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COMMUNICATION SIGNALS

Module №1

Education manual
on telecommunication theory

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This education manual contains:

– main theoretical positions of Telecommunication Theory, chapter Communication Signals;
– methodical guidelines for fulfilling laboratory works;
– individual tasks with input data and methodical instructions for it performance;
– short English-Russian and Russian-English dictionaries on communication signals.

The manual is intended for students training on a direction of Telecommunication studying the module 1 of Telecommunication theory.

APPROVED on Methodical
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1 INTRODUCTION

The branch of science “Theory of telecommunication” studies the common relationships of information transfer on distance. The object of studying is telecommunication system.

Telecommunication system provides transfer of information on distance by electric signals. The problem of information transfer on distance is formulated as: there is a source of the information (a person, a computer, etc.), having some information that is necessary to transfer to the remote recipient. This information should be transferred with the given level of fidelity and with an allowable delay. To discuss this problem we shall define the basic concepts: information, message, signal and communication channel.

Information is a collection of knowledge on any process, events, object. This knowledge reduces uncertainty for the recipient before he obtains knowledge. For transfer or to store information it is used different signs (or symbols), which allow to present it in some form.

Message is a material form of representation of information. First of all it is a set of signs that represent information. Message transfer on a distance is carried out by a signal.

Signal is a physical process in which message is represented and which is used to transfer information on distance. Signal can be electric, sound or light. In the telecommunication theory (by default) signal is an electric current or a voltage that represented the transferred message.

Information system is a system, which functions on the basis of information usage. Information takes place outside and/or inside considered system. Special case of information system is a telecommunication system.

Telecommunication system provides message transfer with a certain quality from a message source to a message recipient. Telecommunication system can provide one-way message transfer (broadcasting) or two-way message transfer (communication). In the first case simplex transfer takes place, in the second case duplex transfer takes place: full duplex– when a system provides simultaneously reception and transfer of messages; half duplex – when a system provides reception and transfer of messages one-by-one.

The generalized block diagram of telecommunication system for one-way message transfer is shown on figure 1. Here $a(t)$ is transmitting message; $\hat{a}(t)$ is receiving message; $b(t)$ is transmitting baseband signal; $\hat{b}(t)$ is receiving baseband signal. In the case of duplex transfer it is necessary two sets of the units shown on figure 1.

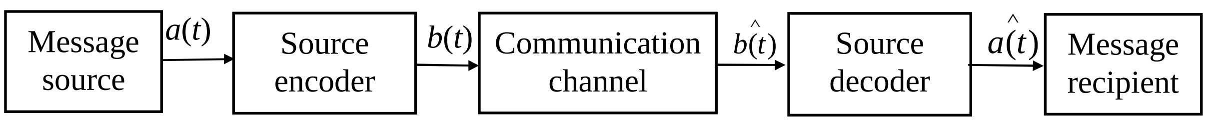


Figure 1 – Generalized block diagram of telecommunication system

A source of information gives out information by messages. The construction of the equipment is mainly defined by characteristics of transferred messages; therefore we speak about “message source”, instead of “information source”; in a similar way we speak about “message recipient”, instead of “information recipient”.

All messages are divided into continuous and discrete messages.

Discrete message consists of sequence of separate signs, which quantity is finite. These signs form an **alphabet of a source**. An example of a typical discrete message is the text. Transformation of discrete messages to electric signals consists in their encoding which is carried out by a source encoder. Encoding of a message is carried out on the basis of a *code*. **Code** is a rule or a table on the basis of which each sign of a message is converted into the code combination (a set of binary symbols) (figure 2). As a result of encoding we receive **digital baseband signal** (figure 3, *a*). Inverse transformation of digital baseband signals to messages consists in decoding, which is carried out in the source decoder (figure 1). The basic characteristic of a digital signal is its rate R, bit per second (bit is the short name of binary symbols). On figure 3, *a* it is shown $T_b = 1/R$, T_b is bit duration.

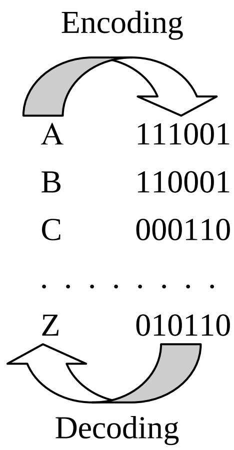


Figure 2 – Illustration of encoding and decoding

Continuous message represents a change of some time continuous magnitude (for example, sound pressure). Transformation of continuous messages to electric signals is carried out by different transducers (for example, a microphone). As a result of transformation we receive **a continuous baseband (analog) signal** (figure 3, *b*). The basic characteristic of a continuous baseband signal is the maximum frequency of its spectrum $F_{\max}$ that characterizes its speed of change.

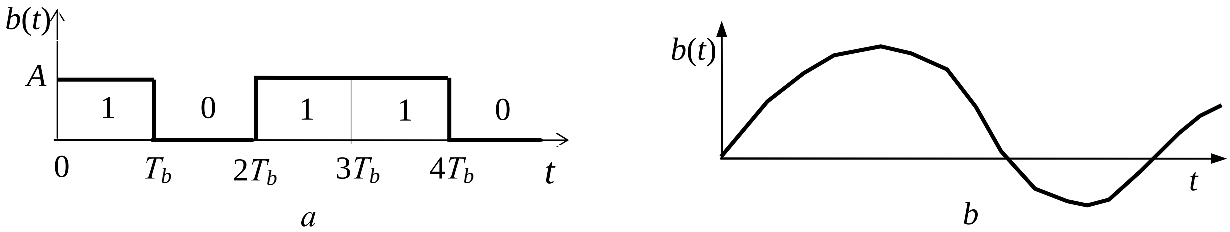


Figure 3 – Baseband signals: *a* – digital signal; *b* – continuous signal

C**ommunication channel** is a set of devices for transfer of electric signals on distance.

If a baseband signal is a digital one then communication channel must be digital. If a baseband signal is continuous then communication channel must be continuous.

A continuous baseband signal can be transformed into a digital signal for transfer by a digital communication channel. In that case there will be analogue-digital conversion. So, in case of digital transfer of continuous messages a source encoder transforms messages into a baseband continuous signal, and then – in a digital signal. Digital transfer has a lot of advantages in comparison with analogue transfer. Digitalisation of transmission systems takes place during several last decades.

Transformations discussed above are shown on figure 1.

A communication channel can be simple or compound.

We shall consider construction of compound channel as part of a telecommunication network.

Network is a set of nodes and links which provide information exchange between users (users are sources and recipients).

There are user nodes (UN) where a terminal equipment is used, and switch nodes (SN) where switching of channels or packages is carried out. On figure 4 the fragment of logic topology of a network is shown. It takes part in transfer of information between the considered user nodes. In one UN there is a message source and a source encoder, and in other UN – a source decoder and a message recipient. User node is called also the terminal equipment.

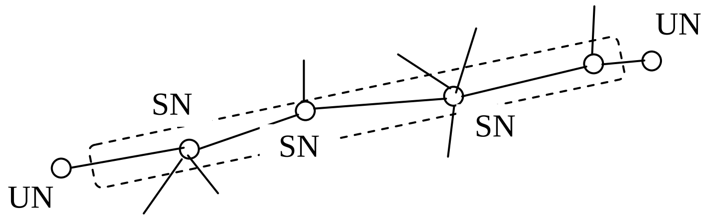


Figure 4 – Fragment of network topology:
SN – switch node; UN – user node

Links that form a communication channel are traced by dotted line.

Links of network are called transmission systems. Transmission system that connects user node with the nearest switch node is called access system.

On figure 5 block diagram of typical digital transmission system is shown.

The transmission system is constructed on the basis of a transmission line. Transmission line is a physical circuit (cable) or a free space (radio communication), used for transfer of a signal on distance.

The baseband digital signal is coded by the error control code allowing at decoding to find out or correct errors, arising at transmission.

The modulator forms a signal, which can be well transferred by transmission line.

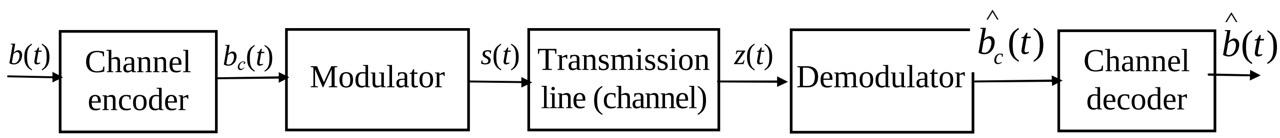


Figure 5 – Block diagram of a typical digital transmission system

Main characteristics of telecommunication system are accuracy of message transfer or quality of transfer and transmission rate or quantity of transferred information per 1 second.

In communication channel signal is distorted because of noise action. A noise is occasional influence on a signal that makes complicate signal transfer.

Distortion of digital signals will consist of occurrence of errors – instead of actually transferred symbols to the recipient other symbols come. Such kind of distortion is quantitatively characterized by probability of a symbol error. Distortion of continuous signals will consist of their form changing because of noise covering. Distortion of continuous signals can be characterized by an average square of difference between accepted and transmitted signals or the signal/noise ratio.

For messages transfer on a telecommunication channel it is necessary to spend a band of frequencies and power of a signal (the basic resources of a telecommunication channel). So, the main tasks of telecommunication theory following:

– how to provide necessary quality of message transfer on a telecommunication channel;

– how to provide necessary transmission rate on a telecommunication channel at limited resources of a telecommunication channel.

2 ELEMENTS OF GENERAL THEORY OF SIGNALS

2.1 Classification of signals

Signal is rather wide concept. Signal is a process of change in time of the physical phenomenon or a state of any technical object. Signals serve for mapping, registration and transmission of message. The common for signals is that in them the information is assumed in it. We shall consider, that a signal is an electric voltage or a current.

The most natural mathematical model of a signal is function of time $b(t)$, $s(t)$, $z(t)$ and etc. Such function of time establishes conformity between any moment of time t and size $s(t)$. Considering mathematical models of signals, we abstract from the concrete physical nature of a signal (a voltage, a current, an intensity of an electro-magnetic field, etc.), considering, that function $s(t)$ completely reflects the important properties of a signal.

Depending on what values of a signal s and values of a variable t are possible, distinguished following:

1. A signal is continuous (figure 6, *a*) if a set of values t is continuous, i.e. the argument accepts any values on an interval of existence of a signal.

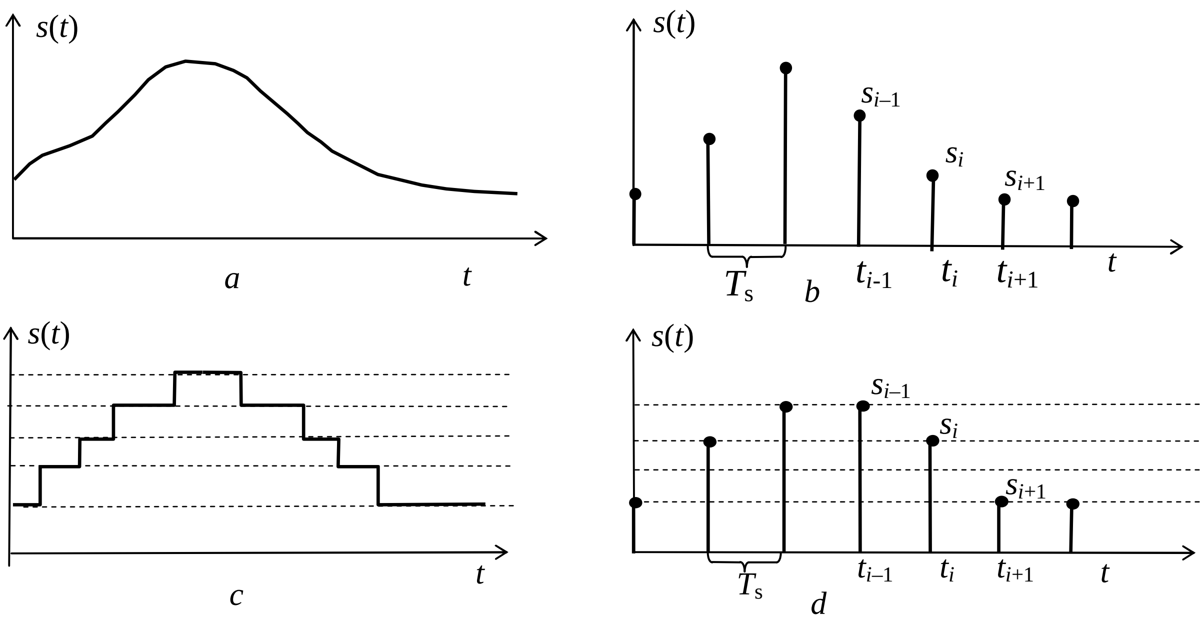


Figure 6 – Types of signals

2. A signal is discrete (figure 6, *b*) if a set of values t is finite. Discrete signal is called also sequence, a time series. Such signal appears as a result of sampling a continuous signal. The step in time, through which discrete signal is given is called sampling interval T_s.

3. A signal is quantized (figure 6, *c*) if value s accept final number of values. Such signal is a result of discretization on a level of a continuous signal.

4. A signal is digital (figure 6, *d*) if it is also discrete, and quantized. Digital signals appear as a result of coding discrete messages, and also as a result of coding continuous signals for their representation by digital signals.

Signals are divided into:

1. Baseband signal is a representation of the message of not electric nature by an electric signal.

2. Modulated signal is a result of transformation of a baseband telecommunication signal in a signal for transfer by transmission line.

All signals are categorized as:

1. Determined signals – mathematical expression of determined (completely known) signal is completely certain function of time set by a formula, a plot or the table of values. For example, $s(t) = A_0\cos(2\pi f_0 t + \varphi_0)$, where A_0, f_0 and φ_0 are defined numbers. All values $s(t)$ are known at any moment.

2. Stochastic signals – mathematical representation of a stochastic signal is stochastic function of time, its values cannot be precisely predicted beforehand. Stochastic function of time (stochastic process) is described by statistical characteristics that characterize those or other properties of this function on the average. Stochastic processes are more full mathematical models of communication signals, than the determined functions of time. But many transformations of signals can be studied, using the determined functions of time. It can be simple function – harmonious waveform, a pulse, etc.

Any real signal $s(t)$ has final duration T_s. In many cases it is convenient to count, that the signal is infinite on duration and exists on an interval $(-\infty, \infty)$.

Signals are real and complex. It is clear, that a signal is the real function of time which represents a state of some object. Sometimes for convenience of the mathematical analysis of signal transformations a complex signal is entered into consideration

$$s(t) = s_1(t) + j\, s_2(t),$$

where $s_1(t)$, $s_2(t)$ is the real functions of time; depending on a solved problem $s_2(t)$ is result of some transformations of function $s_1(t)$ or function, not dependent from $s_1(t)$.

Widely used complex exponent is an example

$$s(t) = e^{j2\pi f_0 t} = \cos 2\pi f_0 t + j \sin 2\pi f_0 t\,.$$

Frequently in devices of signal transformation the auxiliary signals, which are not containing the information, are used. Such signals are auxiliary and refer to as waveforms.

2.2 Energy characteristics of determined signals

The basic energy characteristics of a signal $s(t)$ are its power and energy. Instant power of the real signal is defined as a square of instant value $s(t)$:

$$p(t) = s^2(t),\ \text{V}^2. \qquad (2.1)$$

Power of a signal characterizes signal intensity, its ability to influence on devices, which register a signal.

Average power of a signal of final duration is defined by averaging (2.1) on an interval of existence of a signal $(0, T_s)$

$$P_s = \frac{1}{T_s}\int_0^{T_s} s^2(t)dt. \qquad (2.2)$$

Energy of a signal of final duration is defined as

$$E_s = P_s T_s = \int_0^{T_s} s^2(t)dt. \qquad (2.3)$$

From the last ratio it is clear, that energy of a signal takes into account both signal intensity, and time of its action.

Power and energy of a complex signal $s(t)$ are defined by ratios (2.1)...(2.3) in which instead of $s^2(t)$ it is necessary to substitute $s(t)\cdot s^*(t) = |s(t)|^2$, where $s^*(t)$ is the function in a complex conjugate with $s(t)$; $|s(t)|$ is the module of a signal $s(t)$.

The signal refers to normalized, if its energy

$$E_s = 1. \qquad (2.4)$$

In addition to function of time $s(t)$, which completely defines a signal, other time characteristic – function of correlation of a signal in some cases is used. For a real signal of final duration it is defined

$$K_s(\tau) = \int_0^{T_s} s(t)s(t+\tau)dt, \qquad (2.5)$$

where τ – time shift which accepts both positive, and negative values. At $\tau = 0$

$$K_s(0) = E_s. \qquad (2.6)$$

For a periodic signal with period T which energy indefinitely big, is used the following definition

$$K_{s\,\mathrm{per}}(\tau) = \frac{1}{T}\int_0^{T} s(t)s(t+\tau)dt. \qquad (2.7)$$

Function $K_{s\,\mathrm{per}}(\tau)$ is periodic with period T, and

$$K_{s\,\mathrm{per}}(0) = P_s. \qquad (2.8)$$

There are two signals $s_1(t)$ and $s_2(t)$. Concepts of mutual correlation function is introduced for them

$$K_{s1s2}(\tau) = \int_0^{T_s} s_1(t)s_2(t+\tau)dt, \qquad (2.9)$$

and scalar product are entered

$$(s_1, s_2) = \int_0^{T_s} s_1(t)s_2(t)dt, \qquad (2.10)$$

From last ratio it is clear, that $(s_1, s_2) = K_{s1s2}(0)$.

Signals are orthogonal if scalar product of signals is $(s_1, s_2) = 0$.

Let consider definition of energy characteristics for the rectangular video pulse (figure 7). Analytically record of the rectangular video pulse is:

$$s(t)=\begin{cases}A, & 0\le t<T_s,\\ 0, & t<0,\ t\ge T_s.\end{cases}$$

Average power and energy are defined by ratio (2.1) and (2.3)

$$P_s=\frac{1}{T_s}\int_0^{T_s}A^2dt;\quad E_s=A^2T_s\,.$$

The correlation function of a signal is defined by a ratio (2.5). Let $0<\tau<T_s$. Then

$$s(t)s(t+\tau)=\begin{cases}A^2, & 0\le t<T_s-\tau,\\ 0, & t<0,\ t\ge T_s-\tau.\end{cases}$$

and

$$K_s(\tau)=\int_0^{T_s-\tau}A^2dt=A^2(T_s-\tau).$$

When $\tau\ge T_s$ then $K_s(\tau)=0$. In view of parity property of correlation function final expression is written as

$$K_s(\tau)=\begin{cases}A^2(T_s-|\tau|), & |\tau|<T_s,\\ 0, & |\tau|\ge T_s.\end{cases}$$

The graph of correlation function of the rectangular video pulse is shown on figure 7.

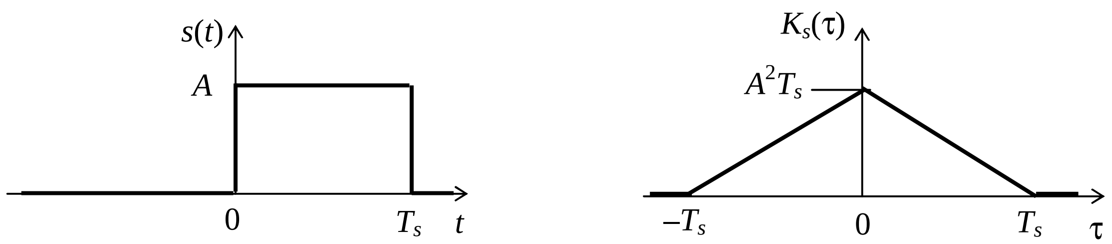


Figure 7 – Rectangular video pulse $s(t)$ and its correlation function $K_s(\tau)$

2.3 Representation of signals in orthogonal bases

Among various mathematical methods, which are used while describing electric circuits and signals, representation of any function as the sum of more simple ("elementary") functions is the most widely applied. Let $s(t)$ is the determined signal of duration T_s. Let present its weighed sum of some basic functions as

$$s(t)=\sum_{n=1}^{\infty}a_n\psi_n(t),\quad 0\le t\le T_s, \tag{2.11}$$

where a_n – factors of decomposition,
$\psi_n(t)$ – basic functions.

Basic functions are selected according to those or other reasons, and then factors of decomposition calculate. But factors of decomposition are calculated in more simple way if basic functions are orthogonal on an interval (0, T_s). Let multiply the left and right parts of equality (1) on $\psi_k(t)$ also we make integration on an interval (0, T_s):

$$\int_0^{T_s} s(t)\psi_k(t)dt = \int_0^{T_s}\sum_{n=1}^{\infty} a_n\psi_n(t)\varphi_k(t)dt = \sum_{n=1}^{\infty} a_n\int_0^{T_s}\psi_n(t)\psi_k(t)dt .$$

As functions $\psi_n(t)$ and $\psi_k(t)$ are orthogonal integrals in the right part are equal to zero except for a case $n = k$ – in this case the integral is equal to energy of basic functions $\psi_k(t)$. Therefore last equality will be written down as

$$\int_0^{T_s} s(t)\psi_k(t)dt = a_k E_{\psi_k} .$$

Let return to an index n and we shall write down a rule of calculation of decomposition factors

$$a_n = \frac{1}{E_{\psi n}}\int_0^{T_s} s(t)\psi_n(t)dt ; \quad n = 1, 2, 3, ... \tag{2.12}$$

If basic functions are orthonormal, so

$$a_n = \int_0^{T_s} s(t)\psi_n(t)dt = (s,\varphi_n), \quad n = 1, 2, 3, ... \tag{2.13}$$

A series (2.11), in which decomposition factors are defined according to the formula (2.12), is called as generalized Fourier series.

While practical using of decomposition (2.11) it is necessary to limit the number of terms

$$\hat{s}(t) = \sum_{n=1}^{N} a_n\psi_n(t) . \tag{2.14}$$

Thus the approximate representation of a signal $s(t)$ is got, which satisfies to the certain measure of accuracy

$$E_\varepsilon = \int_0^{T_s}[s(t) - \hat{s}(t)]^2 dt = E_s - \sum_{n=1}^{N} a_n^2 E_{\psi_n} . \tag{2.15}$$

It is usually considered, that the number N is selected in the way so that to satisfy to the given measure of accuracy, and in writing of decomposition of a signal $s(t)$, a sign of exact equality is used

$$s(t) = \sum_{n=1}^{N} a_n\psi_n(t) . \tag{2.16}$$

Depending on properties of a signal *s*(*t*) different systems of orthogonal functions are used: trigonometrical functions, exponent functions, functions of samples and Walsh functions.

After series expansion of a signal *s*(*t*), decomposition factors completely set a signal *s*(*t*), i.e. according to factors it is possible to recover a signal. Factors of decomposition also allow to define energy characteristics of signal *s*(*t*):

$$E_s = \int_0^{T_s}\left(\sum_{n=1}^{N} a_n \psi_n(t)\right)^2 dt = \sum_{n=1}^{N} a_n^2 E_{\psi_n} \tag{2.17}$$

and scalar product of signals $s_1(t)$ and $s_2(t)$

$$(\mathbf{s_1},\mathbf{s_2}) = \int_0^{T_s}\left(\sum_{n=1}^{N} a_{n1}\psi_n(t)\right)\left(\sum_{n=1}^{N} a_{n2}\psi_n(t)\right)dt = \sum_{n=1}^{N} a_{n1}a_{n2}E_{\psi_n}, \tag{2.18}$$

where a_{n1} and a_{n2} – decomposition factors of signals $s_1(t)$ and $s_2(t)$ appropriately.

Definition of decomposition factors can be made with hardware as shown on figure 8, *a*. This procedure is called the signal analysis. Predicted decomposition factors completely describe a signal. Knowing them, it is possible to synthesize a signal – to recover it according to decomposition factors (figure 8, *b).* The circuits shown on figure 8 find application in communication techniques. In a transmitter the analysis of signals is performed, on a transmission channel the decomposition factors are transferred, in the receiver synthesis of a signal is performed.

