter to the DSB-SC modulator incomes; the got DSB-SC signals are summing up.

2.7 The BFSK signal is forming on the basis of radio pulses, that are differ in frequencies:

$$\begin{aligned} s_0(t) &= aA(t)\cos(2\pi(f_0 - \Delta f/2)\,t), \\ s_1(t) &= aA(t)\cos(2\pi(f_0 + \Delta f/2)\,t), \end{aligned} \qquad (8)$$

where Δf is the frequency separation;

a is the coefficient, that determines the energy of signals.

If the $A(t)$ function is rectangular pulse, it is necessary to provide forming in the modulator BFSK signal without the “break” of phase. It is possible, if frequency separation equals $\Delta f = k/(2T)$, k = 1, 2, 3, ...; $T = T_b$. If k = 1, so $\Delta f = 0{,}5/T$, and modulation is named “minimum shift keying” (MSK). In the case of MSK the normalized spectrum of signal is described by expression:

$$S(f)=\frac{\sqrt{1+\cos\left(4\pi\left(f-f_0\right)T\right)}}{\sqrt{2}\left(1-\left(4\left(f-f_0\right)T\right)^2\right)}. \tag{9}$$

The diagram of dependence (9) is shown on a fig. 4. With increasing of the difference $|f-f_0|$, the spectrum decreases with the speed equal $1/f^2$. If to define the bandwidth F_{MSK} on the first zeros of dependence (9), we have

$$F_{\text{MSK}}=1{,}5/T\,. \tag{10}$$

In order to get the BFSK signal with a narrow spectrum and without intersymbol interference, it is necessary, that a function $A(t)$ would be the Nyquist pulse. In this case it is possible to consider, that the spectrum of signal $s_{\text{BFSK}}(t)$ is the sum of spectrums of two radio pulses with central frequencies $f_0-\Delta f/2$ and $f_0+\Delta f/2$. The normalized spectrum of BFSK signal is shown on a fig. 5. It is seen, that the frequency separation would be minimum, if the spectrums of radio pulses adjoin to each other, and this frequency separation is equal:

$$\Delta f_{\min}=\frac{1+\alpha}{T}. \tag{11}$$

Then bandwidth of BFSK signal:

$$F_{\text{BFSK}}=\Delta f_{\min}+\frac{1+\alpha}{T}=\frac{2(1+\alpha)}{T}, \tag{12}$$

i.e. twice as bandwidth of signals BASK and BPSK.

The forming BFSK signals differs from forming M-PSK signals by working of mapper and that the supporting wave frequencies of generators in DSB-SC modulators differ on the value $\Delta f/2$ from carrier frequency.

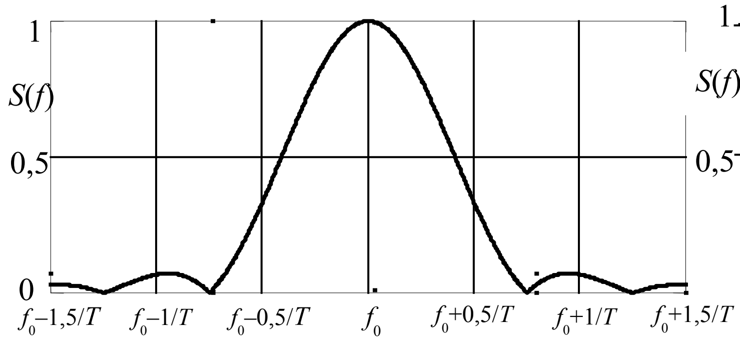

Figure 4 – Spectrum of MSK signal

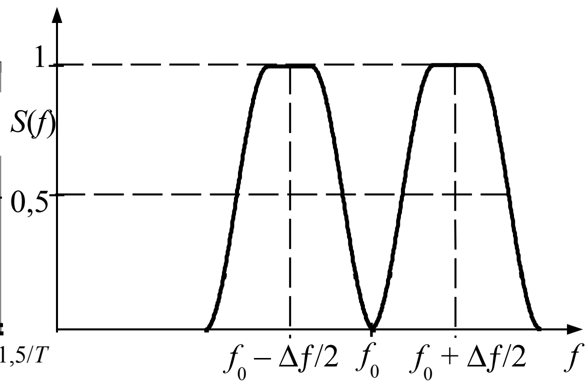

Figure 5 – Spectrum of BFSK signal with $\alpha = 0{,}6$, $\Delta f = 2(1+\alpha)f$