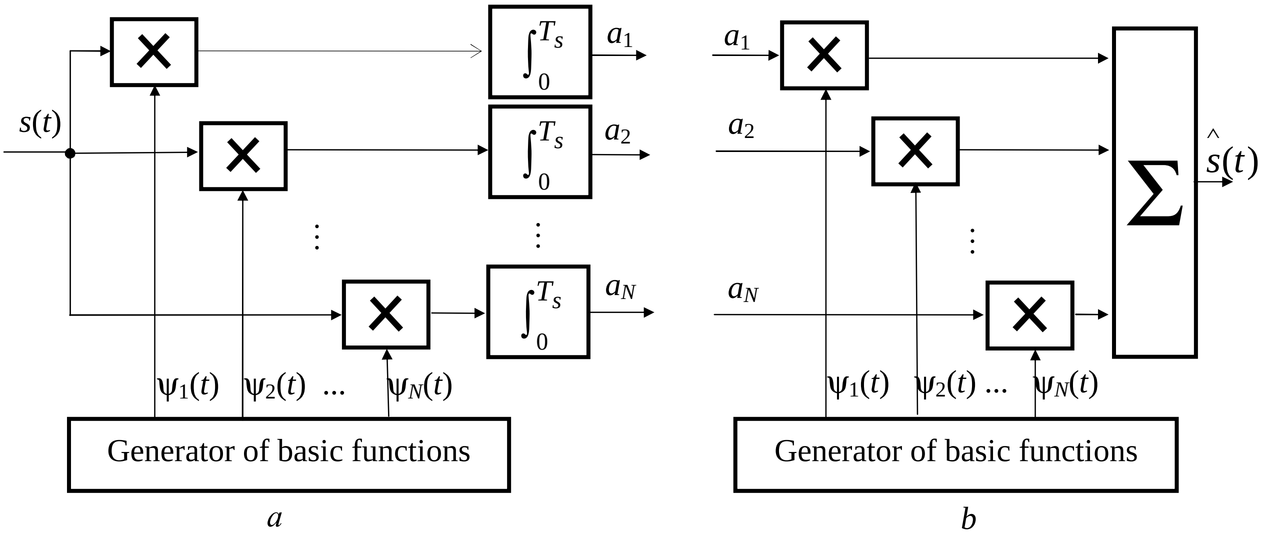


Figure 8 – Circuits of signal analysis (*a*) and synthesis (*b*)

2.4 Geometrical representation of signals

While mathematical describing signals are convenient to consider it as vectors or points in some space (figure 9). Let remember, that a vector is a segment of a given direction and lengths. Usually a vector is set by coordinates of its end. The system of coordinates should be set by unit vectors, angles between

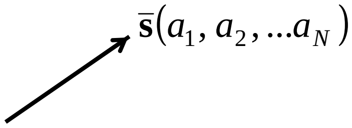


Figure 9 – Vector $\bar{\mathbf{s}}$ in *N*-dimensional space

which are equal 90°. So, to a signal $s(t)$ the vector $\bar{\mathbf{s}}$ is put in conformity.

The basic ratios for N-dimensional linear metric space are:

– length (norm) of a vector $\bar{\mathbf{s}}$

$$\|\bar{\mathbf{s}}\| = \sqrt{\sum_{n=1}^{N} a_n^2}\ , \tag{2.19}$$

– distance between vectors $\bar{\mathbf{s}}_1$ and $\bar{\mathbf{s}}_2$

$$d(\bar{\mathbf{s}}_1, \bar{\mathbf{s}}_2) = \sqrt{\sum_{n=1}^{N} (a_{1n} - a_{2n})^2}\ , \tag{2.20}$$

– scalar product of vectors $\bar{\mathbf{s}}_1$ and $\bar{\mathbf{s}}_2$

$$(\bar{\mathbf{s}}_1, \bar{\mathbf{s}}_2) = \sum_{n=1}^{N} a_{1n} \cdot a_{2n}\ . \tag{2.21}$$

Total sum of all functions of time set on an interval $(0, T_s)$ is called a functional space. These functions are considered as vectors in functional space. Coordinates of these vectors are values of functions of time. It is clear, that $N \to 0$, and ratios (2.19) – (2.21) are passing in the following

– length (norm) of a vector $\bar{\mathbf{s}}$

$$\|\bar{\mathbf{s}}\| = \sqrt{\int_0^{T_s} s^2(t)dt} = \sqrt{E_s}\ , \tag{2.22}$$

it is important to remember, that the length of a vector is equal to a root from energy of a signal;

– distance between vectors $\bar{\mathbf{s}}_1$ and $\bar{\mathbf{s}}_2$

$$d(\bar{\mathbf{s}}_1, \bar{\mathbf{s}}_2) = \sqrt{\int_0^{T_s} (s_1(t) - s_2(t))^2 dt}\ ; \tag{2.23}$$

– scalar product of vectors $\bar{\mathbf{s}}_1$ and $\bar{\mathbf{s}}_2$

$$(\bar{\mathbf{s}}_1, \bar{\mathbf{s}}_2) = \int_0^{T_s} s_1(t) s_2(t) dt\ . \tag{2.24}$$

Let address to decomposition of signals in generalized Fourier series

$$s(t) = \sum_{n=1}^{N} a_n \psi_n(t)\ . \tag{2.25}$$

We take into consideration, that basic functions are orthonormal and are normalized. Let rewrite a ratio in the vector form

$$\bar{\mathbf{s}} = \sum_{n=1}^{N} a_n \bar{\mathbf{ш}}_n\ . \tag{2.26}$$

Hence, if a signal is decomposed into generalized Fourier series it can be presented in N-dimensional space.

2.5 Spectral analysis of periodic signals

In the theory and techniques of communication the trigonometrical basis is the most widely applied to the decomposition of signals in series. Wide application of these functions in the theory and techniques of communication is caused by that while passing through linear circuits the form of each of them does not change – only their levels and phases change (there is a shift in time).

For a periodic signal Fourier series:

$$s(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos 2\pi n f_1 t+b_n\sin 2\pi n f_1 t)\,. \qquad (2.27)$$

where $f_1 = 1/T$, T is period of a signal $s(t)$;

$$a_n=\frac{2}{T}\int_{-T/2}^{T/2}s(t)\cos 2\pi n f_1 t dt,\quad n=0,1,2,...;\quad b_n=\frac{2}{T}\int_{-T/2}^{T/2}s(t)\sin 2\pi n f_1 t dt,\quad n=1,2,...$$

If to enter definitions

$$A_n=\sqrt{a_n^2+b_n^2}\ ,\ \varphi_n=-\text{arctg}\frac{b_n}{a_n}\,,$$

that a series (1) will be transformed in:

$$s(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}A_n\cos(2\pi n f_1 t+\varphi_n)\,. \qquad (2.27a)$$

Decomposition of a signal (2.27) and (2.27a) are called Fourier series in a trigonometrical form. It is more convenient to use a series (2.27a) as it directly establishes, what harmonious components a signal will consist of – what values of their frequencies nf_1, amplitudes A_n and initial phases φ_n. Series (2.27), defines a signal spectrum.

Obvious representation of a spectrum is given by figure 10 which an amplitude spectrum is built on – dependence of amplitudes A_n on frequency and a phase spectrum – dependence of initial phases φ_n from frequency. Frequency f_1 is called a basic frequency of a signal, it is equal to number of the periods of a signal in a second. Harmonious fluctuations with frequencies nf_1 (n = 2, 3, ...) are called harmonics of a signal $s(t)$: $2f_1$ –second harmonic, $3f_1$ –third harmonic, etc.

The amplitude spectrum allows to define bandwidth, as dimension of frequency range where total energy of components equals the certain share of full energy. If a considered signal spectrum adjoins to zero frequency its bandwidth is expressed by number F_{max}. Using F_{max}, assume that the signal does not contain frequencies upper F_{max}.

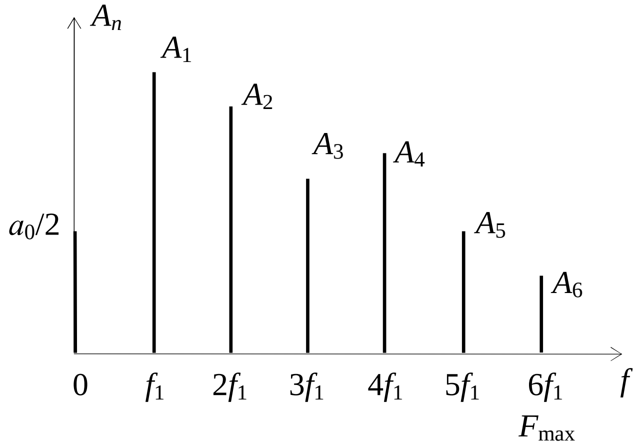


Figure 10 – Amplitude spectrum of a periodic signal

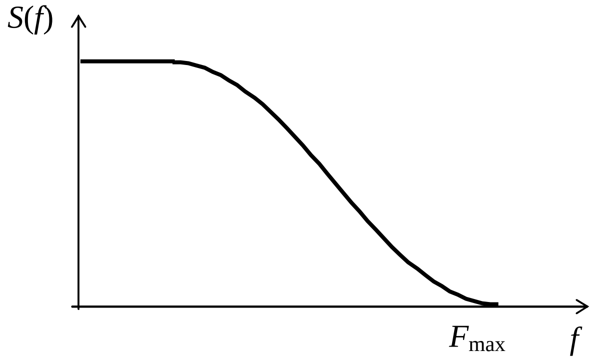


Figure 11 – Amplitude spectrum of a nonperiodic signal

Spectral representation of a periodic signal can be made, using exponential basic functions

$$\varphi_n(t) = e^{j2\pi n f_1 t}, \ n = ..., -1, 0, 1, 2, ...$$

Thus series is written as

$$s(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_1 t} . \tag{2.28}$$

Such decomposition of a signal is called Fourier series in a complex form.

Coefficients of decomposition are defined as:

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} s(t) e^{-j2\pi n f_1 t} dt, \ n = ..., -1, 0, 1, 2, ...$$

Thus:

$$c_n = \frac{a_n}{2} - j\frac{b_n}{2} = \frac{A_n}{2} e^{j\varphi_n} .$$

Feature of Fourier series in a complex form is a compact record of series and coefficients of decomposition. Other feature is using of negative frequencies.

The spectrum corresponding to series (2.28), refers to two-sided spectrum.

2.6 Spectral analysis of nonperiodic signals

Expressions:

$$S(j\omega) = \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt \quad \text{and} \quad s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(j\omega) e^{j\omega t} d\omega \tag{2.29}$$

make a pair of direct and inverse Fourier transforms. Function $S(j\omega)$ refers to as spectral density of a signal. In general case spectral density $S(j\omega)$ is a complex function. It is determined on an interval $(-\infty, \infty)$. Let present it through a module and argument $S(j\omega) = S(\omega) e^{j\varphi(\omega)}$.

Function $S(\omega)$ refers to as an amplitude spectrum of a signal, and $\varphi(\omega)$ – a phase spectrum of a signal. Function $S(\omega)$ – even function of frequency.

Many signals possess even symmetry (it is achieved by a corresponding choice of a reference mark of time). Spectral density of such signals is a real function

$$S(\omega) = 2\int_0^\infty s(t)\cos\omega t dt .$$

Because of evenness of function $S(\omega)$ inverse Fourier transform is

$$s(t) = \frac{1}{\pi}\int_0^\infty S(\omega)\cos\omega t d\omega .$$

Last two integrals make a pair of Fourier cosine-transforms.

Basic difference of spectrum is (figure 10 and 11), that a nonperiodic signal has a continuous spectrum, and periodic signal has a discrete spectrum, it contains harmonics of frequency $f_1 = 1/T$.

The apparatus of Fourier transforms is rather effective mathematical means to solve many problems of theory and techniques of communication. Note only some properties of Fourier transforms.

1. Product of two signals (a general case):

$$s(t) = s_1(t)s_2(t),$$

$$S(j\omega) = S_1(j\omega) * S_2(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_1(j\nu)S_2(j(\omega-\nu))d\nu = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_2(j\nu)S_1(j(\omega-\nu))d\nu$$

– multiplication of signals in time domain corresponds to convolution of their spectrum.

2. Convolution of signals

$$s(t) = s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\tau)s_2(t-\tau)d\tau = \int_{-\infty}^{\infty} s_2(\tau)s_1(t-\tau)d\tau,$$

$$S(j\omega) = S_1(j\omega)\cdot S_2(j\omega).$$

3. Calculation of signals energy

$$E_s = \int_{-\infty}^{\infty} |s(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |S(j\omega)|^2\, d\omega$$

– this ratio is called Parseval relation.

4. Scalar product of signals:

$$(s_1, s_2) = \int_{-\infty}^{\infty} s_1(t)s_2(t)dt; \qquad (s_1, s_2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_1(j\omega)S_2^*(j\omega)d\omega.$$

Having equated last ratio to zero, we shall receive a condition of orthogonality of signals given by spectral density.

2.7 Kotelnikov theorem and series

Kotelnikov theorem states: the signal $s(t)$, not containing frequencies higher than F_{max}, can be strictly recovered on the samples, taken through an interval $T_s \le 1/(2F_{max})$, T_s – sampling interval, $f_s = 1/T_s$ – sampling frequency.

As for any real signal it is possible to specify a highest frequency of a spectrum it is possible to suppose, that Kotelnikov theorem can be applied to all real signals.

It is possible to show, that a spectral density of a discrete signal is periodic repetition with the period f_s of spectral density of a continuous signal from which the discrete signal is received. It is illustrated by graph: on figure 12, *a* an amplitude spectrum of any continuous signal with the maximal frequency F_{max} of a spectrum is shown; on figure 12, *b* its periodic repetition is shown (figure is constructed for a case $T_s < 1/(2F_{max})$ or $f_s > 2F_{max}$. From figure 12, *b* it is understand, that at $f_s \ge 2F_{max}$ on a discrete signal (samples) with LPF it is possible to recover an initial continuous signal (by dotted line it is shown AR of the recovering filter). At $f_s < 2F_{max}$ there is an imposing periodic repetitions of a spectrum, and to recover without an error a continuous signal it is impossible. Thus Kotelnikov theorem is proved.

In time domain connection between a continuous and discrete signal is described by Kotelnikov series

$$s(t) = \sum_{n=-\infty}^{\infty} s(nT_s) \frac{\sin 2\pi F_{max}(t - nT_s)}{2\pi F_{max}(t - nT_s)}.$$

Values $s(nT_s)$ are coefficients of decomposition of a signal $s(t)$ on known system of orthogonal basic functions known from mathematics

$$\varphi_n(t) = \frac{\sin 2\pi F(t - nT_s)}{2\pi F(t - nT_s)}, \ n = ..., -1, 0, 1, 2, ...$$

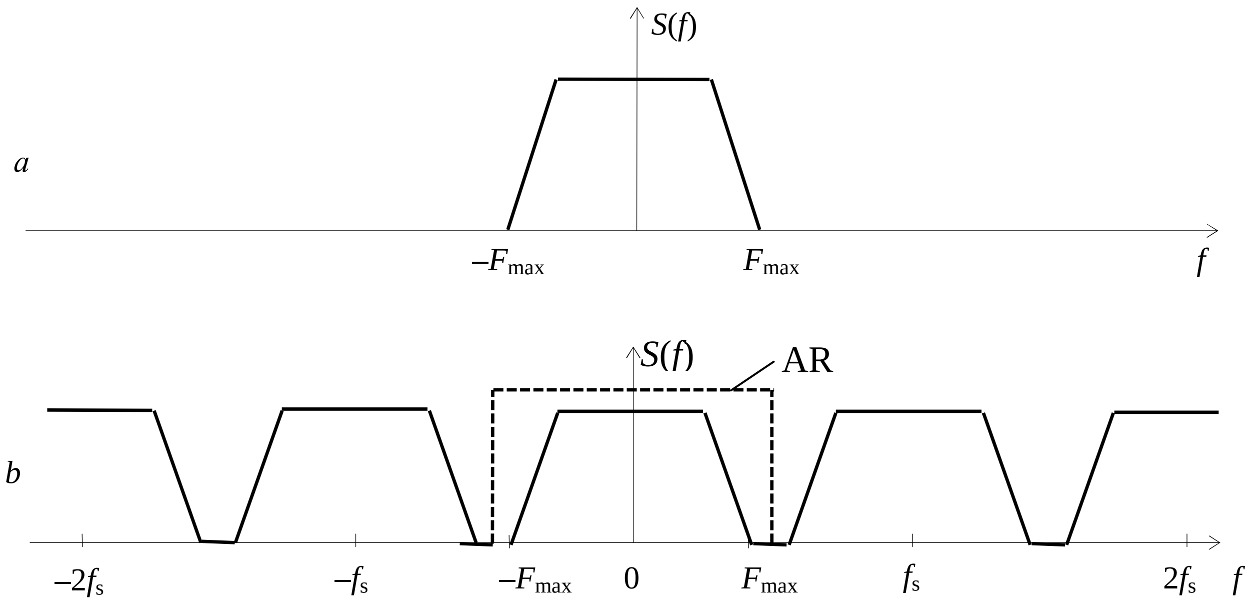


Figure 12 – *a*– spectrum of a continuous signal;
b – spectrum of a discrete signal and AR of a recovering filter

The graphic illustration of Kotelnikov series is given on figure 13

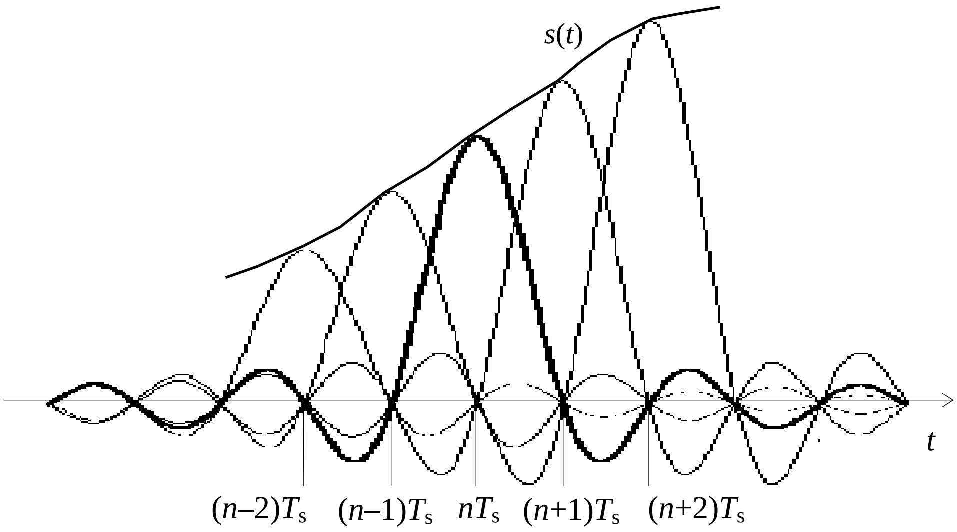


Figure 13 – Signal $s(t)$ and five components of Kotelnikov series

2.8 Representation of bandpass signals

A signal refers to bandpass if its spectrum does not adjoin to zero frequency. Their spectra are concentrated in a frequency band from f_{min} to f_{max}, and $f_{min} > 0$ (figure 14). To describe bandpass signals such parameters are presented: a middle frequency of a spectrum $f_0 = 0,5(f_{min} + f_{max})$ and bandwidth of a spectrum $\Delta F = f_{max} - f_{min}$. As a rule, for bandpass signals the relation $\Delta F << f_0$ is carried out, and then they are refered to narrow-band signals. Narrow-band signals look like quasi-harmonic oscillations with middle frequency f_0 (figure 15) in time domain.

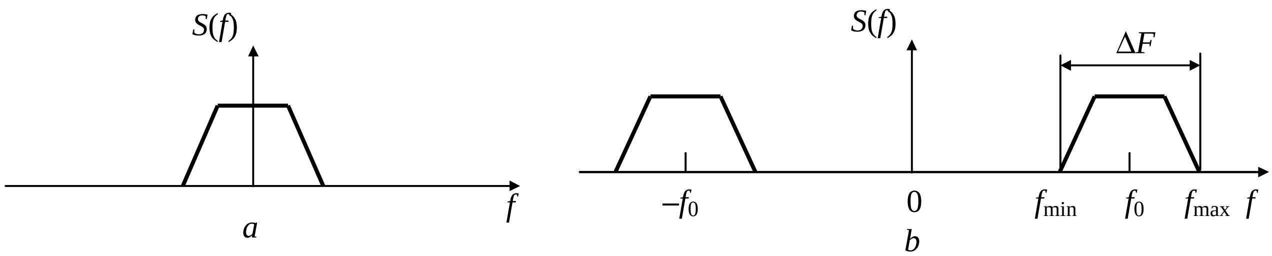


Figure 14 – Spectrum of low-frequency (*a*) and bandpass (*b*) signals

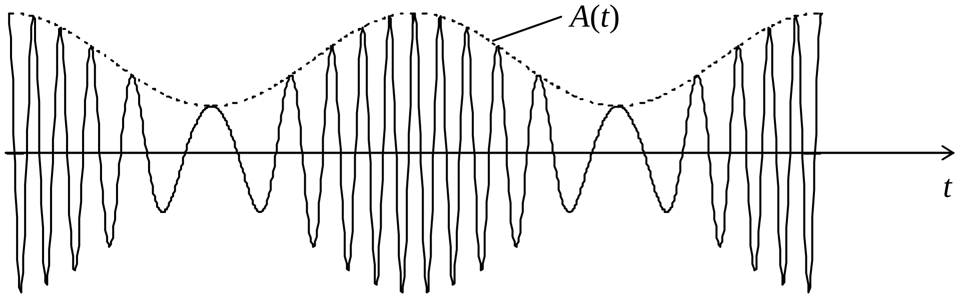


Figure 15 – Time diagram of a bandpass signal

Any bandpass signal can be presented by following mathematical expression:

$$s(t) = A(t)\cos[\psi(t)], \tag{2.30}$$

where $A(t)$ – an envelope of a bandpass signal;

$\psi(t)$ – a full phase of a bandpass signal.

An envelope of a bandpass signal is positively certain function, i.e. $A(t) \geq 0$, not being crossed with a signal, it has with it common points at moments, when instant value of a signal is maximal. A full phase of a bandpass signal will consist of three components:

$$\psi(t) = \omega_0 t + \varphi(t) + \varphi_0, \tag{2.31}$$

where $\varphi(t)$ – increment of a phase;

φ_0 – initial phase.

An increment of a phase causes changing of instant frequency of a signal. By definition frequency of a signal is a speed of its phase changing, i.e.:

$$\omega(t) = \frac{\partial\psi(t)}{\partial t} = \omega_0 + \frac{\partial\varphi(t)}{\partial t}. \tag{2.32}$$

The integral of instant frequency gives a full phase of a signal:

$$\psi(t) = \int_0^t \omega(t)dt + \varphi_0. \tag{2.33}$$

It is widely used quadrature representation of bandpass signals

$$\begin{aligned} s(t) &= A(t)\cos[\omega_0 t + \varphi(t) + \varphi_0] = \\ &= A(t)\cos[\varphi(t) + \varphi_0]\cos[\omega_0 t] - A(t)\sin[\varphi(t) + \varphi_0]\sin[\omega_0 t] = \\ &= I(t)\cos[\omega_0 t] - Q(t)\sin[\omega_0 t], \end{aligned} \tag{2.34}$$

where $I(t) = A(t)\cos[\varphi(t) + \varphi_0]$ – inphase or cosine component;

$Q(t) = A(t)\sin[\varphi(t) + \varphi_0]$ – quadrature or sinus component.

If quadrature components $I(t)$ and $Q(t)$ are known, then it is possible to find an envelope and full phase of a bandpass signal:

$$A(t) = \sqrt{I^2(t) + Q^2(t)};$$

$$\psi(t) = \omega_0 t + \operatorname{arctg}\left(\frac{Q(t)}{I(t)}\right).$$

One more form of representation of bandpass signals is the complex form $\dot{s}(t)$:

$$s(t) = \operatorname{Re}[\dot{s}(t)] = \operatorname{Re}\left[A(t)e^{j\psi(t)}\right] = A(t)\cos[\psi(t)].$$

While analysing of bandpass signals in the complex form a concept of complex envelope of a signal is entered:

$$\dot{s}(t) = A(t)e^{j\psi(t)} = A(t)e^{j(\omega_0 t + \varphi(t) + \varphi_0)} = A(t)e^{j\omega_0 t}e^{j(\varphi(t) + \varphi_0)} = \dot{A}(t)e^{j\omega_0 t}, \tag{2.35}$$

where $\dot{A}(t)$ – complex envelope of a bandpass signal.

Complex envelope has the following form:

$$\dot{A}(t) = A(t)e^{j(\varphi(t)+\varphi_0)} = A(t)\cos[\varphi(t)+\varphi_0] + jA(t)\sin[\varphi(t)+\varphi_0] = I(t) + jQ(t).$$

2.9 Analytical signal

Complex signal $\dot{x}(t) = x(t) + j\tilde{x}(t)$ refers to an analytical one, if $\tilde{x}(t)$ is Hilbert transform from $x(t)$. On figures a complex signal is represented as two circuits, as it is shown on figure 16.

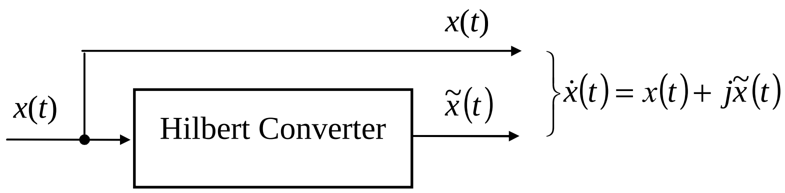


Figure 16 – Production of analytical signal

Hilbert converter is a linear circuit with the impulse response

$$g(t) = \frac{1}{\pi t}, \qquad -\infty < t < \infty.$$

Let take advantage of Duhamel integral

$$\tilde{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}d\tau.$$

This relation refers to Hilbert transform of a signal $x(t)$. Let we find transfer function of Hilbert converter like Fourier transformation from impulse response

$$H(j\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{e^{-j\omega t}}{\pi t}dt = \begin{cases} -j & \text{at} \quad \omega > 0, \\ j & \text{at} \quad \omega < 0. \end{cases} \tag{2.36}$$

or

$$H(j\omega) = -j\,\mathrm{sign}(\omega).$$

Let $S_x(j\omega)$ is a spectral density of a signal $x(t)$. Then spectral density of a signal $\tilde{x}(t)$ is defined as

$$S_{\tilde{x}}(j\omega) = \begin{cases} -j\,S_x(j\omega) & \text{at} \quad \omega > 0, \\ j\,S_x(j\omega) & \text{at} \quad \omega < 0. \end{cases} \tag{2.37}$$

Let we find spectral density of an analytical signal

$$S_{\dot{x}}(j\omega) = S_x(j\omega) + jS_{\tilde{x}}(j\omega) = \begin{cases} 2S_x(j\omega) & \text{at} \quad \omega > 0, \\ 0 & \text{at} \quad \omega < 0. \end{cases} \tag{2.38}$$

We have revealed the important property of an analytical signal – its spectrum on negative frequencies is equal to zero (figure 17).

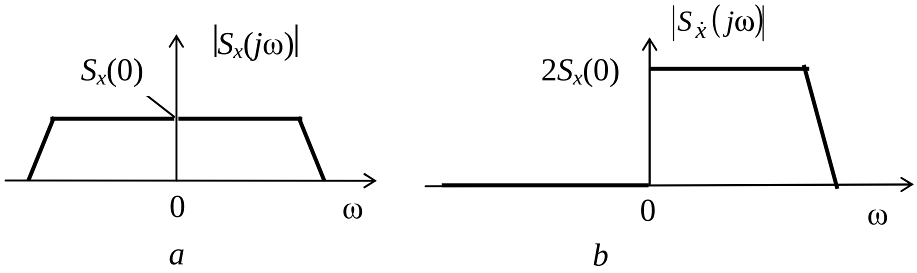


Figure 17 – Spectral density: *a* – a signal $x(t)$;
b – an analytical signal corresponding to it $|S_{\dot{x}}(j\omega)|$

Inverse Hilbert transform is

$$x(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\tilde{x}(\tau)}{t-\tau}d\tau.$$

The module of an analytical signal

$$A(t) = \sqrt{x^2(t) + \tilde{x}^2(t)}$$

is an envelope of a signal $x(t)$, and an argument

$$\varphi(t) = \mathrm{arctg}\frac{\tilde{x}(t)}{x(t)}$$

is a phase of a signal $x(t)$.

From last expressions follows that the analytical signal can be written down as:

$$\dot{x}(t) = A(t)e^{j\varphi(t)} \qquad (2.39)$$

Thus, concepts of an envelope and a phase of a signal can be applied not only to bandpass signal, but also to baseband signals. An envelope satisfies two conditions: $A(t) \geq |x(t)|$ – function $A(t)$ does not cross function $x(t)$ anywhere and in points of contact of functions $A(t)$ and $x(t)$ their derivatives are equal: $A'(t) = x'(t)$, that is functions have the common tangents.

2.10 Sampling of bandpass signals

Representation of bandpass signals by discrete signals is necessary, when transformation of signals (a filtration, detecting, etc.) is carried out by digital signal processors. In case of bandpass signals, and especially narrow-band signals, a sampling frequency can be essentially less $2f_{\max}$.

The spectral density of a discrete signal writes down:

$$S_s(j2\pi f) = f_s\sum_{n=-\infty}^{\infty}S(j2\pi(f - nf_s)), \qquad -\infty < f < \infty, \qquad (2.40)$$

where $S(j2\pi f)$ is spectral density of a continuous signal $s(t)$.

From this expression comes next: the spectrum of a discrete signal is an infinite sum of periodic repetition of a spectrum $S(j2\pi f)$ of a continuous signal $s(t)$ with the period f_s, and scale multiplier f_s.

On figure 18, *a* the amplitude spectrum of any form $S(f)$ of bandpass signal is shown. It is concentrated on an interval (f_{min}, f_{max}). On figure 18, *b* the amplitude spectrum of a discrete signal which can take place at sampling of a signal with a spectrum shown on figure 18, *a* is represented.

Components of a discrete signal spectrum, which are caused by periodic repetition of frequency bands (f_{min}, f_{max}) and ($-f_{max}$, $-f_{min}$) are designated with "filling" different density for descriptive reasons. For impossibility of spectrum components with different *n* aliasing, a choice of value f_s is made on the basis of inequalities

$$\frac{2f_{max}}{k+1} \le f_s \le \frac{2f_{min}}{k}, \quad k = 0, 1, 2, ..., k_{max}. \tag{2.41}$$

In a case, when $k = 0$, a sampling frequency $f_s \ge 2f_{max}$, i.e. this condition of a sampling frequency choice for baseband signals, which satisfies Kotelnikov theorem. When $k > 0$, then representation of a bandpass signal by a discrete signal becomes more economical (smaller number of samples). The most economical representation of a signal will be, when $k = k_{max}$.

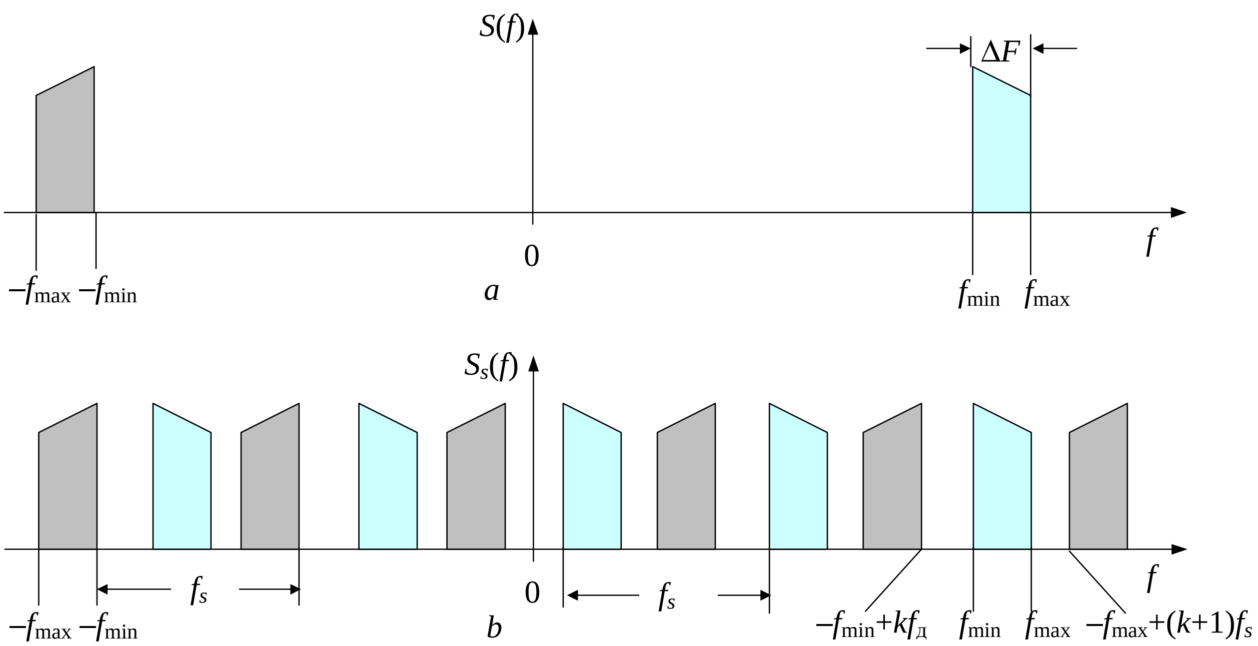


Figure 18 – Spectrum of continuous bandpass signal and discrete signal

A bandpass filter carries out recovering of a continuous bandpass signal on samples. Its lower cutoff frequency $f_{l\,cut}$ is no more than f_{min}, and upper cutoff frequency $f_{u\,cut}$ is not less than f_{max}.

3 DESCRIPTION OF RANDOM PROCESSES

3.1 Classification of random processes

Random (stochastic) process is random function of time. Main feature of random process is that its values cannot be precisely predicted beforehand. Random function of time is described by statistical characteristics that characterize those or other properties of this function on the average.

Random processes are more full mathematical models of communication signals, than the determined functions of time. Many tasks of the theory and techniques of communication can be solved only at the description of signals and noise by random functions. For example, the voltage of a noise on an output of a transmission line or on an output of a microphone is random function of time.

Let designate considered random process as $X(t)$. Separate supervisions of process give different functions $x(t)$ – different **realizations** of random process. Set all possible realizations of the given random process $\{x_k(t)\}$ is called **ensemble** (figure 19).

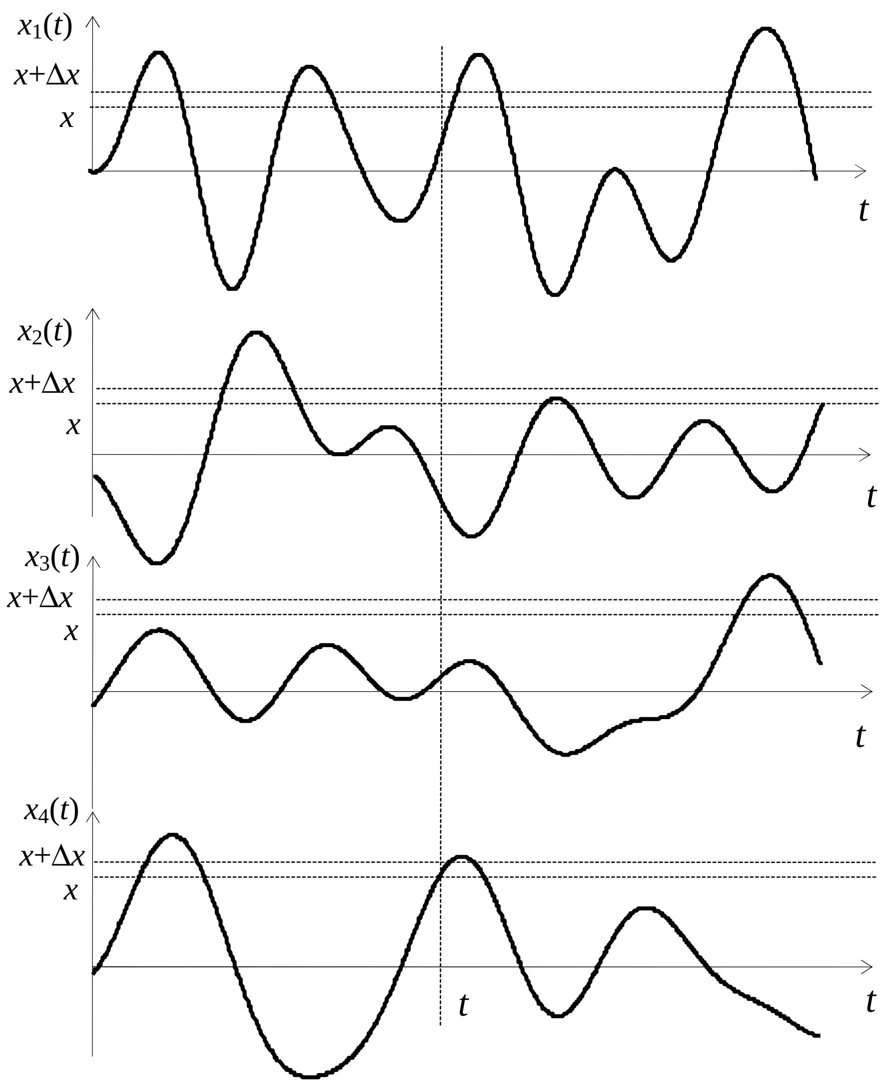


Figure 19 – Ensemble of realizations

The description of random process gives possibility to define some average characteristics of ensemble $\{x_k(t)\}$ as a whole. Such characteristics refer to statistical.

Random process refers to stationary if its statistical characteristics do not change with time.

Stationary random process refers to ergodic if its statistical characteristics are found by averaging on ensemble, coincide with the characteristics found by averaging one realization in time.

Let consider further, if it is not stipulated another, that considered random process is stationary and ergodic.

3.2 Probabilistic characteristics of random processes

Probabilistic characteristics are probability distribution function and probability density function. Probabilistic characteristics are the most used among statistical characteristics of random processes.

By definition, the value of probability distribution function $F(x)$ is equal to the probability of that in the arbitrary time moment process $X(t)$ will take on the value that does not exceed x:

$$F(x) = P\{X(t) \le x\}. \tag{3.1}$$

By definition, the value of probability density function $p(x)$ is equal to the limit of ratio of probability of that in the arbitrary time moment process $X(t)$ will take on the value on interval $(x - \Delta x/2, x + \Delta x/2)$ to the interval length Δx when $\Delta x \to 0$:

$$p(x) = \lim_{\Delta x \to 0} \frac{P\{x - \Delta x/2 < X(t) \le x + \Delta x/2\}}{\Delta x}. \tag{3.2}$$

The properties of $F(x)$ and $p(x)$ functions shown on the table 1 are easy to prove using their definitional formulas (3.1) and (3.2).

Table 1 – The properties of the functions $F(x)$ and $p(x)$

	$p(x)$	$F(x)$
1	$P\{x < X(t) \le x + dx\} = p(x)dx$	$F(x) = P\{X(t) \le x\}$
2	$P\{x_1 < X(t) \le x_2\} = \int_{x_1}^{x_2} p(x)dx$	$P\{x_1 < X(t) \le x_2\} = F(x_2) - F(x_1)$
3	$\int_{-\infty}^{\infty} p(x)dx = 1$	$F(\infty) = 1; \quad F(-\infty) = 0$
4	$p(x) \ge 0$	$F(x_2) \ge F(x_1)$ when $x_2 > x_1$
5	$p(x) = \dfrac{dF(x)}{dx}$	$F(x) = \int_{-\infty}^{x} p(x)dx$

The considered functions (3.1) and (3.2) are one-dimensional distributions of probabilities. They characterize process only during one moment of time. Two-dimensional distribution function and two-dimensional probability density function characterize process during two moments of time t and $t + \tau$.

The two-dimensional probability distribution function of process $X(t)$ is defined as

$$F_2(x_1, x_2, \tau) = P\{X(t) \le x_1;\ X(t+\tau) \le x_2\}. \tag{3.3}$$

The two-dimensional probability density function of process $X(t)$ is defined as

$$p_2(x_1, x_2, \tau) = \frac{\partial^2 F_2(x_1, x_2, \tau)}{\partial x_1 \partial x_2}. \tag{3.4}$$

For n = 3, 4, ... moments of time by analogy with (3.3) and (3.4), n-dimensional distributions of probability can be found. The more value n, the more full random process is described. But consideration of n-dimensional distributions demands complex process of realizations $x_k(t)$. Knowledge of one-dimensional and two-dimensional distributions of probabilities is used for solving of many problems.

3.3 Numerical characteristics of processes

Many problems can be solved, using more "rough" characteristics of processes, than using of probability distributions.

Average value (or expectation) $\overline{X(t)}$ of process is equal to value around of which process accepts the values. Knowing probability density of process, we can define

$$\overline{X(t)} = \int_{-\infty}^{\infty} x p(x) dx. \tag{3.5}$$

Average value on time is designated by wavy line above dependence, which is averaged on time. So, average in time value of process $X(t)$ is defined by averaging on time of realization $x(t)$

$$\widetilde{X(t)} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt. \tag{3.6}$$

Average power of process is average value of a square of process

$$P_X = \overline{X^2(t)} = \int_{-\infty}^{\infty} x^2 p(x) dx. \tag{3.7}$$

or

$$P_X = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt. \tag{3.8}$$

Average value of a square of deviations from average value is a dispersion of process

$$D\{X(t)\} = \overline{[X(t) - \overline{X(t)}]^2} = \int_{-\infty}^{\infty} [x - \overline{X(t)}]^2 p(x) dx. \tag{3.9}$$

or

$$D\{X(t)\} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[x(t) - \overline{X(t)}\right]^2 dt \qquad (3.10)$$

Positive root from dispersion is

$$\sigma_X = \sqrt{D\{X(t)\}} \qquad (3.11)$$

It is called a root-mean-square deviation of a process.

3.4 Correlation function of random process

Dependence between values $X(t)$ and $X(t+\tau)$ (τ – any shift in time) is statistically estimated by correlation function (CF) of process $X(t)$. CF is calculated as average value of product

$$K_X(\tau) = \overline{X(t)X(t+\tau)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 p_2(x_1, x_2, \tau) dx_1 dx_2 \,. \qquad (3.12)$$

or

$$K_X(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t+\tau)\, dt \,. \qquad (3.13)$$

CF properties of stationary process:

1. If in the expression (3.13) put $\tau = 0$ it passes to expression (3.8), therefore

$$K_X(0) = P_X \,. \qquad (3.14)$$

2. As correlation function of stationary process does not depend on time t, average value of product is $\overline{X(t)X(t+\tau)} = \overline{X(t-\tau)X(t)}$, therefore

$$K_X(-\tau) = K_X(\tau) \qquad (3.15)$$

correlation function of random process is even.

3. Let consider an average square of a difference of process values which will be distant in time τ

$$\begin{aligned}\overline{\varepsilon^2(\tau)} &= \overline{[X(t) - X(t+\tau)]^2} = \overline{X^2(t)} - 2\overline{X(t)X(t+\tau)} + \overline{X^2(t+\tau)} = \\ &= 2K_X(0) - 2K_X(\tau).\end{aligned} \qquad (3.16)$$

Average square is always non-negative. Therefore

$$K_X(0) \ge K_X(\tau) \qquad (3.17)$$

value of correlation function of any random process at argument $\tau = 0$ maximal.

4. Let answer a question: what are the difference in process values, which are distant on τ? The answer is in the ratio (3.21):

$$\overline{\varepsilon^2(\tau)} = 2[K_X(0) - K_X(\tau)] \qquad (3.18)$$

the more difference between $K_X(\tau)$ and $K_X(0)$, the more average difference between values of process, which are distant on τ. Thus, correlation function of random process $K_X(\tau)$ characterizes a degree of statistical dependence between values of process, which are distant on τ.

It is obvious, that with growth $|\tau|$, statistical dependence between values $X(t)$ and $X(t+\tau)$ decreases and at rather big $|\tau|$ dependence disappears. At $|\tau| \to \infty$ function $K_X(\tau)$ tends to zero, decreasing monotonously or oscillating around of zero, as shown on figure 20, *a*.

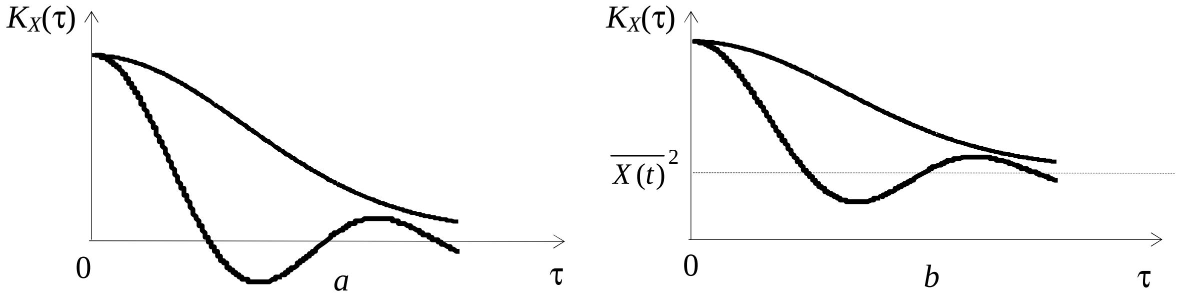


Figure 20 – Correlation functions of random processes:
a – at $\overline{X(t)} = 0$; *b* – at $\overline{X(t)} \neq 0$

5. Definition of statistical dependence is convenient for carrying out the normalized correlation function

$$R_X(\tau) = K_X(\tau)/K_X(0). \tag{3.19}$$

From ratio (3.19) it follows, that $-1 \leq R_X(\tau) \leq 1$. The closer value $R_X(\tau)$ to 1, the more strong correlated values of process, which are distant on τ.

For the rough description of correlation dependence it is introduced the concept of correlation interval (time) of process τ_c: values of process, which are distant on $\tau \leq \tau_c$ are essentially correlated among themselves, and values of process, which are distant on $\tau > \tau_c$ are uncorrelated. Correlation interval is defined differently. One of the ways is the way, as duration of a pulse is estimated. So, it is possible to agree, that

$$\tau_{\text{к}} = \int_0^{\infty} |R_X(\tau)| d\tau. \tag{3.20}$$

Here τ_c is equaled to the basis of a rectangular with height $R_X(0) = 1$, having the same area, as the area under a curve $|R_X(\tau)|$ at $\tau > 0$ and an axis absciss. It is possible to define time of correlation τ_c as duration of function $|R_X(\tau)|$ at $\tau > 0$ at a level, for example, 0,1.

Function of mutual correlation for the characteristic of dependence between values of two random processes $X(t)$ and $Y(t)$, which are distant on τ, is defined in the similar way as correlation function of process

$$K_{XY}(\tau) = \overline{X(t)Y(t+\tau)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyp_2(x,y,\tau)dxdy, \tag{3.21}$$

where $p_2(x,y,\tau)$ – joint probability density of values of stationary processes $X(t)$ and $Y(t)$, which are distant on τ, or

$$K_{XY}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)y(t+\tau)dt. \tag{3.22}$$

3.5 Power spectral density function of stationary random process

Let find Fourier transformation from realization of process $x_k(t)$, i.e. its spectral density

$$\lim_{T\to\infty} S_k(j\omega) = \lim_{T\to\infty}\int_{-T/2}^{T/2} x_k(t)e^{-j\omega t}dt. \tag{3.23}$$

But it will be the spectral characteristic only of realization $x_k(t)$, instead of process as a whole. It is possible to show, that

$$\lim_{T\to\infty}\frac{1}{T}\overline{|S_k(j\omega)|^2} = \int_{-\infty}^{\infty} K_X(\tau)e^{-j\omega\tau}d\tau, \tag{3.24}$$

where the direct line means averaging on ensemble of realizations. As correlation function characterizes process as a whole, the left part in (3.24) is also the spectral characteristic of all process. It is designated as

$$G_X(\omega) = \lim_{T\to\infty}\frac{1}{T}\overline{|S_k(j\omega)|^2}. \tag{3.25}$$

Function $|S_k(j\omega)|^2$ characterizes distribution of energy of process on frequency. As a result of division of this function on T we shall receive distribution of power of process on frequency.