3. Questions

3.1 What is the aim of using the modulation in the telecommunication systems ?

3.2 Give the determination of digital signal.

3.3 Give the determinations of digital modulation signals: M-ASK; M-PSK; M-FSK.

3.4 Why don't the radio-frequency pulses with rectangular envelope used for transmitting digital signals through communication channels? What must pulse envelope be?

3.5 What are the forms of the spectrums of M-ASK; M-PSK; M-FSK signals?

3.6 For what are the multi-level signals used for transmitting digital signals through communication channels?

3.7 What signals of digital modulations are one-dimensional, and what signals are two-dimensional?

4. Home task

4.1 You must study the section "Digital types of modulation" with the summary of lectures and main positions of these pointing. When study this theme you must use the literature [1, p. 196…204, 231…234].

4.2 The clock period is set: $T = 50$ ms. It is necessary to build the time diagrams of elementary radio pulses of frequency $f_0 = 40$ Hz for two cases: with rectangular envelope and with Nyquist pulse envelope.

Note. It is necessary to take into account that elementary radio pulse is the product of rectangular pulse of duration T or Nyquist pulse, and harmonic wave. As a Nyquist pulse it is possible to take a function

$$A(t) = \frac{\sin(\pi t/T)}{\pi t/T}.$$

You are to build the diagram of this function on an interval $(-4T, 4T)$.

4.3 You should be ready for conversation on questions.

5 Laboratory task

5.1 Familiarize with a virtual model on a workplace.

Start the program **1.5**, using the icon **TT(English)** on the desktop. Study scheme model, using the description in section 6 of this LW. Specify with the teacher the laboratory task performance plan.

5.2 Preparation of a virtual model. It is necessary to set a digital signal. For that you should give a decimal number $128 + 10N$ (N is a number of laboratory stand) by binary number. The roll-off factor is equal $\alpha = 1 - 0{,}1N$.

5.3 Research the form and spectrum of BASK and QASK signals as the functions of envelope form. For this purpose it is necessary to set: type of modulation – BASK; envelope form is a rectangular pulse. You should fix in protocol one under one the time diagrams of the followings signals: the digital signal; the output signal of mapper; the modulated signal. You should also fix the spectral diagram of the modulated signal. After that it is necessary to set the second envelope form – the Nyquist pulse. You should fix in protocol the time and spectral diagrams of the modulated signal.

The same research should be performed for the QASK signal.

In conclusions, on the basis of comparison of spectral diagrams, you should indicate the expedience of using the radio-frequency pulses with Nyquist pulse envelope and the expedience of using the multi-positional signals for decreasing the occupied frequency band.

5.4 Research the form and spectrum of BPSK and QPSK signals as the functions of envelope form. You should repeat the researches, performed in p.5.3, for the BPSK and QPSK signals. Compare the spectrums of M-ASK and M-PSK signals.

5.5 Research the form and spectrum of BFSK signal as the functions of envelope form. You should repeat researches, conducted in pp. 5.3 and 5.4, for the MSK and BFSK signals. Compare the spectrums of BASK, MSK and BFSK signals.

6 Description of laboratory model

The laboratory work is performed on a computer program in the HP VEE environment with using the virtual model. The structure scheme of model is shown on a fig. 6.

A model is universal modulator of digital modulated signals. It includes the digital signal generator with duration equals $8T_b$, signal symbols can be changed. The bit duration is set: $T_b = 50$ ms. Modulator consists of the followings blocks: mapper; shaping filters; carrier generators; two multipliers and adder. The setting of modulation type affects on the mapping code of an encoder and carrier generators and permits to set the followings types of modulation: BASK, QASK, BPSK, QPSK, and QFSK. The signals from two encoder outputs incomes to the filter inputs, shaping the radio pulse envelopes in the form of the Nyquist pulse. The scheme contains a switch, allowing to exclude the shaping filters from the scheme, and so radio pulse would have the rectangular envelope. The formed pulses are multiplied with carriers. The carrier frequency f_0 is set equal 40 Hz. In the case of BFSK the frequency separation Δf is set in accordance with a formula (9), and in the case of MMS the frequency separation $\Delta f = 0{,}5/T$. A model contains oscillographs and spectrum analyzer.

7 Requirements to the report

7.1 The **name** of laboratory work.
7.2 The **purpose** of work.
7.3 The **results** of the home task processing.
7.4 The **structure schemes** of the every laboratory task processing.
7.5 The **results** of performing the laboratory tasks at points (oscillograms and spectrograms, with signing).
7.6 The **conclusions** on every item of task, in which it is necessary to make the analyses of the results, that were got (coincidence of theoretical and experimental data, showing of properties of signals, etc.).
7.7 The **date**, **signature** of student, **visa** of the teacher with mark in a 100-point scale.

Literature

1. **Скляр Б.** Цифровая связь. Теоретические основы и практическое применение. 2-е издание.: Пер. с англ. – М.: Издательский дом «Вильямс», 2003. – 1104 с.

Individual task № 2.1
CALCULATION OF RANDOM PROCESS CHARACTERISTICS

Input data:

The white Gaussian noise $N(t)$ (Volts) with the one-sided spectral power density N_0 on the input of low-pass filter (LPF) with given frequency response (FR) $H(f)$, $0 \le f < \infty$.

It is necessary:

1. Write input data of your variant.
2. Find expression for the noise spectral power density $X(t)$ on the LPF output $G_X(f)$ and build the graph of this function.
3. Define average power of the noise $X(t)$.
4. Define the effective bandwidth Δf_{eff} of noise $X(t)$ and show it on the graph of the $G_X(f)$ function.
5. Find expression for the correlation function of noise $X(t)$ on the LPF output $K_X(\tau)$ and build the graph of this function.
6. Define the correlation time τ_c of noise $X(t)$ and show it on the graph of the $K_X(\tau)$ function.
7. Calculate the product of $\Delta f_{\text{eff}}\tau_c$.
8. Define probability of that in the arbitrary time moment noise $X(t)$ will take on the value from the given interval $(x_1,\ x_2)$.
9. Bring a list of the used literature; there must be references on used literary source with pointing of subsections or numbers of pages in the text of the executed individual task.

Table 1– Given types of filter (the number of variant is determined by the two last number of your student's book number)

№ variant	Type of filter
00...24	Ideal LPF with FR $H(f) = \begin{cases} 1, & 0 \le f \le F_{\text{cut}}, \\ 0, & f > F_{\text{cut}}, \end{cases}$ where F_{cut} is the LPF cut off frequency
25...49	RC-filter with FR $H(f)= \dfrac{1}{\sqrt{1+(2\pi f \tau_f)^2}}$, where τ_f is the LPF time constant
50…74	Butterworth filter with FR $H(f)= \dfrac{1}{\sqrt{1+(f/F_{\text{cut}})^{2n}}}$, where n is the order filter, assume $n = 2$; F_{cut} is the filter cut off frequency
75...99	Gaussian filter with FR $H(f)= \exp(-a^2 f^2)$, where a – the coefficient determining the LPF FR slope

Table 2 – Given the numerical values (the number of variant is determined by the last num-ber of your student's book number)

№ variant	0	1	2	3	4	5	6	7	8	9
N_0, 10^{-6} V^2/Hz	0,1	5	2	1	40	10	200	100	5000	1000
F_{cut}, 10^5 Hz	100	4	20	40	1	6	0,3	0,8	0,02	0,1
τ_f, 10^{-6} s	0,04	0,6	0,2	0,06	2	0,4	7	3	100	20
a, 10^{-7} s	0,5	15	3	1,5	60	10	200	75	3000	600
x_1, V	$-\infty$	– 0,5	0	0	1	2	$-\infty$	2	4	0
x_2, V	1	0,5	∞	3	3	∞	0	4	∞	4

Methodical instructions to performance IT № 2.1

Look up [1, p. 133...145; 2, p. 49...60]. Next sequence of the Individual task № 2.1 performance is recommended.