The expression (3.24) can be rewritten as direct and inverse Fourier transformations

$$\left.\begin{aligned} G_X(\omega) &= \int_{-\infty}^{\infty} K_X(\tau)e^{-j\omega\tau}d\tau, \\ K_X(\tau) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} G_X(\omega)e^{j\omega\tau}d\omega. \end{aligned}\right\} \tag{3.26}$$

On the basis (3.26) it is possible to write down

$$K_X(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_X(\omega)d\omega = \int_{-\infty}^{\infty} G_X(f)df. \tag{3.27}$$

But $K_X(0) = P_X$. It follows from (3.27), that function $G_X(\omega)$ really characterizes distribution of power of process on frequency on an interval $(-\infty, \infty)$, and value of function $G_X(\omega)$ or $G_X(f)$ is equal power of process in bands in 1 Hz near of frequencies $+f$ and $-f$. Therefore function $G_X(\omega)$ refers to as power spectral density function of process. Thus, the power spectral density function and correlation function of stationary random process are connected by Fourier transformations. This statement is known as Khinchin-Wiener's theorem. Dimension of function $G_X(\omega)$ is V^2/Hz or Watt/Hz, coincides with dimension of energy and, probably, therefore sometimes function $G_X(\omega)$ is called energy spectrum of process.

As functions $K_X(\tau)$ and $G_X(\omega)$ are even instead of pair transformations (3.26) it is possible to write down a pair of Fourier cosine-transformations which are, as a rule, more simple in calculations, than ratio (3.26)

$$\left.\begin{aligned} G_X(\omega) &= 2\int_0^{\infty} K_X(\tau)\cos\omega\tau\, d\tau, \\ K_X(\tau) &= \frac{1}{\pi}\int_0^{\infty} G_X(\omega)\cos\omega\tau\, d\omega. \end{aligned}\right\} \qquad (3.28)$$

Knowing function $G_X(\omega)$, it is possible to define width of a spectrum of process from some condition, for example, length of area of positive frequencies, outside of which value of function is not exceeded with values $0{,}1\max\{G_X(\omega)\}$. If the spectrum adjoins to zero bandwidth of a spectrum is defined as $F_{\max}$ (figure 21, *a)*, and if a spectrum is bandpass, bandwidth of a spectrum is defined as ΔF (figure 21, *b)*.

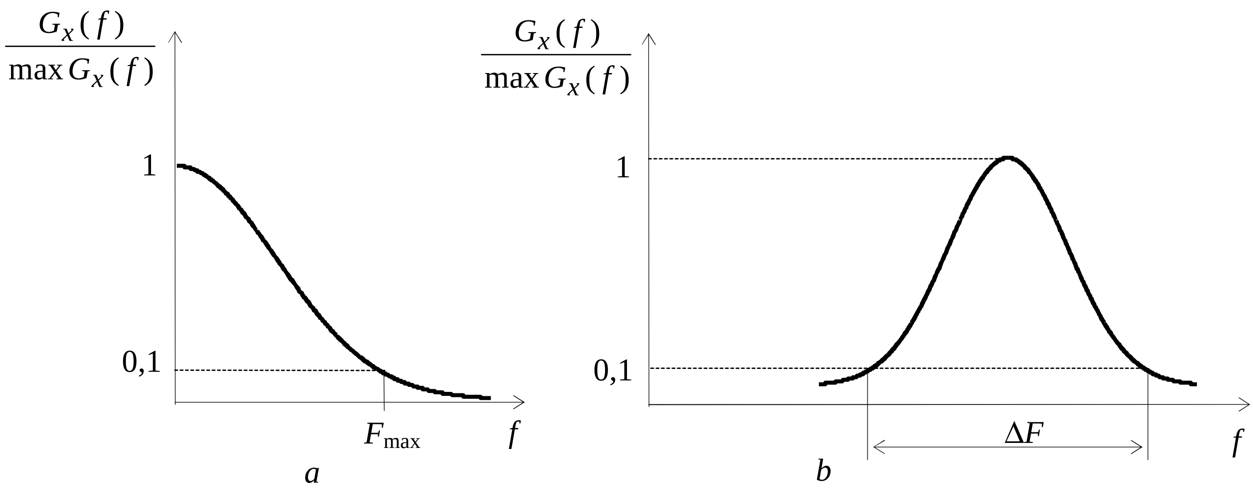


Figure 21 – Definition of bandwidth of process: *a* – the spectrum adjoins to zero frequency; *b* – bandpass spectrum

As functions $G_X(\omega)$ and $K_X(\tau)$ are connected by Fourier transformation according to property of change of scale, than the less correlation interval, the more wide a spectrum of process and on the contrary. In other words, the correlation interval and bandwidth of process are inversely proportional values.

3.6 Gaussian random process

Most frequently in the theory and techniques of communication meets so-called gaussian (or normal) random process. Random stationary process $X(t)$ refers to gaussian process if its one-dimensional and two-dimensional probability density functions are described by the following expressions

$$p(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-a)^2}{2\sigma^2}\right), \tag{3.29}$$

$$p_2(x_1,x_2,\tau)=\frac{1}{2\pi\sigma^2\sqrt{1-R_X^2(\tau)}}\exp\left(-\frac{(x_1-a)^2-2R_X(\tau)(x_1-a)(x_2-a)+(x_2-a)^2}{2\sigma^2[1-R_X^2(\tau)]}\right), \tag{3.30}$$

where σ^2 – dispersion of process $X(t)$;

a – average value of process $X(t)$;

$R_X(\tau)$ – value of the normalized correlation function of process $X(t)$.

To define two-dimensional probability density of normal random stationary process, it is enough to know only its correlation function. Thus, normal stationary processes can differ one from another with kind of correlation function and power spectral density.

The one-dimensional probability distribution function of normal process is described

$$F(x)=1-Q\left(\frac{x-a}{\sigma}\right), \tag{3.31}$$

where

$$Q(z)=\frac{1}{\sqrt{2\pi}}\int_z^\infty \exp(-\frac{t^2}{2})dt \tag{3.32}$$

– Q-function or addition to probability distribution function. Graphs of functions (3.29) and (3.31) are shown on figure 22.

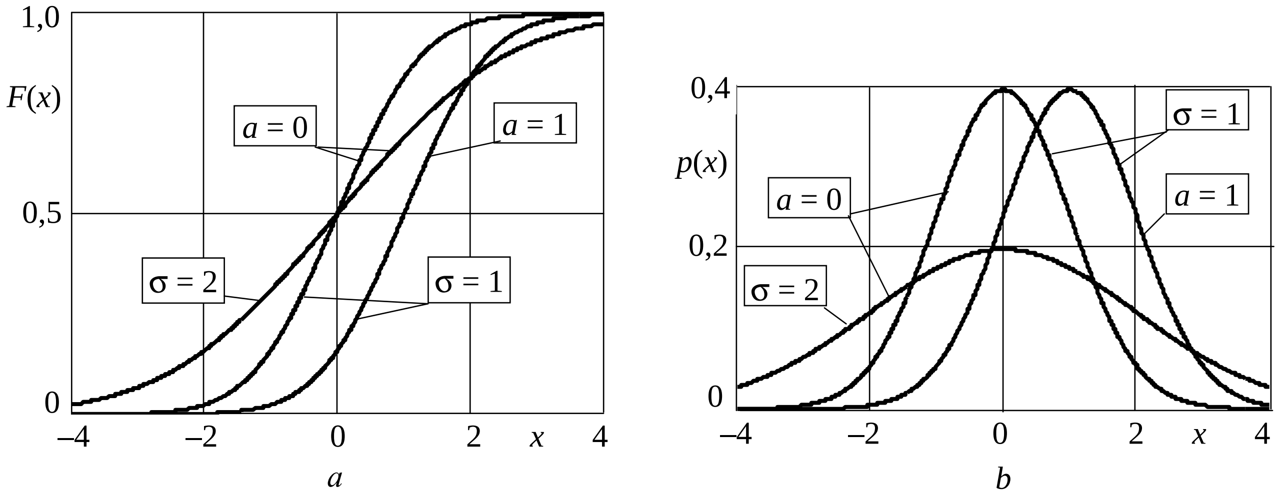


Figure 22 – Gaussian distribution:
a – probability distribution function; *b*– probability density function

Gaussian bandpass process is convenient to present through quadrature components

$$X(t)=A(t)\cos(\omega_0 t+\Phi(t))=A_c(t)\cos\omega_0 t+A_s(t)\sin\omega_0 t=X_c(t)+X_s(t), \tag{3.33}$$

where $X_c(t)$ and $X_s(t)$ are quadrature components of process ;

ω_0 – some frequency belonging to a band of process $X(t)$.

Quadrature components are uncorrelated processes having gaussian probability distribution. Their dispersions are identical and equal to half of dispersion of process $X(t)$.

Envelope $A(t)$ and phase $\Phi(t)$ are also uncorrelated processes. Envelope $A(t)$ has Reyleigh probability distribution (figure 23, *a*)

In expressions (3.34) σ^2 is the dispersion of process $X(t)$. Average value of process $\overline{A(t)} = \sqrt{\frac{\pi}{2}}\sigma$, dispersion $D[A(t)] = (2 - \frac{\pi}{2})\sigma^2$, average power $P_A = 2\sigma^2$.

$$\left.\begin{aligned} p(a) &= \begin{cases} \frac{a}{\sigma^2}\exp\left(-\frac{a^2}{2\sigma^2}\right), & a > 0, \\ 0, & a \le 0; \end{cases} \\ F(a) &= \begin{cases} 1 - \exp\left(-\frac{a^2}{2\sigma^2}\right), & a > 0, \\ 0, & a \le 0. \end{cases} \end{aligned}\right\} \qquad (3.34)$$

The phase $\Phi(t)$ has uniform probability distribution on interval $(0, 2\pi)$ (figure 23, *b*)

$$p(\varphi) = \begin{cases} \frac{1}{2\pi}, & 0 \le \varphi < 2\pi, \\ 0, & \varphi < 0, \ \ \varphi \ge 2\pi. \end{cases} \qquad (3.35)$$

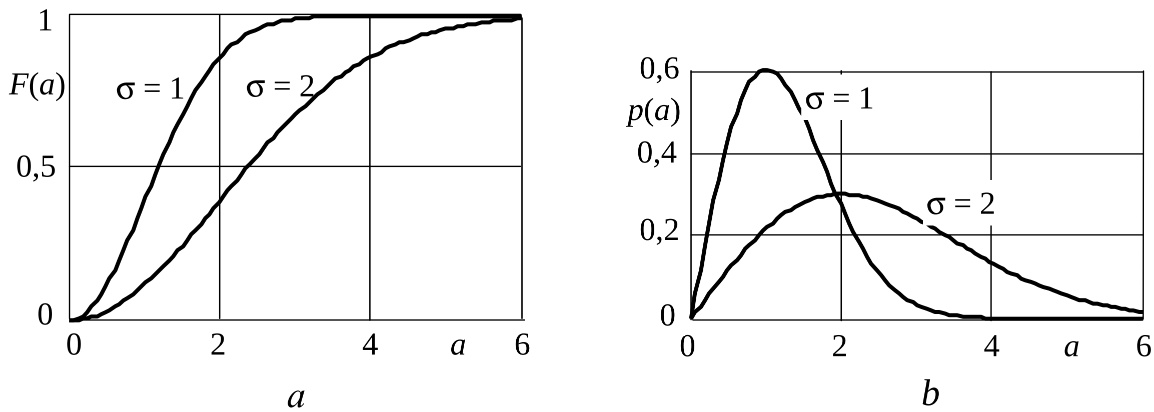


Figure 23 – Reyleigh distribution:
a – probability distribution function; *b*– probability density function

3.7 White noise

Random process refers to as white noise, if power spectral density function is a constant

$$G(\omega) = \frac{N_0}{2}, \qquad -\infty < \omega < \infty, \qquad (3.36)$$

where N_0 is a power of process in a band equals 1 Hz.

Graphic dependences shown on figure 24 correspond to expression (3.36).

The correlation function of a white noise is defined as inverse Fourier transform from (3.36)

$$K(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{N_0}{2}e^{j\omega\tau}d\omega=\frac{N_0}{2}\delta(\tau) \qquad (3.37)$$

On figure 25 the graph of correlation function of white noise is represented.

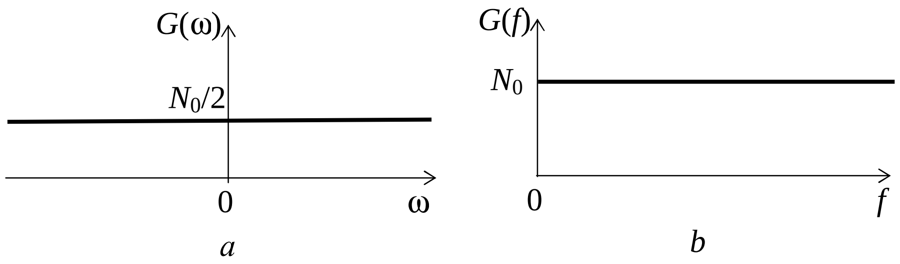


Figure 24 – Power spectral density function of white noise: *a* – two-sided spectrum; *b*– one-sided spectrum

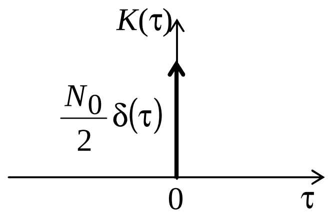


Figure 25 – Correlation function of white noise

3.8 Transformation of random processes by linear electric circuits

While studying of passage of random processes through linear circuits it is considered, that statistical characteristics of input random process $X(t)$ are known; transfer function of a linear circuit $H(j\omega)$ is known also. It is necessary to find characteristics of output process $Y(t)$.

Power spectral density function (PSDF) of process on output of a linear circuit is connected with PSDF of input process through square AR of a circuit

$$G_Y(\omega) = G_X(\omega)H^2(\omega). \qquad (3.38)$$

In particular, if input process is white noise, then PSDF of output process repeats square AR of a linear circuit.

Correlation function (CF) of process on output of a linear circuit is defined as Fourier transform from PSDF of process

$$K_X(\tau)=\frac{1}{\pi}\int_{0}^{\infty}G_X(\omega)\cdot\cos(\omega\tau)d\omega \qquad (3.39)$$

Let $X(t)$ is white noise with one-sided PSDF $G_X(f) = N_0$, $0 \le f < \infty$, it acts on an input of ideal LPF with AR

$$H(f) = \begin{cases} H_0, & 0 \le f < F_{\text{cut}}, \\ 0, & f \ge F_{\text{cut}}, \end{cases} \qquad (3.40)$$

where F_{cut} – cut off frequency of LPF. Then PSDF of process $Y(t)$:

$$G_Y(f) = G_X(f)\cdot H^2(f) = \begin{cases} N_0 H_0^2, & 0 \le f < F_{\text{cut}}, \\ 0, & f \ge F_{\text{cut}}. \end{cases} \qquad (3.41)$$

PSDF of process $Y(t)$ is shown on figure 26, *a*.
Average power of process $Y(t)$ is:

$$P_Y = \int_0^{\infty} G_Y(f)\,df = \int_0^{F_{\text{cut}}} N_0 H_0^2\,df = N_0 H_0^2\,F_{\text{cut}}. \qquad (3.42)$$

Correlation function of process $Y(t)$ is:

$$K_Y(\tau) = N_0\,H_0^2\,F_{\text{cut}}\frac{\sin 2\pi F_{\text{cut}}}{2\pi F_{\text{cut}}}. \qquad (3.43)$$

On figure 26, *b* the normalized correlation function $R_Y(\tau) = K_Y(\tau)/K_Y(0)$ is shown. Correlation interval of process $Y(t)$ $\tau_c = 1/(2F_{\text{cut}})$.

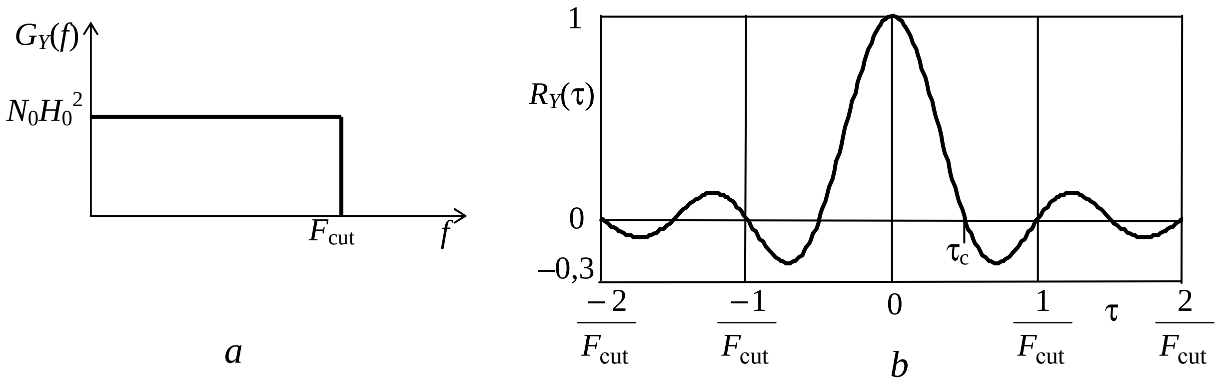


Figure 26 – Characteristics of process $Y(t)$ at a filtering with ideal LPF:
a – PSDF; *b* – CF

The concept noise band of a linear circuit is entered. Noise band of a circuit is equal to integral from a square normalized AR of circuit

$$F_n = \int_0^{\infty} \frac{H^2(f)}{H_{\max}^2}\,df, \qquad (3.44)$$

where $H_{\max}$ is the maximal value of AR.

An ideal LPF has noise band $F_n = F_{\text{cut}}$. Noise band of a circuit allows easy to define power of process on an output of a circuit if on an input of a circuit white noise with one-sided PSDF N_0 acts:

$$P_Y = N_0 \cdot F_n \cdot H_{\max}^2 \qquad (3.45)$$

Consider probability distribution of process on output of a linear circuit. If on an input of a linear circuit Gaussian process acts, then output process will be also Gaussian – a type of distribution is not changed, only its parameters are changed. If on an input of a circuit process is not gaussian, then the distribution kind is changed, and output process has probability distribution closer to gaussian, then distribution of input process.

The filtering is narrow-band if bandwidth of circuit is much less than width of a spectrum of input process. At a narrow-band filtering the phenomenon of normalization of process takes place, which consists in the following – irrespective of a kind of distribution of input process, probability distribution of process on an output of a circuit is Gaussian.

3.9 Transformation of random processes by non-linear electric circuits

While researching of random processes passing through non-linear inertial circuit it is considered, that statistical characteristics of input process $X(t)$ and dependence $y=f(x)$ between instant values of input and output processes are known. It is necessary to find characteristics of output process $Y(t)$.

The most widespread function $f(x)$ for the description of non-linear transformations is the polynomial of degree n

$$f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n, \qquad (3.46)$$

where a_0, a_1, a_2,..., a_n are coefficients of polynomial.

Factors and degree of a polynomial are defined as a result of approximation of the characteristic of a real electric circuit or proceeding from some assumptions. There are other dependences also used, except polynomial dependence (3.46).

Each of composed functions (3.46) brings the contribution to formation of values of reaction of a non-linear circuit on input action. So, a_0 describes occurrence of a constant component at $x = 0$; $a_1 x$ is linear composed element which provides proportional mapping of values x in y; $a_2 x^2$ is square-law composed element, $a_3 x^3$ is cubic composed, that are provided by the contributions proportional to x^2, x^3, etc.

The elementary action is a harmonious fluctuation $x(t) = A_1\cos 2\pi f_1 t$. In this case

$$y(t) = a_0 + a_1 A_1 \cos 2\pi f_1 t + a_2 A_1^2 \cos^2 2\pi f_1 t + ... + a_n A_1^n \cos^n 2\pi f_1 t. \qquad (3.47)$$

If to take advantage of formulas of multiple arguments we shall receive

$$y(t) = Y_0 + Y_1 \cos 2\pi f_1 t + Y_2 \cos 2\pi 2 f_1 t + ... + Y_n \cos 2\pi n f_1 t, \qquad (3.48)$$

where Y_0 is constant component of response;

Y_1, Y_2, ..., Y_n are amplitudes of the first, the second..., n-th harmonics of action.

Thus, response to harmonious action contains a constant component and harmonics of frequency of action – it essentially distinguishes non-linear circuits from linear in which new components do not arise.

In the case of biharmonic action

$$x(t) = A_1 \cos 2\pi f_1 t + A_2 \cos 2\pi f_2 t. \qquad (3.49)$$

The approach to definition of response is the same, as well as used above, expression for $x(t)$ is substituted in a polynomial (3.46). While raising the sum (3.49) to such power as square, a cube, etc., degrees of cosine frequencies f_1 and f_2 will appear, that after transformations it gives expression of such kind (3.48) for fluctuations of frequencies f_1 and f_2. But products of cosines and their degrees are also appeared. Product of cosines gives components of summing and differential frequencies.

Components of combinational frequencies generally will take place

$$f_{comb} = |pf_1 \pm qf_2|, \quad (3.50)$$

where p, q are integers 0, 1, 2, ..., but such, that $p + q \leq n$. Their sum $N = p + q$ is called the order of combinational frequency.

So, if $n = 3$, that in a spectrum of response there can be components of frequencies f_1, f_2, $2f_1$, $2f_2$, $|f_1 \pm f_2|$, $3f_1$, $3f_2$, $|2f_1 \pm f_2|$, $|f_1 \pm 2f_2|$. Amplitudes of components depend on amplitudes A_1 and A_2 and coefficients of a polynomial (3.46).

While passing of random process through a non-linear circuit the type of distribution of momentary values are significantly changed.

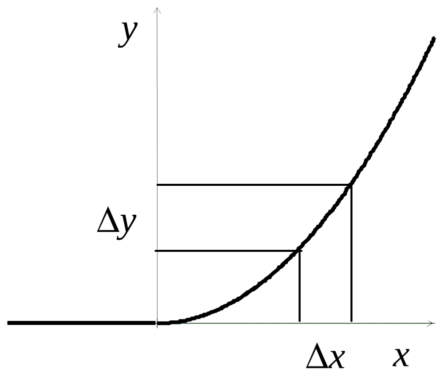


Figure 27 – The characteristic of nonlinearity

On figure 27 non-linear dependence is shown as $y = f(x)$. All values of process $X(t)$, got on the interval Δx, are mapped in values of process $Y(t)$, got on the interval Δy. Therefore equality $p(x)\Delta x \approx p(y)\Delta y$ is correct. As for the infinitesimal increments dx and dy, we shall receive, that

$$p(y) = \frac{p(x)}{|dy / dx|} \quad (3.51)$$

It also is the general rule of calculation of probability density of output process.

To define power spectral density function of output process $G_Y(f)$ the next way is possible: to define correlation function of output process $K_Y(\tau)$, and then to perform with it Fourier transform. It follows from definition of correlation function

$$K_Y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1) f(x_2) p_2(x_1, x_2, \tau) dx_1 dx_2, \quad (3.52)$$

where $f(x)$ is the function describing a non-linear circuit;

$p_2(x_1, x_2, \tau)$ is two-dimensional probability density of input process.

Methods of definition of characteristics of output process are stated. Certainly, in particular cases there can be mathematical difficulties.

4 METHODS OF ANALOG MODULATION

4.1 Classification of analog modulation types

In most transmission systems, telecommunication baseband signals cannot be passed directly by communication channels without transformation into other signals. Transformations have as an object to co-ordinate signals characteristics with communication channels characteristics. One of such transformations is modulation.

It is distinguished that if a modulating signal is continuous it belongs to analog modulation, and if modulating signal is digital it belongs to digital modulation.

4.2 General information about analog modulation

At analog modulation one of carrier parameters $u_{car}(t)$ gets increases, that are proportional to the values of modulating signal $b(t)$.

Carrier is auxiliary harmonic oscillation, necessary for implementation of modulation process.

$$u_{car}(t) = A_0\cos(2\pi f_0 t + \varphi_0),$$

At such oscillation amplitude, frequency or initial phase can get increases

Name of parameter which gets increases determines the name of type of modulation (amplitude, phase, and frequency).

While considering analog types of modulation we will consider that a modulating signal is a telecommunication baseband signal $b(t)$ with such characteristics:

-maximal frequency of signal spectrum F_{max} is given;

-a signal is normalized so, that maximal on the module value $|b(t)|_{max} = 1$;

-average value of signal $\overline{b(t)} = 0$;

-the coefficient of amplitude K_A, is given. It determines, in how many times the maximal on the module value of signal is exceeded its average quadratic value (root out of average power P_b):

$$K_A = \frac{|b(t)|_{max}}{\sqrt{P_b}}. \qquad (4.1)$$

If a signal is normalized by the method indicated above

$$P_b = 1/K_A^2. \qquad (4.2)$$

4.3 Amplitude modulation and its versions

At amplitude modulation of amplitude increases of harmonious carrier are proportional to the instantaneous values of modulating signal, i.e. amplitude of the modulated signal is

$$A(t) = A_0 + \Delta A b(t), \qquad (4.3)$$

where ΔA is a coefficient of proportionality, which is chosen so that amplitude $A(t)$ does not take on negative values, i. e. $\Delta A \le A_0$. As $|b(t)|_{max} = 1$, then ΔA determines the most maximal increase of carrier amplitude on the module. Frequency and initial phase of carrier remain constant.

It is comfortable to pass to the relative maximal increase of amplitude - amplitude modulation factor

$$m_{AM}=\Delta A/A_0. \qquad (4.4)$$

It is clear, that

$$0 < m_{AM} \le 1. \qquad (4.5)$$

In the case of random modulating signal, analytical expression of AM signal looks like

$$s_{AM}(t) = A_0[1 + m_{AM}b(t)]\cos(2\pi f_0 t + \varphi_0). \qquad (4.6)$$

The time base diagram of AM signal is shown on figure 28. Envelope of the modulated signal repeats the form of modulating signal.

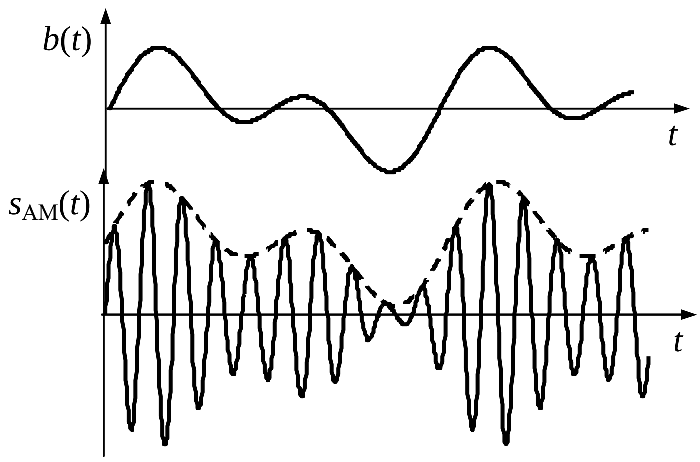


Figure 28 - Modulating $b(t)$ and modulated $s_{AM}(t)$ signals

We will pass to determination of spectral characteristics of AM signal. Let the modulating signal $b(t)$ be harmonious oscillation of frequency $F < f_0$. We will write down expression for single-tone AM signal

$$s_{AM}(t) = A_0[1 + m_{AM}\cos(2\pi Ft)]\cos(2\pi f_0 t +\varphi_0). \qquad (4.7)$$

If to use the trigonometric formula of cosine product, we will get from a formula (4.7) following

$$s_{AM}(t) = A_0\cos(2\pi f_0 t) + 0{,}5A_0 m_{AM}\cos[2\pi(f_0 + F)t] +$$
$$+ 0{,}5A_0 m_{AM}\cos[2\pi(f_0 - F)t]. \qquad (4.8)$$

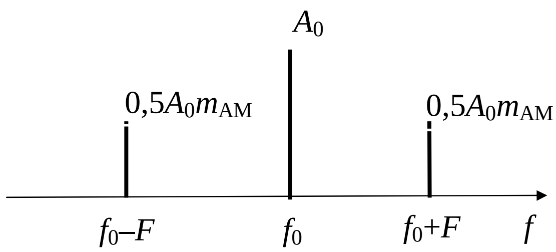


Figure 29 – Amplitude spectrum of single-tone AM signal

It follows from formula (4.8), that the spectrum of single-tone AM signal contains three harmonious oscillations: with frequency of carrier (carrier oscillation) f_0; upper sideband oscillation with frequency $f_0 + F$ and lower sideband oscillation with frequency $f_0 - F$. Amplitude spectrum of single-tone AM signal is shown on figure 29. Amplitudes of sideband oscillations are identical and even when m_{AM} = 1 does not exceed the half of amplitude of carrier A_0.

We will pass to consideration of spectrum of AM signal in a case of complex modulating signal which will to a great extent answer the real signals of telecommunication. The complex signal *b(t)* has finite or infinite sum of harmonious components. Every component causes appearance in the spectrum of the modulated signal two components – summing and differential frequencies. Their total sums create accordingly upper and lower sidebands of frequencies.

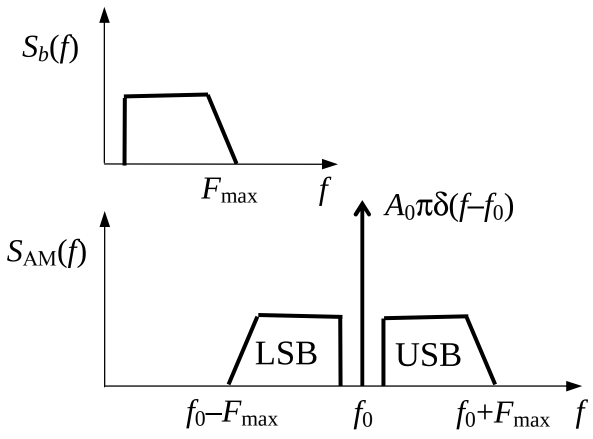


Figure 30 – Spectrums modulating and AM signals

On figure 30 random amplitude spectrum of modulating signal and its proper amplitude spectrum of AM signal are shown. It consists of harmonious oscillation of carrier frequency, upper sideband of frequencies (USB) and lower sideband of frequencies (LSB). Thus USB is the scale copy of spectrum of modulating signal, which is shifted on frequency on the value f_0. LSB is the mirror reflection of USB according to carrier frequency f_0. *F*igure 30 gives an important result: the width of spectrum of AM signal F_{AM} equals the doubled value of maximal frequency of spectrum of modulating signal F_{max}, i.e.

$$F_{AM} = 2F_{max}. \quad (4.9)$$

It is possible to show that average power of carrier $P_{car} = \frac{A_0^2}{2}$, and sideband $P_{SB} = \frac{A_0^2 m_{AM}^2}{2K_A^2}$. Then ratio of sideband power to total power of AM signal is $\frac{P_{SB}}{P_{AM}} = \frac{m_{AM}^2}{K_A^2 + m_{AM}^2}$. If to take on a maximally possible value m_{AM} = 1, and value of amplitude modulating signal coefficient K_A = 5 (voice signal), so part of sidebands power is P_{SB}/P_{AM} = 0,04 or 4%.

We see that while using of AM for transmission of telecommunication signals, prevailing part of power of AM signal is spent on oscillation of carrier frequency, although this oscillation does not carry information, as its level in the process of modulation remains constant – information is contained in the sidebands of frequencies. Therefore it is efficient to form a signal with a spectrum, consisting only of two sidebands of frequencies (without oscillation of carrier frequency), – such signal is called the signal of double-sideband with suppressed carrier modulation.

Such type of modulation, when the modulated signal is a product of modulating signal $b(t)$ and carrier is called double-sideband with suppressed carrier modulation. Analytical expression of signal double-sideband with suppressed carrier modulation (DSB-SC) looks like

$$s_{DSB\text{-}SC}(t) = A_0 b(t)\cos(2\pi f_0 t). \qquad (4.10)$$

The time base diagrams of modulating and modulated signals are shown on fig. 31. From figure 31 it is evident, that envelope of DSB-SC signal $A(t) = A_0 |b(t)|$ (it is shown by dotted line) does not repeat modulating signal.

From comparison of mathematical expressions, describing AM and DSB-SC signals it is clear that the spectrum of DSB-SC signal differs from the spectrum of AM signal by absence of oscillation of carrier frequency.

Random amplitude spectrum of modulating signal and its proper amplitude spectrum of DSB-SC signal are shown on figure 32. It consists of upper sideband of frequencies (USB) and lower sideband of frequencies (LSB). Thus USB is the scale copy of spectrum of modulating signal, shifted on frequency on the value f_0. LSB is the mirror reflection of USB relatively to frequency of carrier oscillation f_0.

From fig. 5 it flows, that the width of spectrum of DSB-SC signal $F_{DSB\text{-}SC}$ equals the doubled value of maximal frequency of modulating signal spectrum F_{max}, i.e.

$$F_{DSB\text{-}SC} = 2F_{max}. \qquad (4.11)$$

The width of spectrum of DSB-SC signal is the same as width of spectrum of AM signal.

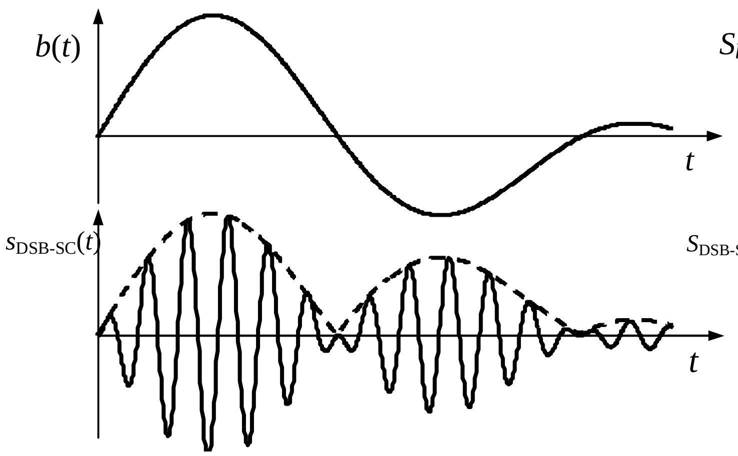


Figure 31 – Modulating $b(t)$ and modulated $s_{DSB\text{-}SC}(t)$ signals

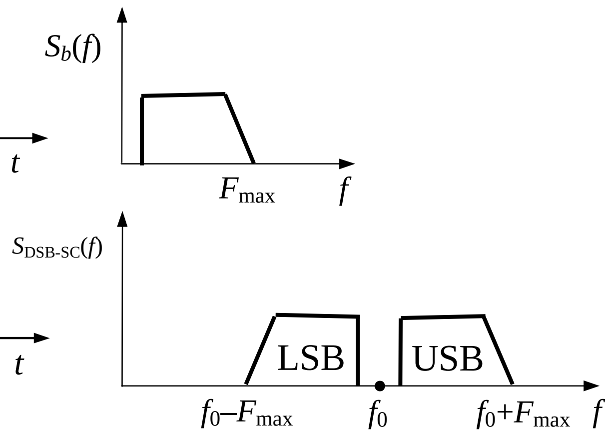


Figure 32 - Spectra of modulating and DSB-SC signals

Important advantage of DSB-SC signals in comparison with the AM signals is the advanced efficiency of the use of transmitter power, as considerable part of signal power is not spent on carrier oscillation, which is in the spectra of AM signals.

Without the losses of information about the signal $b(t)$ it is possible to take out one sideband (upper or lower) from the spectrum of DSB-SC signal. Thus we will get single sideband modulation (SSB).

In general case (for the random signal $b(t)$) SSB signal is written down as

$$s_{SSB}(t) = A_0 b(t)\cos(2\pi f_0 t + \varphi_0) \mp A_0 \tilde{b}(t)\sin(2\pi f_0 t + \varphi_0), \qquad (4.12)$$

where sign "–" refers to description of signal with the upper sideband of frequencies, and sign "+" – with a lower sideband; $\tilde{b}(t)$ is a signal, conjugated on Gilbert with a signal $b(t)$:

On figure 33 the time base diagrams of random modulating signal $b(t)$, the conjugated on Gilbert signal $\tilde{b}(t)$ and SSB signal calculated for it are shown. From figure 33 it is evident, that SSB signal envelope $A(t) = A_0\sqrt{b^2(t)+\tilde{b}^2(t)}$ (shown by dotted line) does not repeat a modulating signal.

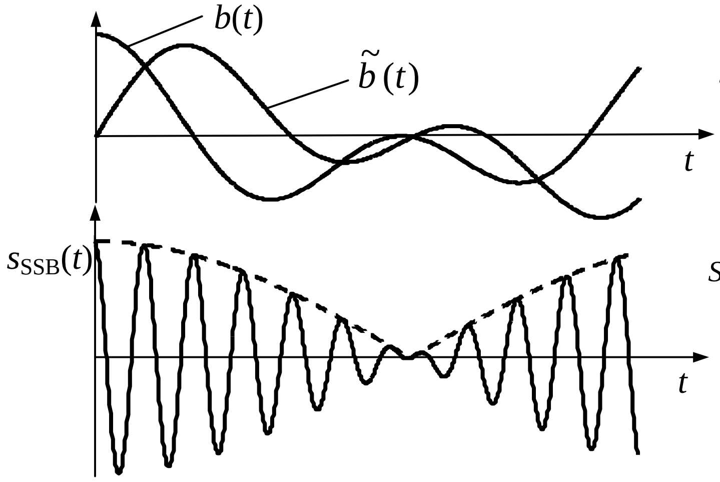


Figure 33 - Modulating $b(t)$ and modulated $s_{SSB}(t)$ signals

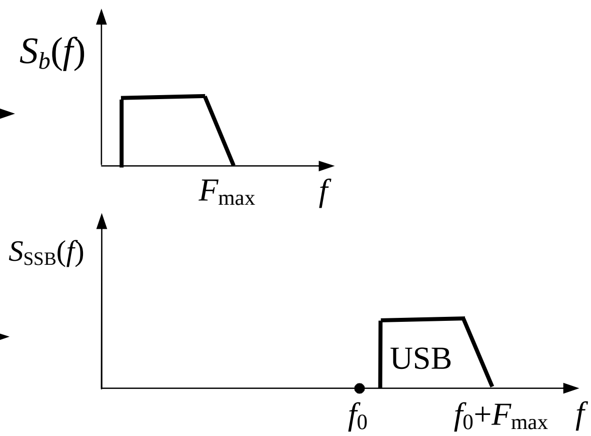


Figure 34 - Spectra of modulating and SSB signals

On figure 34 amplitude spectrum of SSB signal, got from the spectrum of DSB-SC signal by the exception of lower sideband of frequencies (it is possible to eliminate the upper sideband of frequencies) is shown. So, such type of modulation, when the spectrum of the modulated signal coincides with the spectrum of modulating signal, shifted on carrier frequency, or is the inversion of the shifted spectrum respectively carrier frequency is called single sideband.

From figure 34 it flows, that spectrum width of SSB signal F_{SSB} equals maximal frequency of spectrum of modulating signal

$$F_{SSB} = F_{max}. \qquad (4.13)$$

Important advantage of SSB signal in comparison with DSB-SC and AM signals is twice decreased width of modulated signal spectrum, which allows the signals amount to increase twice in the set frequencies band. Therefore SSB is widely used in the systems of multichannel transmission with a frequency division. SSB is a single type of analog modulation, when the band of frequencies of signal is not broaden while modulation. Except of considered "clean" SSB in communication networks it was found the use of SSB signals with carrier (with pilot signal) and with partial suppression of one sideband of frequencies. It creates certain comforts at forming and detection of the modulated signals.

4.4 Frequency and phase modulation

These two methods of modulation are reffered to the angular types of modulation (AnM) – amplitude of the modulated signal remains constant, and argument (angle) of trigonometric function – carrier $u_{car}(t) = A_0\cos(2\pi f_0 t + \varphi_0)$ gets the increase $\Delta\varphi(t)$, conditioned by the process of modulation. Therefore signal AnM can be written down as

$$s_{AnM}(t) = A_0\cos(2\pi f_0 t + \Delta\varphi(t) + \varphi_{0)} = A_0\cos\Phi(t). \tag{4.14}$$

The function $\Phi(t)$ is called an angle, complete phase, instantaneous phase or simply phase of signal, and φ_0 is called the initial phase of signal. Instantaneous frequency of signal at the set function $\Phi(t)$ is determined

$$f(t) = \frac{1}{2\pi}\cdot\frac{d\Phi(t)}{dt} = f_0 + \frac{1}{2\pi}\cdot\frac{d(\Delta\varphi(t))}{dt} = f_0 + \Delta f(t), \tag{4.15}$$

where
$$\Delta f(t) = \frac{1}{2\pi}\cdot\frac{d(\Delta\varphi(t))}{dt} \tag{4.16}$$

is frequency increase.

Carrier $u_{car}(t)$ has instantaneous frequency $f(t) = f_0$ which is a constant, and an instantaneous phase depends linearly on time: $\Phi(t) = 2\pi f_0 t + \varphi_0$.

At the set function $\omega(t)$ the instantaneous phase of signal is determined

$$\Phi(t) = \int_{-\infty}^{t} 2\pi f(t)dt + \varphi_0 = 2\pi\int_{-\infty}^{t}(f_0 + \Delta f(t))dt + \varphi_0 = 2\pi f_0 t + 2\pi\int_{-\infty}^{t}\Delta f(t)dt + \varphi_0, \tag{4.17}$$

i.e. increase of phase is
$$\Delta\varphi(t) = 2\pi\int_{-\infty}^{t}\Delta f(t)dt. \tag{4.18}$$

The initial phase j_0 can be considered as permanent integrations.

At phase modulation the increase of phase is proportional to the instantaneous values of modulating signal

$$\Delta\varphi(t) = \Delta\varphi_d\, b(t), \tag{4.19}$$

where $\Delta\varphi_d$ is a coefficient of proportion, which is called phase deviation. As maximal on the module value $|b(t)|_{max} = 1$, deviation of phase at PM is the most maximal deviation of phase from linear dependence in time.

Mathematical description of PM signal

$$s_{PM}(t) = A_0\cos(2\pi f_0 t + \Delta\varphi_d\, b(t) + \varphi_0). \tag{4.20}$$

At phase modulation instantaneous frequency depends on a modulating signal by next way

$$\Delta f(t) = \frac{1}{2\pi}\cdot\frac{d(\Delta\varphi_d b(t))}{dt} = \frac{\Delta\varphi_d}{2\pi}\cdot\frac{d(b(t))}{dt}. \tag{4.21}$$

At frequency modulation the increase of frequency is proportional to the instantaneous value of modulating signal

$$\Delta f(t) = \Delta f_d\, b(t), \tag{4.22}$$

where Δf_d is coefficient of proportionality, which is called frequency deviation and determines the maximal deviation of instantaneous frequency of the modulated signal from carrier frequency f_0.

The increase of phase is at frequency modulation

$$\Delta\phi(t) = 2\pi \int_{-\infty}^{t} \Delta f(t) dt = 2\pi \Delta f_d \int_{-\infty}^{t} b(t) dt\,. \tag{4.23}$$

Will get mathematical description of FM signal by the substitution of expression (4.23) in formula (4.14):

$$s_{\mathrm{FM}}(t) = A_0 \cos(2\pi f_0 t + 2\pi \Delta f_{\mathrm{d}} \int_{-\infty}^{t} b(t) dt + \varphi_0)\,. \tag{4.24}$$

From the given above descriptions of signals it follows, that FM and PM have a lot in common. Both FM and PM have the increases of frequency and phase. The name of modulation type is determined by which of parameters gets increases, proportional to modulating signal.

At angular modulations connection between the spectra of modulating and modulated signals is considerably more difficult, than at AM and its varieties. Therefore an analysis of spectra of angular modulations here is not given. We will discuss a final result. The analysis shows that theoretically the bandwidth of amplitude spectrum is infinite. However basic part of power of signal is concentrated in some limited frequency interval around f_0, which is considered as the width of signal spectrum. The spectrum width of FM and PM signals is calculated on formulas:

$$F_{\mathrm{FM}} = 2\,(m_{\mathrm{FM}} + 1)\, F_{\max}, \tag{4.25}$$

$$F_{\mathrm{PM}} = 2\,(m_{\mathrm{PM}} + 1)\, F_{\max}. \tag{4.26}$$

where

$$m_{\mathrm{FM}} = \Delta f_{\mathrm{d}} / F_{\max} \tag{4.27}$$

– is an index of frequency modulation, which is determined by ratio of frequency deviation of FM signal to frequency of modulating signal;

$$m_{\mathrm{PM}} = \Delta\varphi_{\mathrm{d}} \tag{4.28}$$

– is an index of phase modulation, which equals phase deviation of PM signal.

Only frequency modulation has got wide distribution. It is distinguished narrowband (in the case of $m_{\mathrm{FM}} < 1$) and broadband (in the case of $m_{\mathrm{FM}} >> 1$) modulations. Narrowband FM signal spectrum width is comparable with the AM signal spectrum width. If FM is broadband, then signal spectrum width approximately equals doubled deviation of frequency.

4.5 Forming of the modulated signals (modulators)

Two features presently characterize construction of modulators diagrams:

1) modulators are performed on the processors of digital signals; thus diagrams work with samplings of signals, and frequency of sampling is chosen according to the relation, considered in s. 2.10;

2) the modulator diagram realize a forming algorithm which follows from mathematical description of the modulated signal.

Let consider the construction of DSB-SC signal modulator diagram. Mathematical description of signal looks like

$$s_{\text{DSB-SC}}(t) = A_0 b(t)\cos(2\pi f_0 t + \varphi_0). \tag{4.29}$$

The diagram of DSB-SC signal modulator, constructed on the basis of this relation, is shown on figure 35. It has a generator carrier oscillation *G* and multiplier.

Modulator of AM signal is constructed on the basis of correlation (4.30):

$$s_{\text{AM}}(t) = A_0[1 + m_{\text{AM}} b(t)]\cos(2\pi f_0 t + \varphi_0). \tag{4.30}$$

According to this relation modulator is realized by a diagram, represented on figure 36.

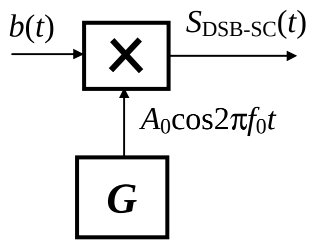


Figure 35 – Block diagram of DSB-SC modulator

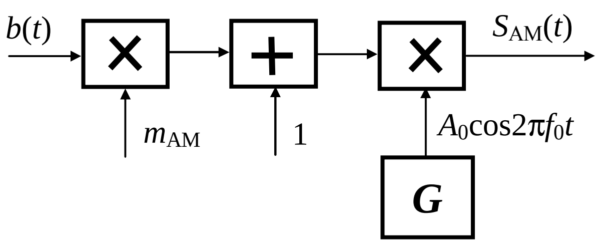


Figure 36 – Block diagram of amplitude modulator

Modulator of SSB signal can be performed by a filter method or phase method. On figure 37 the diagram of SSB signal by a filter method forming is shown. Necessary (upper or lower) sideband of frequencies is selected from the double-sideband with suppressed carrier modulation signal spectrum by a bandpass filter.

Phase method of forming of SSB signal is based on basic property of analytical signal: its spectrum is concentrated on positive frequencies. We will form an analytical signal $\dot{b}(t) = b(t) + j\tilde{b}(t)$. On figure 38, *a* the spectrum of random signal $b(t)$ is shown, and on figure. 38, *b* it is shown spectrum of signal $\tilde{b}(t)$. Product of $\tilde{b}(t)$ and $A_0 e^{j2\pi f_0 t}$ signals gives an analytical signal $\dot{m}(t)$. Its spectrum is the spectrum shifted to the right on f_0 of signal $\dot{b}(t)$ (figure 38, *c)*. To pass to the real signal it is necessary to take real part of signal $\dot{m}(t)$:

$$\begin{aligned} S_{\text{SSB}}(t) &= \text{Re}\{\dot{m}(t)\} = \text{Re}\{(b(t) + j\tilde{b}(t)) \cdot A_0(\cos 2\pi f_0 t + j\sin 2\pi f_0 t)\} = \\ &= A_0 b(t)\cos 2\pi f_0 t - A_0\tilde{b}(t)\sin 2\pi f_0 t, \end{aligned} \tag{4.31}$$

that gives expression for SSB signal with USB. It is easy to make sure, that the increase of the complex conjugated analytical signal product $\dot{b}^*(t) = b(t) - j\tilde{b}(t)$ and $A_0 e^{j2\pi f_0 t}$ and selection of real part give SSB signal with LSB:

$$S_{SSB}(t) = A_0 b(t)\cos 2\pi f_0 t + A_0 \tilde{b}(t)\sin 2\pi f_0 t\,. \qquad (4.32)$$

Diagram of SSB signal modulator (figure 39) is given from relation (4.31).

The diagram of Gilbert converter can be synthesized on its impulse response or according to the necessity AR and PR (figure 40). Here F_{min} and F_{max} are boundary frequencies of modulating signal spectrum.