1. Spectral power density of noise $X(t)$ on the LPF output is determined by expression

$$G_X(f) = G_N(f)H^2(f) = N_0H^2(f),$$

it is necessary to build the graph of the $G_X(f)$ function for the interval of frequency values from 0 to the value, at which $G_X(f) << G_X(0)$.

2. Average power of noise $X(t)$ is determined by the integral

$$P_X = \int_0^\infty G_X(f)df .$$

3. The effective bandwidth Δf_{eff} of noise $X(t)$ is determined

$$\Delta f_{\text{eff}} = \frac{1}{G_X(0)}\int_0^\infty G_X(f)df \quad \text{or} \quad \Delta f_{\text{eff}} = \frac{P_X}{G_X(0)},$$

value Δf_{eff} must be shown on the graph of the $G_X(f)$ function.

4. The correlation function of the noise $X(t)$ is determined

$$K_X(\tau) = \int_0^\infty G_X(f)\cos 2\pi f\tau \, df .$$

It is necessary to build the graph of the $K_X(\tau)$ function for the interval of values τ from 0 to the value, at which $|K_X(\tau)| << K_X(0)$. It is useful to check up implementation of main properties of correlation function:

– $K_X(\tau)$ – is even function;
– $K_X(0) = P_X$, where P_X is average power of process;
– $K_X(0) \ge K_X(\tau)$.

5. Correlation time τ_c of the noise $X(t)$ it is possible to define by one of the following methods:

– as a value of τ, at which the $K_X(\tau)$ function first time takes on a zero value (it is comfortable in the case of ideal LPF);
– as a value of τ, at which function $K_X(\tau) = 0{,}1K_X(0)$;
– as a result of calculation of integral

$$\tau_c = \frac{1}{K_X(0)}\int_0^\infty |K_X(\tau)|d\tau .$$

The value of τ_c must be shown on the graph of the $K_X(\tau)$ function;

6. Calculate the product $\Delta f_{\text{eff}}\tau_c$, the value of this product is a value order of magnitude 0,5.

7. For determination of probability of that in the arbitrary time moment noise $X(t)$ will take on the value from interval (x_1, x_2), it is necessary to use expression

$$P\{x_1 < X(t) \le x_2\} = F(x_2) - F(x_1),$$

where $F(x)$ is probability distribution function of the noise $X(t)$. If a Gaussian process acts at the input of linear electric circuit, an output process also has Gaussian probability distribution. For Gaussian processes the function of probability distribution is written down:

$$F(x) = 1 - Q\left(\frac{x - \overline{X(t)}}{\sigma_X}\right),$$

where $Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^{\infty} \exp\left(-\frac{t^2}{2}\right)dt$ is Q-function or addition to Gaussian probability distribution function;

$\overline{X(t)}$ – is the average value or the expectation of a noise $X(t)$ (in our task $\overline{X(t)} = 0$);

σ_X –root-mean-square deviation of a random process, it is determined as $\sigma_X = \sqrt{D[X(t)]}$;

$D[X(t)]$ – variance of a noise $X(t)$, as $\overline{X(t)} = 0$, then $D[X(t)] = P_X$.

In the absence Q-function table it is possible to take advantage of approximation formula:

$$Q(z) \cong 0,65 \exp[-0,44(z + 0,75)^2] \text{ when } z > 0;$$

$$Q(z) = 1 - Q(|z|) \text{ when } z < 0,\ Q(0) = 0,5,\ Q(\infty) = 0.$$

For the P_X, $K_X(\tau)$, and τ_c determination you can use following expressions:

$\int_0^{F_{cut}} \cos 2\pi f\tau\, df = F_{cp} \frac{\sin 2\pi F_{cut}\tau}{2\pi F_{cut}\tau}$;	$\int_0^{\infty} e^{-a^2x^2} \cos bx\, dx = \frac{\sqrt{\pi}}{2a} e^{-b^2/4a^2}$ when $a > 0$;
$\int_0^{\infty} \frac{\cos mx}{x^4 + 4a^4} dx = \frac{\pi e^{-ma}}{8a^3}(\sin ma + \cos ma)$;	$\int_0^{\infty} \frac{\cos ax}{1 + x^2} dx = \frac{\pi}{2} e^{-\lvert a \rvert}$;
$\int e^{ax} dx = \frac{1}{a} e^{ax}$.	