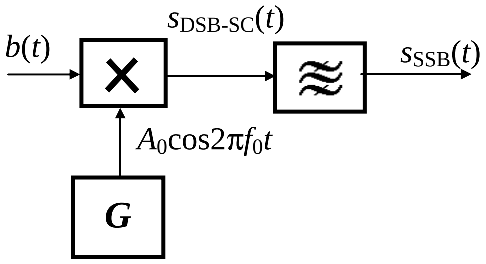


Figure 37 - Forming of SSB signal by a filter method

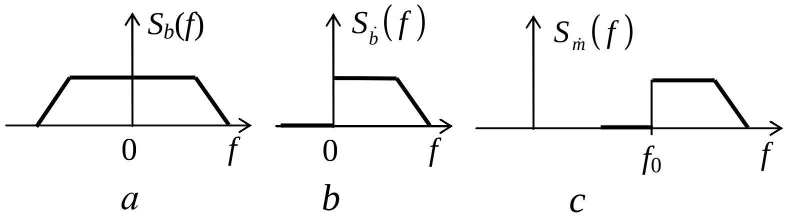


Figure 38 - Amplitude spectrums of signals $b(t)$, $\dot{b}(t)$ and $\dot{m}(t)$

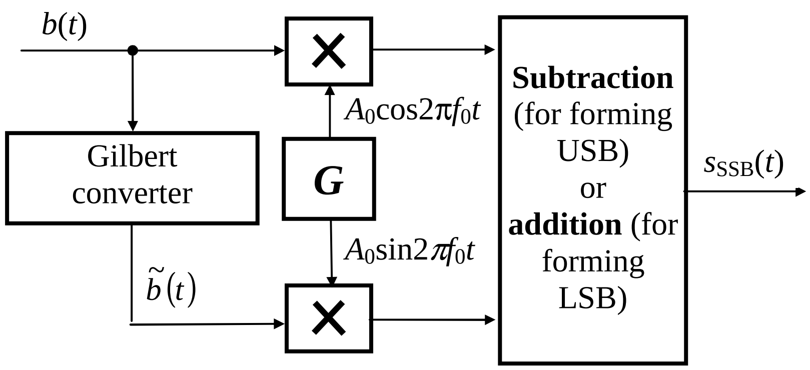


Figure 39 - Forming of SSB signal by a phase method

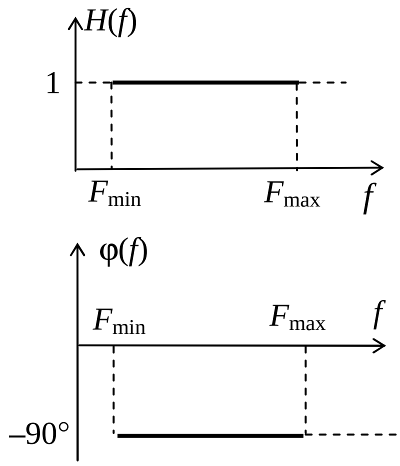


Figure 40 - Gilbert converter AR $H(f)$ and PR $\varphi(f)$

The diagrams of PM and FM signals modulators can be found in literature.

4.6 Detecting of signals

Device which output voltage is proportional to some parameter of input band-pass signal is called detector. On the base of this determination, it is necessary to use definitions: amplitude, frequency and phase detectors.

An amplitude detector can be performed on the diagram of synchronous detector or envelope detector.

The diagram of synchronous detector is shown on figure 41. This detector is called also coherent. Both names are related with the fact that supporting oscillation must be coherent with carrier oscillation of the input modulated signal. Such oscillation is produced by the system of phase lock loop PLL (in the case of SSB it is necessary to pass pilot signal). A synchronous detector is used for detection of AM, DSB-SC and SSB signals. It is possible to check efficiency of the detection process analysing signals passing of mentioned modulation types through the diagram of detector.

A envelope detector is shown on a figure 42. Such detector does not require coherent supporting oscillation and PLL system. It explains the name of incoherent detector. Property of signal, that its spectrum is oscillation of carrier frequency of signal is used for detection of AM signal. The diagram of incoherent detector of AM signal is constructed on the basis of non-linear electric circuit (NEC), which is described by polynomial, containing the element $a \cdot x^2$. Oscillation of differential frequencies on the output of non-linear circuit: upper sideband and carrier, and also carrier and lower sideband of frequencies are conjugated in the signal spectrum. These differential frequencies are selected by LPF, which is provided by detection of AM signal. Such diagram was used for analogue realization of equipment.

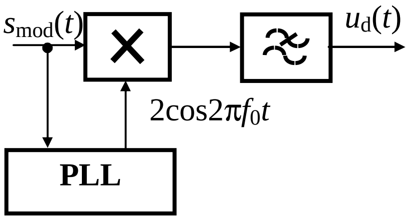


Figure 41 – Block diagram of synchronous detector

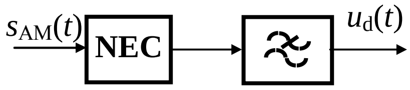


Figure 42 – Block diagram of incoherent detector on the basis of nonlinear circuit

Detector on the basis of quadrature splitter (figure 43) is used during processor realization of detector envelope. Here Δφ is a random initial phase of supporting oscillation. Detector is used and in those cases, when there is not carrier oscillation in the spectrum of detecting signal. Output voltage of detector $u_d(t)$ is proportional to bandpass signal envelope $s(t)$, that explains the name of detector – detector of envelope.

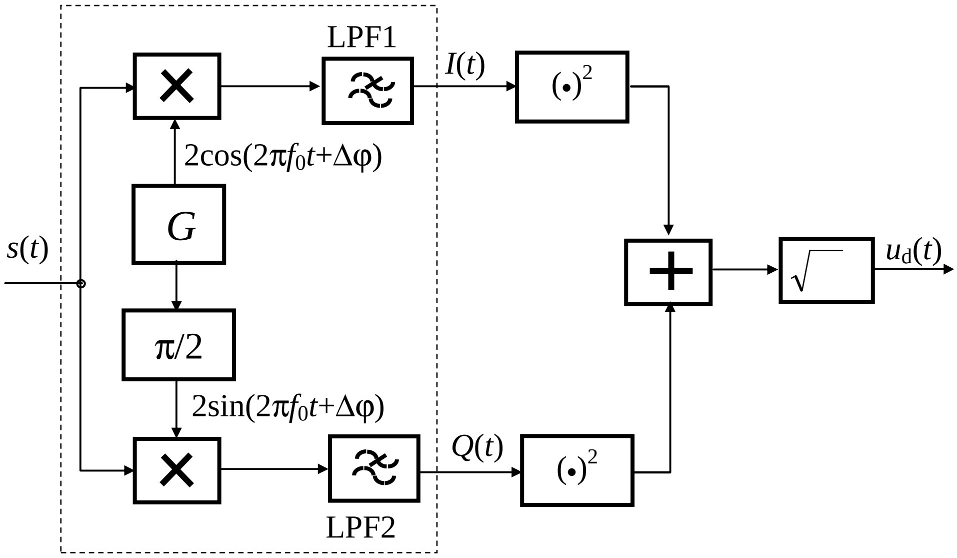


Figure 43 - Diagram of incoherent detector on the basis of quadrature splitter

The diagrams of frequency and phase detectors can be found in literature.

5 METHODS OF DIGITAL MODULATION

5.1 General information on digital modulation

Modulation refers to digital if a modulating signal is the digital signal.

Digital signal (DS) is a sequence of digital symbols, which belong to the certain alphabet. As a rule, symbols are binary and designated 1 and 0, them name bits, and they act through interval T_b. DS appears as a result of coding signs on discrete messages or samples of continuous messages. DS is possible to present, writing down sequence, for example, 10110..., and specifying value T_b. Basic characteristic of DS is rate of a signal or bit rate $R = 1/T_b$, bit/s.

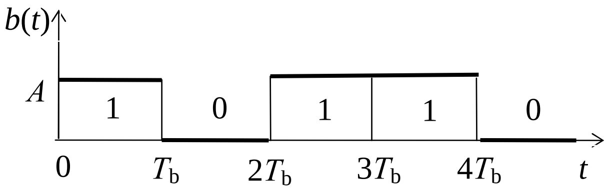


Figure 44 – Digital signal

In the terminal equipment of transmission systems, executed on logic microcircuits or processors, digital signals look like sequence of rectangular pulses. The example of such signal is resulted on figure 44. Carrier of information is the rectangular pulse, and the information is displayed in amplitude of a pulse which accepts values *a* and 0.

For transfer of digital signals by communication channels methods of digital modulation, basically, with serial transfer are used.

The modulated signal for transfer by a communication channel is formed from signals $s_i(t)$ – elementary pulse signals belonging to ensemble $\{s_i(t)\}$, $i = 0, 1..., M - 1$, where M – number of elementary signals ($M \geq 2$). Elementary signals name also channel symbols.

At formation of the modulated signal the sequence of binary symbols is broken into blocks from $n = \log_2 M$ bits. To each such block (the quantity of possible various blocks is $M = 2^n$) is put in conformity an elementary signal $s_i(t)$. Duration n bits makes clock interval $T = T_b \cdot \log_2 M$. The received sequence of the elementary signals acting through time T, forms the modulated signal

$$s(t) = \sum_{k=-\infty}^{k=\infty} s_i^{(k)}(t - kT), \tag{5.1}$$

where $s_i^{(k)}(t - kT)$ – i-th signal transmitting on a k-th clock interval.

On figure 45 transition from DS to the modulated signal ($n = 4$, $M = 16$) is shown. On the plot $s(t)$ transmitting elementary signals $s_{13}(t)$, $s_1(t)$, $s_3(t)$, $s_6(t)$, ... are shown. Digital modulation is a display of blocks of bits in pulses-carriers.

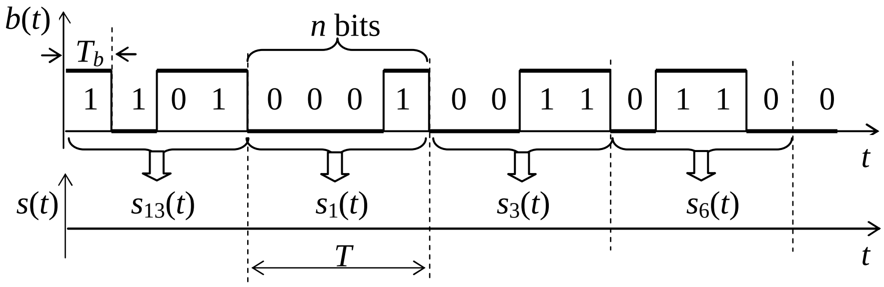


Figure 45 – Arrangement of transformation DS into the modulated signal

If $M = 2$, a signal $s(t)$ is binary; if $M > 2$, a signal $s(t)$ is multilevel or M-ary.

Serial transmission is considered. Also there is a parallel-serial transmission (OFDM). It will be considered later.

Key parameter of the modulated signal is the bandwidth. It depends on rate R and ensemble of elementary signals $\{s_i(t)\}$. The problem of a choice of digital modulation kind is reduced to a choice of ensemble of elementary signals.

5.2 Choice elementary pulse forms

The information is displayed in amplitudes of pulses, instead of in their form. Therefore the form of a pulse-carrier is necessary for choosing under spectral and other characteristics.

Shown on figure 44 DS does not approach for direct transmission on communication channels because of its spectral properties. On figure 44 elementary signal is the rectangular pulse

$$A(t) = \begin{cases} 1, & 0 \le |t| \le T/2, \\ 0, & |t| > T/2; \end{cases} \tag{5.2}$$

the symbol 1 is represented by a pulse $aA(t)$, and a symbol 0 – a pulse with zero amplitude.

Let's find spectral density of function $A(t)$:

$$S_A(jf) = \int_{-T/2}^{T/2} A(t) e^{-j2\pi ft} dt = T \frac{\sin \pi ft}{\pi ft}. \tag{5.3}$$

The amplitude spectrum of function $A(t)$ is

$$S_A(f) = T |\sin \pi ft / \pi ft|. \tag{5.4}$$

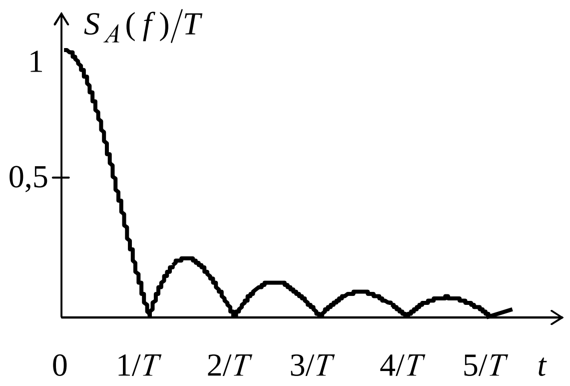


Figure 46 – Normalized spectrum of the rectangular pulse

On figure 46 the plot of the normalized amplitude spectrum $S_A(f)/T$ is resulted. The spectrum of the rectangular pulse decreases extremely slowly – with a speed $1/f$. With the purpose of economy of a band of frequencies of a communication channel it is necessary to use pulses of the smoothed form.

The pulse $A(t)$ should satisfy to a condition

$$A(t) = \begin{cases} 1, & t = 0, \\ 0, & t = kT, \; k = \pm 1, 2, ... \end{cases} \tag{5.5}$$

The condition (5.5) is a sampling condition or a condition of absence of intersymbol interference (ISI). After sampling of a pulse $A(t)$, satisfying a condition (5.5), the discrete signal is formed $A(n)$ = ..., 0, 0, 1, 0, 0, ...

It is possible to show, that the spectrum of a pulse $A(t)$ should be skew symmetric

$$S_A(\omega_N - \omega) + S_A(\omega_N + \omega) = T, \quad 0 \le \omega \le 2\pi/T, \tag{5.6}$$

where $\omega_N = \pi/T$ – Nyquist frequency.

On figure 47 examples of functions $S_A(\omega)$ with skew symmetry are resulted.

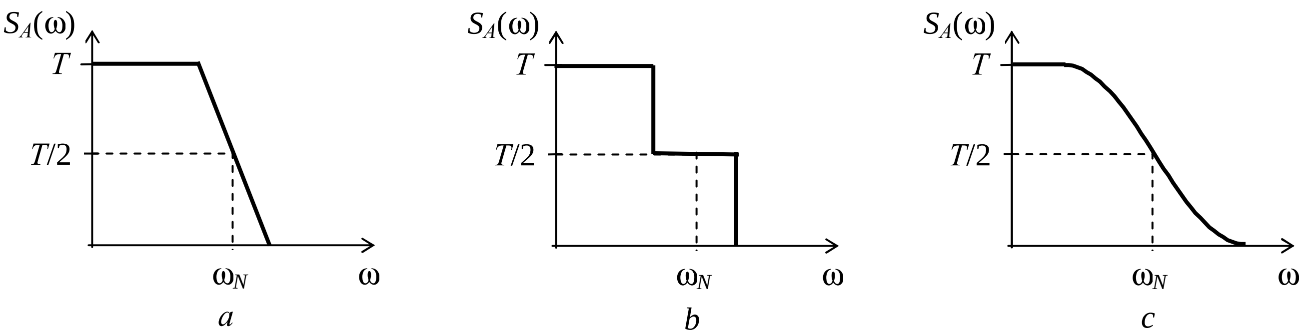


Figure 47 –Examples of functions $S_A(\omega)$ with skew symmetry

The spectrum of pulse signals satisfying a sampling condition (5.5), refer to as Nyquist spectrum. Them designate as $N(f)$. More often Nyquist spectrum describe by function

$$N(f) = \begin{cases} T, & 0 \le |f| \le (1-\alpha) f_N, \\ 0{,}5T\left[1 + \sin\left(\dfrac{\pi}{2\alpha}\left(1 - \dfrac{|f|}{f_N}\right)\right)\right], & (1-\alpha) f_N < |f| < (1+\alpha) f_N, \\ 0, & |f| \ge (1+\alpha) f_N. \end{cases} \tag{5.7}$$

where $f_N = 1/(2T)$ is Nyquist frequency.

α – roll-off factor of a signal spectrum , $0 \le \alpha \le 1$.

Dependence (5.7) refers to „raised cosine“. On figure 48 such dependences are resulted for α = 0; 0,2; 0,5 and 1. From figure 48 it is visible, that bandwidth of a pulse $F = (1 + \alpha)f_N$. Minimal possible bandwidth min$F = f_N = 1/(2T)$.

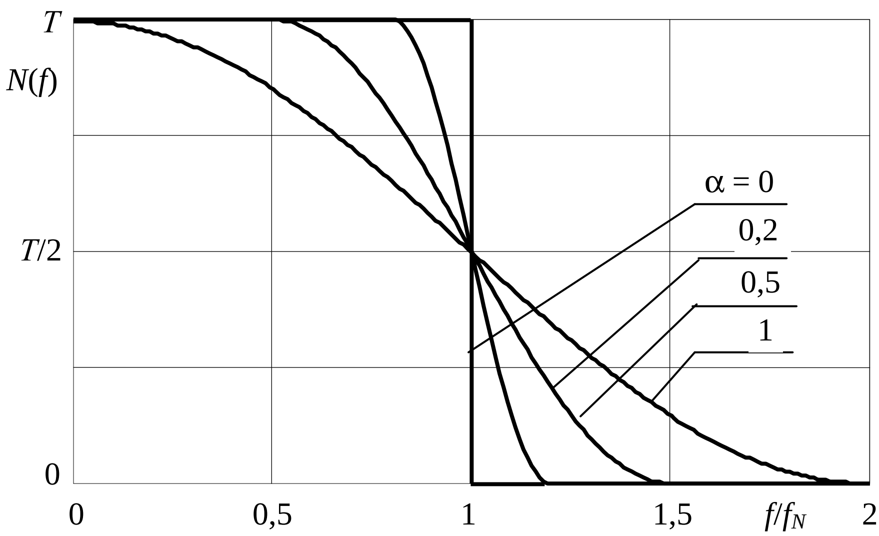


Figure 48 – Nyquist spectrum

Typical values of factor α lay in limits from 0,2 up to 0,4.
Function $A(t)$ can be received as inverse Fourier transform from $N(f)$

$$A(t) = \frac{\sin 2\pi f_N t}{2\pi f_N t} \cdot \frac{\cos 2\pi\alpha f_N t}{1-(4\alpha f_N t)^2}. \tag{5.8}$$

Pulses $A(t)$ name Nyquist pulses (figure 49).

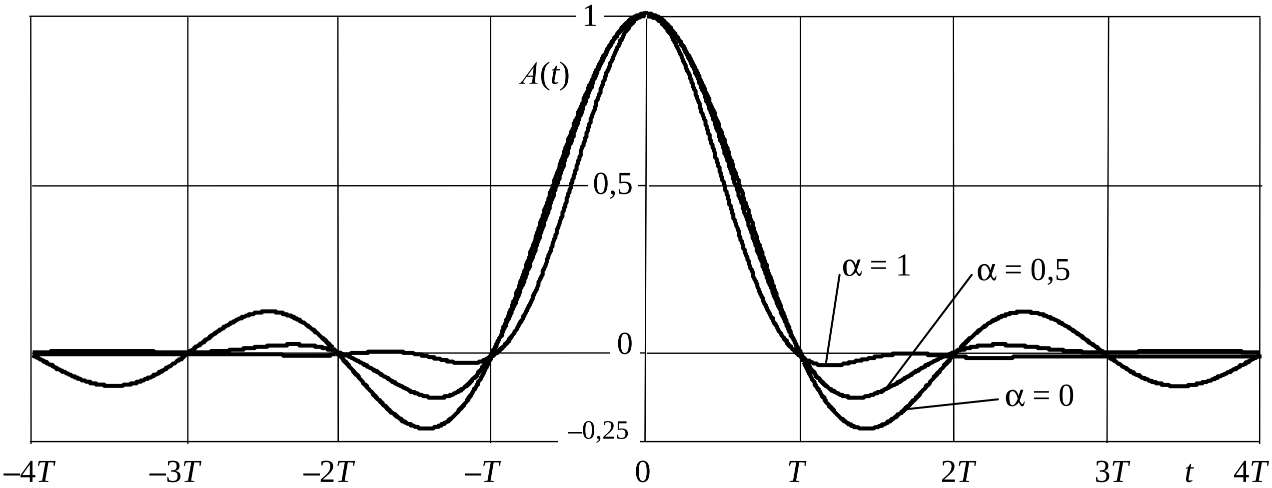


Figure 49 – Nyquist pulses

The type of pulse at the transmission of a baseband signal is considered. If the spectrum should be bandpass radio impulses $A(t)\cdot\cos(2\pi f_0 t)$ and $A(t)\cdot\sin(2\pi f_0 t)$ are used. Their amplitude spectrum have two side band which are copies of a spectrum of pulse $A(t)$ (figure 50). If $A(t)$ has Nyquist spectrum the bandwidth of a radio pulse is defined $F = 2(1 + \alpha)f_N$.

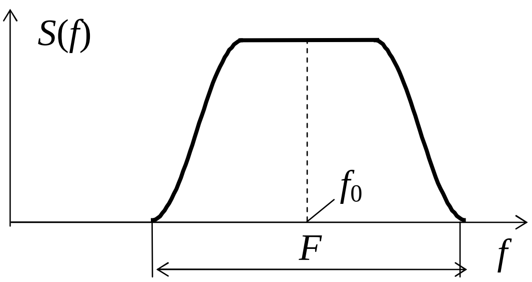


Figure 50 – Spectrum of radio pulses

5.3 Pulse-amplitude modulation

Modulation is named pulse-amplitude, if by channel symbols, used for forming of the modulated signal, are low-frequency impulses, i.e. their spectrum joins to a zero frequency. Channel symbols are described as

$$s_i(t) = a_i A(t), \quad i = 0, 1, ..., M - 1, \tag{5.9}$$

where $A(t)$ – impulse with certain time and spectral descriptions, the maximal value of which is equal 1; that intersymbol interference was not, impulse $A(t)$ must be the Nyquist impulse;

a_i is a coefficient, representing information.

The signals of pulse-amplitude modulation are designated as *M*-ary PAM, where *M* is a number of channel symbols.

Evident presentation of signals of digital modulation is signal constellation. On signal constellation each of channel symbols is represented a point, the co-ordinates of points are coefficients, which are, describe channel symbols. In the case of MPAM signals every channel symbol is described only one coefficient a_i, therefore for presentation of MPAM signals one-dimensional space is used. On a figure 51 signal constellation of 2PAM or BPAM signal is shown. A mapping code, setting accordance between binary characters and coefficients, is specified also a_i.

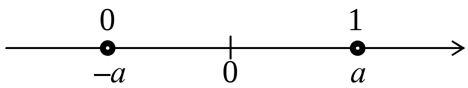


Figure 51 – BPAM signal constellation

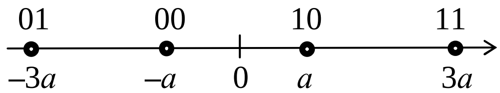


Figure 52 – QPAM signal constellation

On a figure 52 signal constellation of 4PAM or QPAM signal is shown. A mapping code for the QPAM signal sets accordance between the pair of binary symbols (n = 2) and by the coefficients a_i. These pair also determines the signal number - binary symbols are the record of signal number in the binary notation scale. A mapping code must be a Gray code blocks from n bits, which correspond nearby signals, must differ only in one bit. Gray code minimizes the amount of erroneous bits in the case of origin of error of decision about the passed channel symbol at demodulation.
Both at BPAM and at QPAM number a determines energies each of channel symbols and middle power of the modulated signal.

Like the considered examples it is possible to build signal constellations for M= 8, 16, ...

The chart of MPAM signal forming is resulted on a figure 53. DS acts at the input. Mapper takes $n = \log_2 M$ is bits and gives out the coefficients of a_i rectangular pulses duration T. From these impulses a forming filter produces the impulses $a_i A(t)$. This procedure repeats oneself on every clock interval.
For different values M work of chart differs only a mapping code.

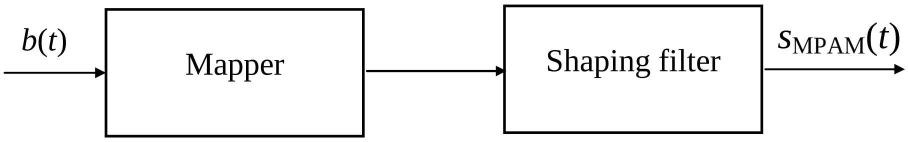


Figure 53 – Modulator of MPAM signal

MPAM signals bandwidth is determined by the width of impulses spectrum $A(t)$. As $T = T_b\log_2 M$, then

$$F_{\text{MPAM}} = \frac{1+\alpha}{2T_{\text{b}}\log_2 M} = \frac{R(1+\alpha)}{2\log_2 M}. \tag{5.10}$$

5.4 One-dimensional bandpass signals of digital modulation

One-dimensional bandpass signals of digital modulation are MASK signals – M-ary amplitude shift keying ($M \geq 2$) and BPSK signals – binary phase shift keying.

At the MASK and BPSK signals channel symbols are radiopulses and they are written down:

$$s_i(t) = a_i\sqrt{2}A(t)\cos(2\pi f_0 t), \quad i = 0, 1, ..., M-1, \tag{5.11}$$

where a_i – number, representing $n = \log_2$M is bit, passed a signal $s_i(t)$;
$A(t)$ – function, determining the form of radiopulses, its maximal value is equal 1;
f_0 – frequency of radiopulse.

As channel symbols differs only the coefficient a_i, signal constellations of these types of modulation appear in one-dimensional space, and the modulated signals are named one-dimensional. On a figure 54 signal constellations of BPSK signals, BASK and QASK with pointing of mapping codes are resulted.

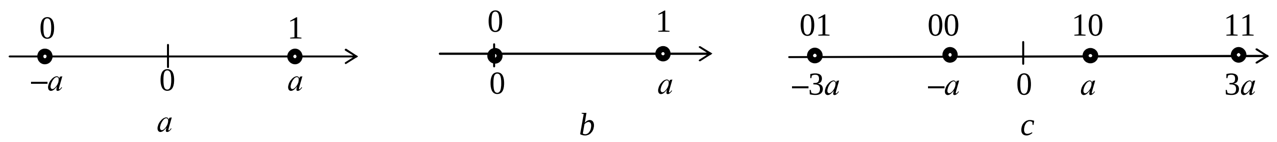


Figure 54 - Signal constellations of BPSK (a); BASK (b); QASK (c) signals

Energy of i-th channel symbol calculated

$$E_i = \int_{-\infty}^{\infty} s_i^2(t)dt = a_i^2.$$

Comfortably to compare the different methods of modulation on the size of minimum distance between the signals d, characterizing the difference of signals is quantitative. The size d is determined on signal constellation. The size d must be expressed through the physical parameters of signals. Comfortably to consider E_{b} such parameter is energy, expended on the transmission of one bit: $E_b = P_s T_b = P_s / R$. The last sizes are set on the system of transmission. Energy E_b is expressed through middle energy of signals E_{ev} and amount bit, passed one signal n

$$E_b = \frac{E_{ev}}{n}; \quad E_{ev} = \frac{1}{M}\sum_{i=0}^{M-1} E_i; \; n = \log_2 M .$$

We will conduct calculations. For BASK:

$$d = a,\ E_0 = 0,\ E_1 = a^2,\ E_{\text{ev}} = 0{,}5a^2,\ E_b = E_{\text{ev}} = 0{,}5a^2,\ d = \sqrt{2E_b} .$$

For BPSK:

$$d = 2a,\ E_0 = E_1 = a^2,\ E_{ev} = a^2,\ E_b = E_{ev} = a^2,\ d = 2\sqrt{E_b}\ .$$

For QASK:

$$d = 2a,\ \ E_0 = E_2 = a^2,\ \ E_1 = E_3 = 9a^2,$$

$$E_{ev} = 0{,}5(a^2 + 9a^2) = 5a^2,\ \ E_b = 0{,}5E_{ev} = 2{,}5a^2,\ \ d = 2\sqrt{2/5 \cdot E_b} = 1{,}25\sqrt{E_b}\ .$$

Time diagrams of the examined channel symbols are resulted on a figure 55. For obviousness it is accepted at a construction, that $A(t)$ – rectangular pulse of duration, equal to the clock interval. Like considered it is possible to build signal constellations and time diagrams for the 8ASK, 16ASK signals etc.

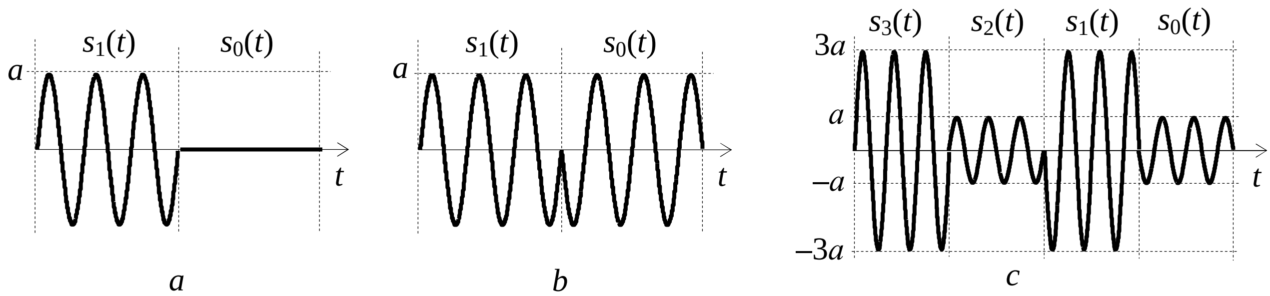


Figure 55 – Time diagrams of elementary signals: *a* – BASK; *b* – BPSK; *c* – QASK

We will consider the spectrum of carrier pulse $\sqrt{2}\,A(t)\cdot\cos(2\pi f_0 t)$, which channel symbols are built on the basis of (5.11). This carrier pulse is the signal of analogue DSB-SC, and that is why his spectrum consists of two sidebands, concentrated near frequency of radiopulse f_0, which can be considered frequency of carrier oscillation. Frequencies sidebands is the reflection of spectrum of impulse $A(t)$. So, spectral properties any of channel symbols $s_i(t)$ wholly determined the function $A(t)$.

Frequencies sidebands are the copies of Nyquist spectrum (figure 56), and the spectrum width of MASK and BPSK signals is determined:

$$F = 2 f_{\mathrm{N}}(1+\alpha) = \frac{R(1+\alpha)}{\log_2 M}. \qquad (5.12)$$

An important conclusion follows from expression (5.12) – the increase of levels number of MASK signal at the fixed speed R allows decreasing the spectrum width of channel symbols.

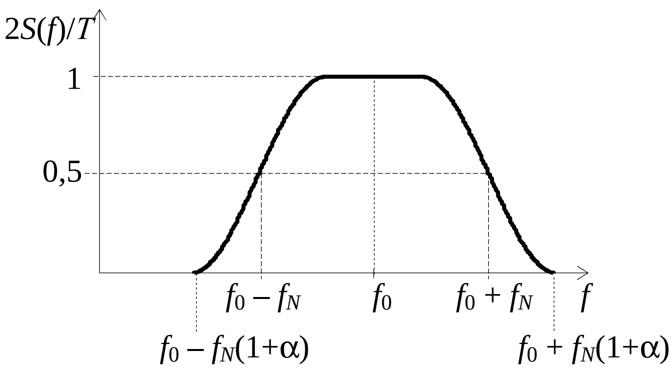


Figure 56 - Spectrums of MASK and BPSK channel symbols

We will consider the chart of forming of MASK and BPSK signals. From comparison of expressions (5.9) and (5.11) follows, that at the MPAM signals carrier pulse $A(t)$, and at the bandpass signals carrier pulse $\sqrt{2}\,A(t)\cdot\cos(2\pi f_0 t)$. Thus, the chart of forming of one-dimensional bandpass signals (modulator) is built on the basis of chart of figure 53 with addition the generator of carrying waveform *G* $\sqrt{2}\cos(2\pi f_0 t)$ and multiplier ✕ (figure 57).

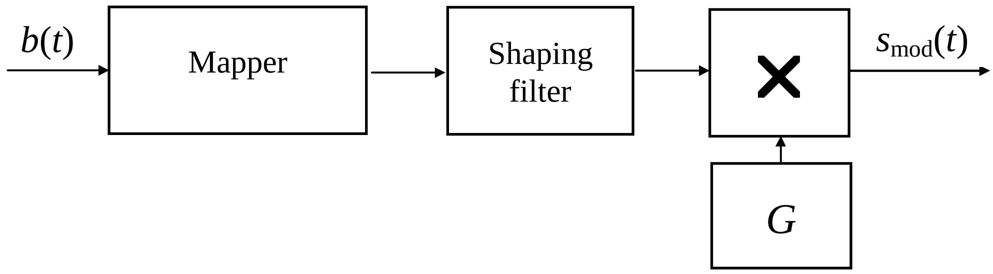


Figure 57 - Modulator of one-dimensional bandpass signals

So, on the basis of analysis of one-dimensional bandpass signals of digital modulation became obvious, that the name of modulation method specifies, what channel symbols parameter differs: MASK – *M* signals differ in amplitudes, BPSK – two signals differ in initial phases (0 and 180°).

5.5 Two-dimensional bandpass digitally modulated signals

For two-dimensional bandpass signals of digital modulation the signals *M*-ary PSK ($M \geq 4$) and *M*-ary APSK (amplitude-phase shift keying) uses. At these types of modulation channel symbols are described the sum of cosine and sine radiopulses:

$$s_i(t) = a_i\sqrt{2}A(t)\cos 2\pi f_0 t + b_i\sqrt{2}A(t)\sin 2\pi f_0 t, \quad i = 0,\ 1, ...,\ M-1, \qquad (5.13)$$

where a_i, b_i – pair of coefficients, which jointly represents a sequence from $n = \log_2 M$ bits, passed an channel symbol $s_i(t)$;

$A(t)$ – function, determining the form of radiopulses, its maximal value is equal 1;

f_0 – radiopulses frequency.

As every channel symbol is described two coefficients a_i and b_i, signal constellations of these types of modulation appear in two-dimensional space, and the modulated signals are named two-dimensional.

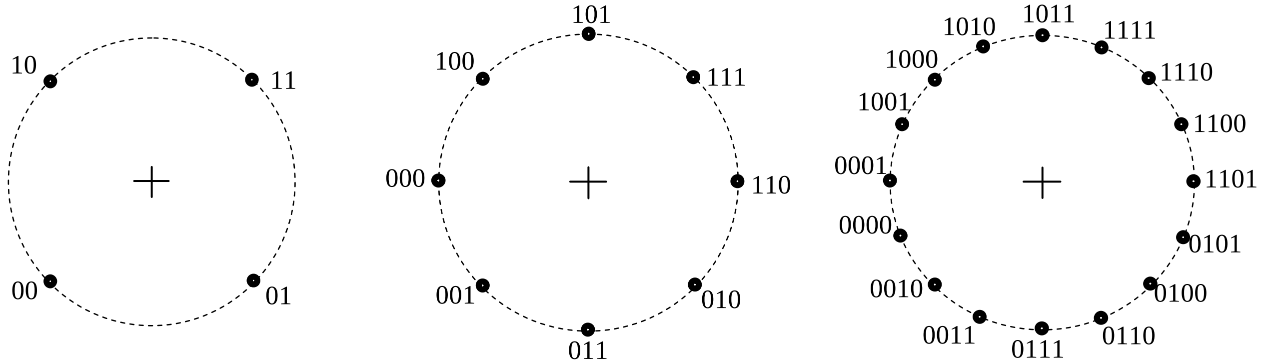
Sum of cosine and sine radiopulses of monotonous forms in (5.13) can be transferable one radiopulse of the same form with the amplitude multiplier A_i and initial phase j$_i$, determined:

$$A_i = \sqrt{2\left(a_i^2 + b_i^2\right)}, \qquad \varphi_i = -\text{arctg}\frac{b_i}{a_i}, \qquad i = 0, 1, ..., M-1. \qquad (5.14)$$

The identical amplitude multipliers $A_i = a$ have elementary MPSK signals for all i, and their initial phases φ_i differ with the step 2π/M. On a figure 58 signal constellations of MPSK signals are resulted with pointing of mapping codes. Evidently, that mapping codes are Gray codes.

The elementary MAPSK signals of differ or by the amplitude multipliers A_i, or initial phases φ_i, or amplitude multipliers and initial phases simultaneously. On a figure 59 constellations of 16-ary of quadrature amplitude modulation (16QAM) are resulted. Signals of MQAM are the separate cases of MAPSK signals.

For signals MQAM attribute the signals of MAPSK, if points of signal constellation are in the knots of square grate. Such structure of constellation gives certain comforts at demodulation.



a *b* *c*

Figure 58 - Signal constellations of QPSK (*a*); 8PSK (*b*); 16PSK (*c*) signals

For QPSK:

$$d = \sqrt{2}a\,,\ E_0 = E_1 = E_2 = E_3 = a^2,\ E_{ev} = a^2,\ E_b = 0{,}5E_{ev} = 0{,}5a^2,\ \ a = \sqrt{2E_b}\ ,$$

$$d = 2\sqrt{E_b}\ ,\ \ \max d = 2a = 2\sqrt{2E_b}\ .$$

For 8PSK:

$$d/2 = a{\cdot}\sin 22{,}5°,\ \ d = 0{,}765a,\ \ \ E_i = a^2,$$

$$E_{ev} = a^2,\ \ E_b = E_{ev}/3 = a^2/3,\ \ d = 0{,}765\sqrt{3E_b} = 1{,}36\sqrt{E_b}\ .$$

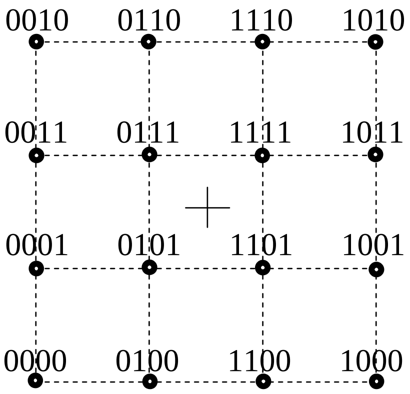


Figure 59 – 16QAM signal constellation

The followings MQAM signals of are used in practice: 4QAM (the same, that QPSK), 8QAM, 16QAM, 64QAM, 256QAM, 1024QAM.

Signals, described expression (5.13), are a sum two DSB-SC signals with identical amplitude spectra which are determined the signal $A(t)$ spectrum. In case if $A(t)$ – the Nyquist pulse, amplitude spectrum to each of constituents, and also their sum, looks like, resulted on a figure 56. Therefore the spectrum width of channel symbols in the case of MPSK and MAPSK is described expression (5.12).

We will consider the chart of forming of MPSK and MAPSK signals. From comparison of expressions (5.13) and (5.11) flows out, that the chart of two-dimensional bandpass signals forming (modulator) is built on the basis of figure 57 chart with addition of second subchannel and summarizing (figure 60). Mapping code coder puts in accordance $n = \log_2 M$ to the entrance bits two rectangular impulse with amplitudes a_i and b_i; rectangular impulse is filtered forming filters, to get the Nyquist impulses; the impulses $a_iA(t)$ and $b_iA(t)$ act at the inputs of double sideband suppressed carrier modulator; the gotten modulated signals are added up.

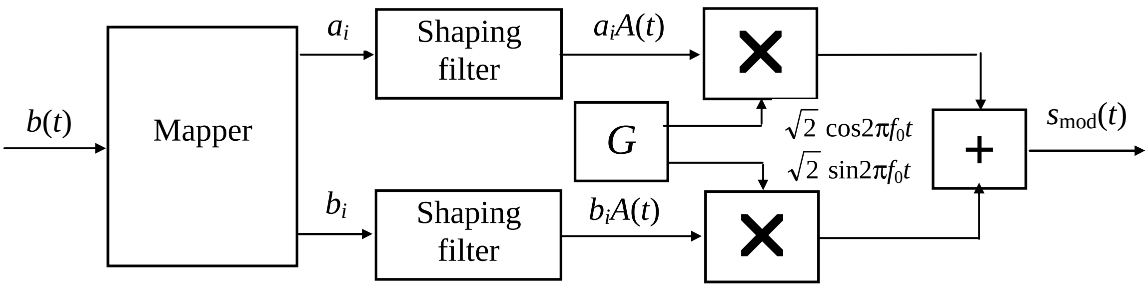


Figure 60 – Modulator of two-dimensional bandpass signals

Thus, made sure again, that the name of method of digital modulation specified what parameter (or by what parameters) channel symbols differ: MPSK – M signals differ in initial phases, MAPSK – M signals differ in amplitudes and/or initial phases. To the two-dimensional signals the signals of binary frequency modulation (BFSK) belong also. From the name of modulation follows that channel symbols are radio-pulses, different frequencies:

$$\begin{aligned} s_0(t) &= aA(t)\cos\left(2\pi\left(f_0 - \Delta f/2\right)t + \varphi_0\right), \\ s_1(t) &= aA(t)\cos\left(2\pi\left(f_0 + \Delta f/2\right)t + \varphi_1\right), \end{aligned} \tag{5.15}$$

where $s_0(t)$ – signal for the transmission of symbol 0;

$s_1(t)$ – signal for the transmission of symbol 1;

f_0 – middle frequency of radiopulses;

Δf – frequency spacing;

$A(t)$ – function, determining the form of radiopulses, its maximal value is equal 1;

a – coefficient which determines energy of signals;

φ_0, φ_1 – initial phases of impulses.

The modulated signal is written down

$$s_{\text{BFSK}}(t) = \sum_{k=-\infty}^{\infty} s_i^{(k)}(t - kT). \tag{5.16}$$

In order that at demodulation radiopulses can it was be divided on condition that their phases j$_0$ and j$_1$ are arbitrary, the spectra of radiopulses $s_0(t)$ and $s_1(t)$ must not be recovered. If the spectrums of signals are not recovered, such signals orthogonal. We will pass to vector representation of channel symbols

$$\bar{s}_i = a_i \bar{\Psi}_0 + b_i \bar{\Psi}_1, \tag{5.17}$$

where

$$\begin{aligned} \psi_0(t) &= A(t)\cos\left(2\pi\left(f_0 - \Delta f/2\right)t + \varphi_0\right), \\ \psi_1(t) &= A(t)\cos\left(2\pi\left(f_0 + \Delta f/2\right)t + \varphi_1\right), \end{aligned} \tag{5.18}$$

and a mapping code which does equivalent records (5.17) and (5.15) is resulted in table 2. On the basis of expression (5.17) and table 2 BFSK signal constellation looks like, shown on a figure 61. Here the impulses $y_0(t)$ and $y_1(t)$ form the base of signals space.

Table 2 – BFSK mapping code

i	a_i	b_i
0	a	0
1	0	a

In order that a width of spectra of radiopulses was minimum and there was not intersymbol interference (ISI), an impulse $A(t)$ must be the Nyquist impulse. At that rate it is possible to consider that a spectrum of signal $s_{\text{BFSK}}(t)$ is the sum of spectra of two radiopulses of frequencies $f_0 - \Delta f/2$ and $f_0 + \Delta f/2$. On a figure 62 the rationed spectrum of BFSK signal, from which follows, is presented, that frequency spacing of will be minimum, when the spectra of radiopulses join one to other, and evened:

$$\Delta f_{\min} = \frac{1+\alpha}{T}, \tag{5.19}$$

where T – a clock interval is equal T_b.

Then spectrum width of BFSK signal:

$$F_{\text{BFSK}} = \Delta f_{\min} + \frac{1+\alpha}{T} = \frac{2(1+\alpha)}{T}, \tag{5.20}$$

it is twice greater spectrum widths of BASK and BPSK signals.

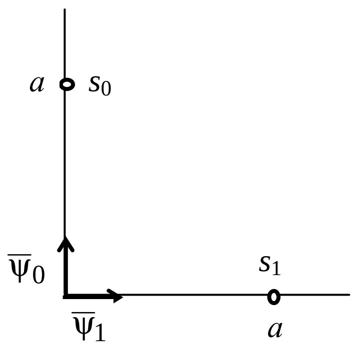


Figure 61 – BFSK signal constellation

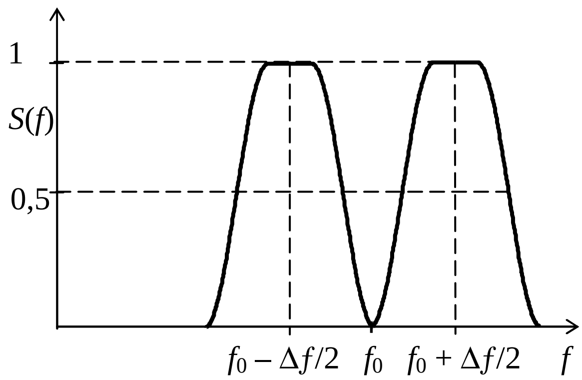


Figure 62 - Spectrum of BFSK signal, on the basis of Nyquist impulses

From expression (5.17) and table 2 the chart of BFSK modulator flows (figure 63). Forming of BFSK signals differs from forming of MPSK signals work of mapping code coder and that generators of carrying waveform frequencies in double sideband suppressed carrier modulators differ on the size $\Delta f/2$ from frequency of bearing oscillation.

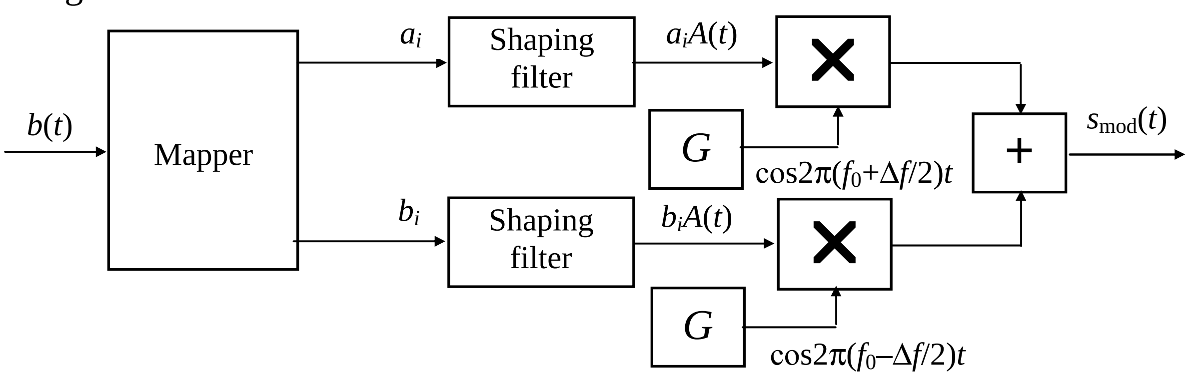


Figure 63 – BFSK signal modulator, if channel symbols are Nyquist impulses

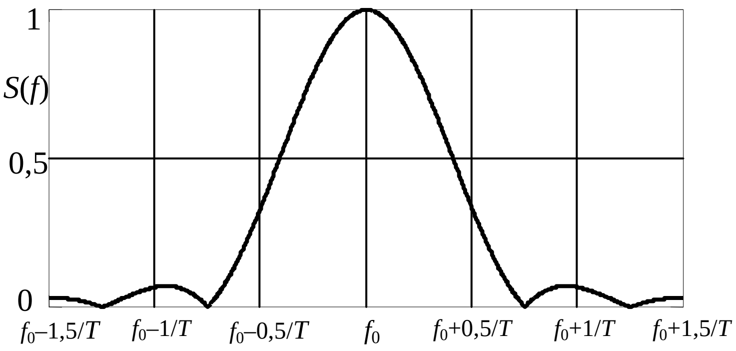


Figure 64 - MSK signal spectrum

If the function $A(t)$ is rectangular impulse, it is necessary to provide forming of BFSK signal the chart of modulator without the «open-phase fault». It is possible, when frequency spacing $\Delta f = k/(2T)$, where $k = 1, 2, 3, ...$; $T = T_b$. When $k = 1$, $\Delta f = 0{,}5/T$ and modulation is named modulation of minimal shift keying (MSK). In the case of MSK the rationed spectrum of signal is described expression

$$S(f) = \frac{\sqrt{1+\cos\left(4\pi(f-f_0)T\right)}}{\sqrt{2}\left(1-\left(4(f-f_0)T\right)^2\right)} \qquad (5.21)$$

Dependence (5.21) is resulted on a figure 64. With the increase $|f - f_0|$ spectrum decreases at a speed $1/f^2$. If to define the F_{MSK} spectrum width on the first zeros of dependence (5.21) then

$$F_{MSK} = 1{,}5/T. \qquad (5.22)$$

5.6 Signals with spread spectrum

Broadband signals have been used in the systems of telecommunication, since 50-s. The properties of these signals for the first time were used in the systems of radio communication for removal effect of radiowaves multipath propagation.