Literature

1. **Стеклов В. К.,** Беркман Л.Н. Теорія електричного зв’язку: Підручник для студентів ВУЗів. За ред. В.К. Стеклова – К.: Техніка, 2006.

2. **Теория** электрической связи: Учебник для вузов / А.Г. Зюко, Д.Д. Кловский, В.И. Коржик, М.В. Назаров; Под ред. Д.Д. Кловского. – М.: Радио и связь, 1998.

Individual task № 2.2
DESCRIPTION AND CALCULATION
OF DIGITAL MODULATIONS SIGNALS CHARACTERISTICS

Input data:

– two types of digital modulation (table 1);
– rate of modulating digital signal (table 2);
– roll-off factor of spectrum of the modulated signal (table 2);

It is necessary:

1. Write input data of your variant.
2. Paint on the same figure two time diagrams of:
a) realization of digital modulating signal (8-9 binary symbols – two last numbers of your record-book, written in the binary numeration);
б) modulated signals of given modulation types; accept that, radio pulse envelop is rectangular.
3. Build signal constellations of given modulation types, on signal constellation show mapping code.
4. Write analytical expressions of channel symbols of given modulation types.
5. Consider that average energy of signals, expended on the transmission of one binary symbol, E_b = const; to calculate for given modulation types minimal and maximal distances between channel symbols, expressed through E_b.
6 Calculate and to draw the modulated signals amplitude spectrum for given modulation types; calculate bandwidth of signals of given modulation types and show them on the spectrogram.
7. Paint the functional diagram of modulators for given types of digital modulation and explain principles of their operation.
8. Formulate conclusions to the executed task; point to the advantages (or imperfections) of the multi-level modulation type given to you, in comparison with binary modulation type.
9. Bring a list of the used literature; there must be references on used literary source with pointing of subsections or numbers of pages in the text of the executed individual task.

Table 1– Given types of modulation (the number of variant is determined by the last number of your student's book number)

№ var.	0	1	2	3	4	5	6	7	8	9
Digital mod.	BASK, QPSK	BASK, 8PSK	BASK, QASK	BFSK, QPSK	BFSK, 8PSK	BFSK, QASK	BPSK, QPSK	BPSK, QPSK	BPSK, 8PSK	BPSK, QASK

Table 2 - Given R and α (the number of variant is determined by the last but one number of your student's book number)

№ var.	0	1	2	3	4	5	6	7	8	9
R, kbits/s	9,6	19,2	24	32	64	128	256	384	512	2048
α	0,20	0,25	0,30	0,35	0,20	0,25	0,30	0,35	0,20	0,25

Methodical instructions to performance IT № 2.2

Data on signals of digital modulation see in methodical instructions to performance of laboratory work 1.5 (p. 20) and [1, p. 196…204, 231…234].

The mapping code should be a code Gray.

The amplitude spectrum of the modulated signal is described by Nyquist spectrum. The baseband Nyquist spectrum is defined by an expression

$$N(f) = \begin{cases} T, & 0 \le |f| \le (1-\alpha) f_N, \\ 0{,}5T\left[1 + \sin\left(\frac{\pi}{2\alpha}\left(1 - \frac{|f|}{f_N}\right)\right)\right], & (1-\alpha) f_н < |f| < (1+\alpha) f_N, \\ 0, & |f| \ge (1+\alpha) f_N, \end{cases}$$

where $f_N = 1/T$ is Nyquist frequency;

T is clock period;

α is a roll-off factor of spectrum.

Literature

1. **Скляр Б.** Цифровая связь. Теоретические основы и практическое применение. 2-е издание.: Пер. с англ. – М.: Издательский дом «Вильямс», 2003. – 1104 с.