The next stage of implementation of broadband signals in telecommunication systems was early 90-s, when these signals were used in mobile communication of second generation of standard of IS-95 networks for Code Division Multiple Access realization (CDMA). Code Division Multiple Access has appeared extraordinarily effective in mobile communication networks, therefore the systems of third generation of cdma2000 and UMTS also use broadband signals.

Since middle of 90-s implementation of wireless ports of access, which allow to connect various devices, for example, mobile telephone and computer was begun. The example of such port is Bluetooth, which provides connection of various devices in a radius of 100 m. The known property of broadband signals is their high stability to the narrow-band hindrances which are made by other devices of radio contact.

Broadband signals (BS) are called such signals, which spectrum width is more than minimum band of frequencies, necessary to pass a digital signal of the set rate:

$$\Delta f_{BS} >> \Delta f_{min}, \qquad (5.23)$$

where Δf_{BS} is a width of spectrum of broadband signal;

Δf_{min} is a minimum band of frequencies, necessary for a transmission; that is equal to the limit of Nyquist, which for binary bandpass signals is equal to transmission rate, i.e. $\Delta f_{\min} = R_{\text{b}}$.

One of widespread methods of broadband signals forming is Direct Sequence Spread Spectrum - DSSS. Signals, formed by such method, are used in mobile communication of standards of IS-95 and UMTS networks.

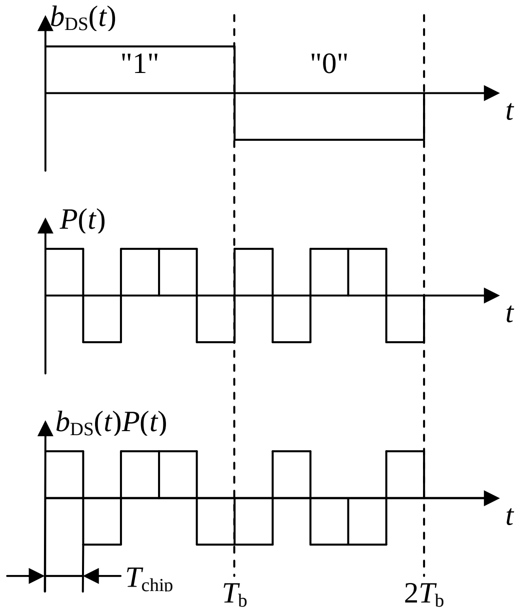


Figure 65 – Direct spread of spectrum: $b_{\text{DS}}(t)$ – a digital signal; $P(t)$ – PNS

The principle of direct expansion is that information character is multiplied by a so called Pseudo-noise sequence (PNS) the period of which is equal to duration of character T_b (figure 65). Expansion of spectrum takes a place due to that duration of element of PNS, which is called a chip, is less then duration of information character, i.e. $T_{\text{chip}} << T_{\text{b}}$.

PNS is a sequence of binary characters which values correspond a certain law. Such PNS are the function of Walsh, *m*-sequence etc.

After multiplying by PNS a signal is given on the input of standard modulator of BPSK, on the output of which a broadband signal appears with Direct Sequence Spread Spectrum:

$$s_{\text{DSSS}}(t) = b_{\text{DS}}(t) P(t) \sqrt{2} A(t) \cos(2\pi f_0 t), \qquad (5.24)$$

where $b_{\text{DS}}(t)$ is a digital signal;

$P(t)$ is a PNS;

$A(t)$ is a envelope signal with Direct Sequence Spread Spectrum;

f_0 is a carrier frequency.

The chart of signal modulator with Direct Sequence Spread Spectrum is presented on figure 66. Modulator contains forming LPF, which task is to form impulses for the transmission of chips with a compact spectrum. This filter forms envelope signal. It is chosen such AR LPF, that on his output a spectrum of chip (element of PNS) is the spectrum of Nyquist. In this case, the width of spectrum of signal with direct spread is calculated as:

$$\Delta f_{\text{DSSS}} = (1+\alpha) R_{\text{chip}} = (1+\alpha) N R_{\text{b}}, \qquad (5.25)$$

where $R_{\text{chip}} = 1/T_{\text{chip}}$ – chip rate, i.e. transmission rate of PNS elements;

$R_{\text{b}} = 1/T_{\text{b}}$ – rate of digital signal;

N is a number of chips on single informative symbol;

α is a Nyquist spectrum roll-off factor $0 \le \alpha \le 1$.

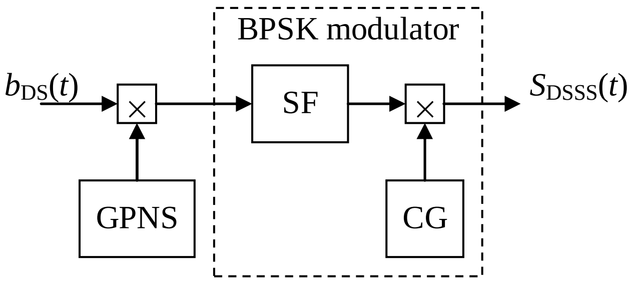


Figure 66 – Direct Sequence Spread Spectrum signal modulator: GPNS – PNS generator; CG – carrier generator

From the expression (5.25) follows, that signal spectrum with direct expansion is exactly in *N* times wider, than signal without spreading of spectrum, therefore a number is often called the coefficient of spectrum spreading.

Spreading of spectrum allows getting some useful properties.

Figure 67 demonstrates spectral properties of broadband signals. From the figure follows, that under spreading of spectrum of a signal of value of his spectral density is reduced. Values of spectral density of a broadband signal under considerable spreading of a spectrum (hundreds and thousands times) become near to the value of spectral density of noise. In this case it is difficult to distinguish a signal from noise, if the parameters of signal are unknown, for example, bearing frequency is unknown. Broadband signals are in general called noise-like, as their properties are similar property of white noise.

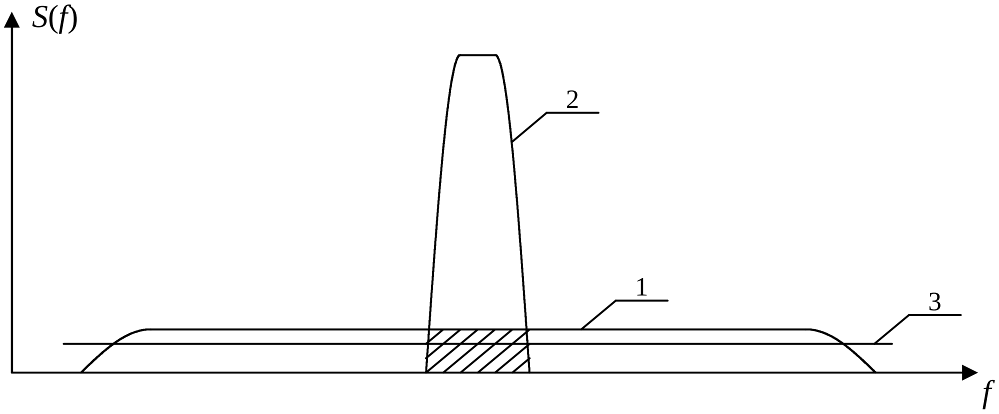


Figure 67 – Spectrums of: 1 – broadband signals; 2 – narrowband signal; 3 – white noise

Similarity of values of spectral density of broadband signals and value of spectral density of noise means, that these signals do not create considerable hindrances to signals without spectrum spreading. On the other side figure 67 demonstrates another property of broadband signals, namely firmness to the narrow-band hindrances. On a figure it is possible to see, that the spectrum of narrow-band signal (2) destruct insignificant part of spectrum of broadband signal (shaded part), that allows effectively to remove influencing of such hindrance. In fact both systems in which broadband signals are used and the systems in which signals are used without spreading of spectrum can work in one band of frequencies.

At demodulation of signals with the spreading spectrum the so-called matched filters are used. Their properties are:

– if on the input of a filter is given a signal which is matched with a filter, output response repeats the function of correlation of this signal.

– if on the input of a filter is given a signal which is not matched with a filter, output response repeats the function of mutual correlation of input signal and signal which a filter is matched with.

On figure 68 and figure 69 correlation properties of broadband signals are demonstrated. The function of correlation of any signal $s(t)$ is determined by expression:

$$K(\tau) = \int_{-\infty}^{\infty} s(t)s(t-\tau)dt, \tag{5.26}$$

and function of mutual correlation of two signals $s_1(t)$ and $s_2(t)$ is determined by expression:

$$K_{12}(\tau) = \int_{-\infty}^{\infty} s_1(t)s_2(t-\tau)dt. \tag{5.27}$$

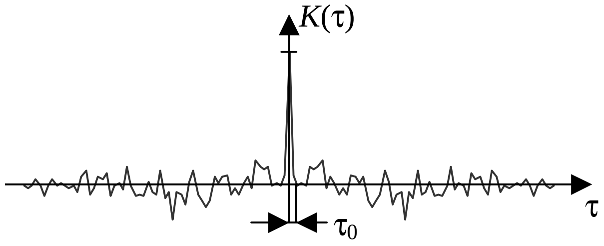


Figure 68 – Broadband signal correlation function

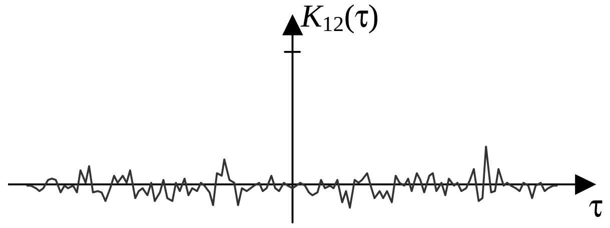


Figure 69 – Two PNS mutual correlation function

As it follows from figure 68 a correlation function of broadband signal has main narrow signal spike and small on values sideband signal spikes. Duration of the main signal spike of correlation function τ_0 is inversely proportional the band of frequencies of broadband signal, i.e.:

$$\tau_0 \approx \frac{1}{\Delta f_{\text{BS}}}. \tag{5.28}$$

A function of mutual correlation of two broadband signals has small on values signal spikes (figure 69). It allows realizing the code division of signals (wave-form separation).

Multipath propagation of radiowaves is characteristic for systems of radio contact and leads a few copies of a passed signal *s(t)*, which appear as a result of reflection of electromagnetic wave from various objects, enter into the input of receiver. These copies of a passed signal enter into input of receiver with different delays t_l:

$$s_{\text{rec}}(t) = \sum_{l=1}^{L} s(t-t_l), \tag{5.29}$$

where l is a ray number; a way of distribution of electromagnetic wave is called ray;
L is an amount of rays.

There will be observed sum of functions of correlation on the output of a matched filter (figure 70). As a function of correlation of broadband signals has the narrow main bursts and small on values sideband bursts, signals of separate rays can be put together laid down – processed by Rake-receiver. Realization of Rake-receiver for processing of signals of two rays is represented on a figure 71.

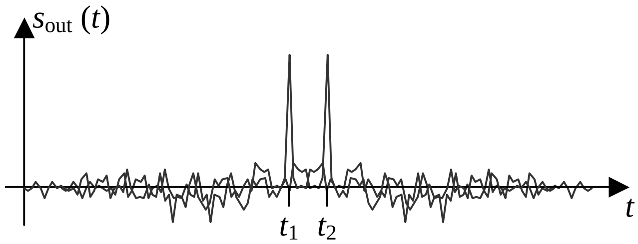


Figure 70 – MF output signal in a case of input sum of two PNS

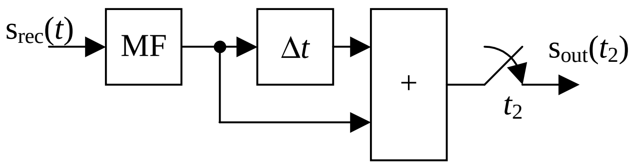


Figure 71 – Rake-receiver

In communication networks with multiple access with code division on the input of MF a few broadband signals of different subscribers which are transmitted simultaneously and in one band of frequencies, enter into input of matched filter:

$$s_{rec}(t) = \sum_{i=1}^{M} s_i(t), \tag{5.30}$$

where M is an amount of active subscribers.

For waveform separation of broadband signals property 2 of matched filters is used. As a value of function of mutual correlation of broadband signals tends to zero, signals, which are not matched with a filter, will not create considerable hindrances for a signal, which is matched with a filter.

5.7 OFDM

Orthogonal Frequency Division Multiplex (OFDM) realize a parallel-serial transmission (figure 72). A communication channel is simultaneously transmit L of modulated signals with a serial transfer which has been examined before. For this purpose sequence of binary characters is demultiplexed in L parallel sequences $b_{(1)}(t)$, $b_{(2)}(t)$, …, $b_{(L)}(t)$... On the basis of each of such sequences the modulated signals *of s(1)(t)* are formed, $s_{(1)}(t)$, $s_{(2)}(t)$, ..., $s_{(L)}(t)$ as well as under a serial transfer. Sum of signals $s_{(l)}(t)$, $l = 1, 2, ..., L$ forms the modulated signal of parallel-serial transmission, which is written down as

$$s(t) = \sum_{l=1}^{L} s_{(l)}(t) = \sum_{l=1}^{L} \sum_{k=-\infty}^{k=\infty} s_{(l)i}^{(k)}(t - kT), \quad (5.31)$$

where $s_{(l)i}^{(k)}(t - kT)$ – i-th signal, passed by l-th subchannel on k-th clock interval.

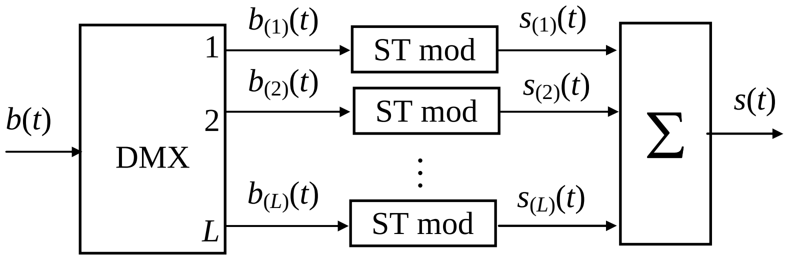


Figure 72 – Modulator of parallel-serial transfer:
DMX – demultiplexer;
ST mod – serial transfer modulator

The feature of this method of transmission are:

- rate of digital signals $b_{(l)}(t)$ lower, than signal $b(t)$ in L times;
- on the output of modulator L of the modulated signals $s_{(l)}(t)$ simultaneously present;
- the modulated signals of separate subchannels $s_{(l)}(t)$ must be such, that they can be divided out of a sum (5.31) for separate demodulation; these signals occupy different bands of frequencies.

In real systems the number L can make thousands. If AR and PR of communication channel are distorting, in the bands of frequencies for separate signals $s_{(l)}(t)$ a channel is practically nondistorted. At first glance complication of method of transmission actually requires simple transformations: all L of modulators are realized by one procedure of inverse fast Fourier transform. For demodulation direct fast Fourier transform is used. OFDM is used in systems of radio contact and digital sound and television broadcasting.

6 METHODICAL GUIDELINES FOR FULFILLING LABORATORY WORKS

LW 1.2 Researching of random processes probability distributions

1. Objectives

Studying and experimental investigation of one-dimensional probability distribution functions and the probability density functions of random processes.

2. Main principles

2.1. It is considered that studied processes are stationary and ergodic. In such processes one-dimensional probability distribution function and one-dimensional probability density function do not depend on time.

2.2. By definition the values of one-dimensional probability distribution function $F(x)$ are equal to the probability of that in the arbitrary time moment process. $X(t)$ will take on the value that does not exceed x:

$$F(x) = P\{X(t) \le x\}. \quad (1)$$

The value of one-dimensional probability density function $p(x)$ is equal to the limit of ratio of probability that in the arbitrary time moment, the process $X(t)$ will take on the value on the interval $(x - \Delta x/2,\ x + \Delta x/2)$ to the interval length Δx when $\Delta x \to 0$:

$$p(x) = \lim_{\Delta x \to 0} \frac{P\{x - \Delta x/2 < X(t) \le x + \Delta x/2\}}{\Delta x}. \quad (2)$$

The properties of $F(x)$ and $p(x)$ functions shown on the table below are easy to prove using their definitional formulas (1) and (2).

Table 1 – The properties of the functions $F(x)$ and $p(x)$

	$p(x)$	$F(x)$
1	$P\{x < X(t) \le x + dx\} = p(x)dx$	$F(x) = P\{X(t) \le x\}$
2	$P\{x_1 < X(t) \le x_2\} = \int_{x_1}^{x_2} p(x)dx$	$P\{x_1 < X(t) \le x_2\} = F(x_2) - F(x_1)$
3	$\int_{-\infty}^{\infty} p(x)dx = 1$	$F(\infty) = 1; \quad F(-\infty) = 0$
4	$p(x) \ge 0$	$F(x_2) \ge F(x_1)$ when $x_2 > x_1$
5	$p(x) = \dfrac{dF(x)}{dx}$	$F(x) = \int_{-\infty}^{x} p(x)dx$

The functions $F(x)$ and $p(x)$ are used to calculate the hit probabilities of the process values on the given interval (line 2, table 1), to perform statistical averaging while determination of process characteristics or of the result of certain operation with random process.

2.3. For processes which are often used, analytical expressions of functions $F(x)$ and $p(x)$ are known.

For **Gaussian (normal) process** (for example, fluctuation noise):

$$p(x)=\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(x-a)^2}{2\sigma^2}}, \quad (3)$$

where $a=\overline{X(t)}$ is the average value or mathematical expectation of a random process

$$a=\int_{-\infty}^{\infty} x\,p(x)dx; \quad (4)$$

σ – root-mean-square deviation of a random process, it is determined as $\sigma=\sqrt{D[X(t)]}$;

$D[X(t)]$ – dispersion of a random process (an average value of a squared deviation of a value of random process out of its average value)

$$D[X(t)]=\int_{-\infty}^{\infty}(x-a)^2\,p(x)dx. \quad (5)$$

The probability distribution function of normal process has following expressions:

$$F(x)=1-Q\left(\frac{x-a}{\sigma}\right), \quad (6)$$

where

$$Q(z)=\frac{1}{\sqrt{2\pi}}\int_{z}^{\infty}\exp\left(-\frac{t^2}{2}\right)dt$$

– Q-function or addition to Gaussian probability distribution function.

On figure 1, *a*, the graphs of the probability distribution are given at a = 1 and σ = 0,5.

Probability distribution of **harmonic oscillation** $X(t) = A\cdot\cos(2\pi f t + \varphi)$, where A and f are constants, and φ is a random value, is described by the expressions:

$$p(x)=\begin{cases}\dfrac{1}{\pi\sqrt{A^2-x^2}}, & |x|\le A,\\ 0, & |x|>A;\end{cases} \qquad F(x)=\begin{cases}0{,}5+\dfrac{1}{\pi}\arcsin\dfrac{x}{A}, & |x|\le A,\\ 0, & |x|>A.\end{cases} \quad (7)$$

The average value of harmonic oscillation is equal to zero, and root-mean-square deviation is equal to $A/\sqrt{2}$. On figure 1 *b*, the graphs of probability distribution of harmonic oscillation are given at A = 2. If $x = A$, then the value of probability density tends towards infinity.

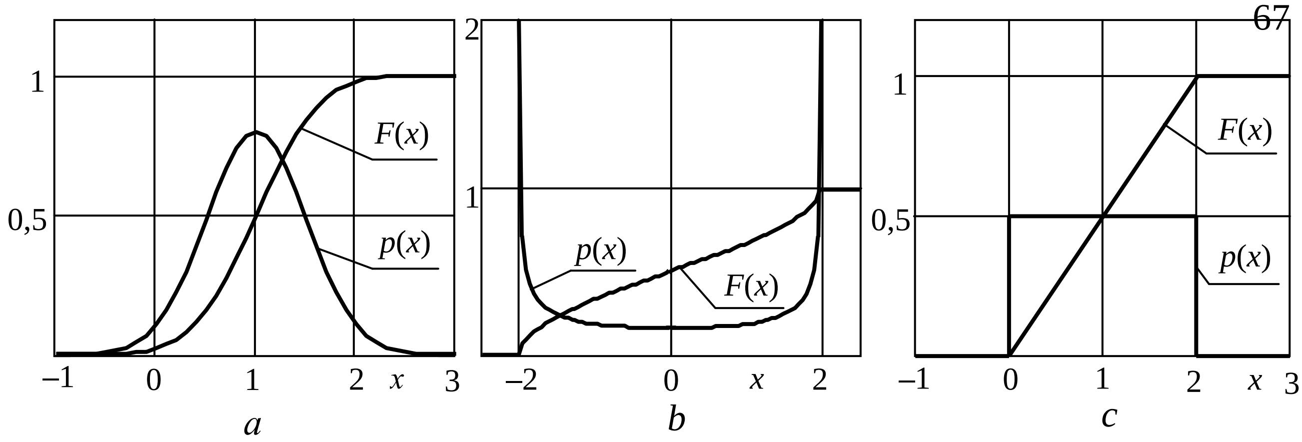


Figure 1 – The probability distributions: *a* –of the Gaussian random process; *b*– of the harmonic oscillation; *c* – random process with a uniform distribution

Functions $F(x)$ and $p(x)$ for the random process with a **uniform distribution** on the interval ($x_{\min}$, $x_{\max}$) are written down as:

$$p(x) = \begin{cases} \dfrac{1}{x_{\max} - x_{\min}}, & \text{at } x_{\min} < x \le x_{\max}, \\ 0, & \text{at } x \le x_{\min}, \quad x > x_{\max}; \end{cases}$$

$$\mathrm{F(x)} = \begin{cases} 0 & \text{at} \quad x < x_{\min}; \\ \dfrac{x - x_{\min}}{x_{\max} - x_{\min}} & \text{at} \quad x_{\min} \le x \le x_{\max}; \\ 1 & \text{at} \quad x > x_{\max}. \end{cases} \qquad (8)$$

The average value of the random process with a uniform distribution is equal to $(x_{\min} + x_{\max})/2$ and root-mean-square deviation is equal to $(x_{\max} - x_{\min})/\sqrt{12}$. The graphs of a uniform probability distribution for $x_{\min} = 0$ and $x_{\max} = 2$ are given on figure 1, *c*.

3. Questions

3.1. What processes are called stationary and ergodic?

3.2. Give the definition of the one-dimensional probability distribution function of random process and prove its properties.

3.3. Give the definition of the one-dimensional probability density function of the random process and prove its properties.

3.4. How can you find the hit probability of process values on defined interval, using the probability distribution function or the probability density function?

3.5. Write down the expressions for the expectation and dispersion of a random process. What is their physical meaning?

3.6. Write down expression for the normal probability distribution function and explain the meaning of values considered with it.

3.7. Explain the type of the graphs of probability distribution function of the harmonic oscillation with an accidental phase, fluctuation noise, and the random process with a uniform distribution.

3.8. Describe the principle of operation of devices to measure the probability distribution function and probability density function of random process.

4. Home task

4.1. Learn chapter "Probabilistic characteristics of a random processes" from the compendium of lectures and literature.

4.2. Perform calculations and build probability distribution function $F(x)$ and probability density function $p(x)$ graphs of the normal (Gaussian) random process, $a = 0$ and root-mean-square deviation $\sigma = 1 + 0{,}1N$(where N is a number of work-group) for the values $-3\sigma < x < 3\sigma$. In the absence of the probability integral table it is possible to use the approximation formula:

$$Q(z) \cong 0{,}65 \exp[-0{,}44(z + 0{,}75)^2] \text{ under } z > 0;$$

$$Q(z) = 1 - Q(|z|) \text{ under } z < 0,\ Q(0) = 0{,}5,\ Q(\infty) = 0.$$

Results of calculations should be presented in the form of tables and graphs.

4.3 Be ready to discuss key questions.

5 Laboratory task

5.1 Acquaintance with a virtual model on a workplace

Start the program 1.2, using the icon TT(English) on the desktop. It is necessary to study the structure of a virtual model using its description in part 6 of this LW and to master entering of parameters. Specify with the teacher the laboratory task performance plan.

5.2 Research of the random process with a uniform distribution probability

Click in the menu “Choice of process“ item “With a uniform distribution”. Place in proper windows values $x_{min} = -1$ and $x_{max} = 1$. Ultimate values of argument at the analysis of distributions are $x_{low} = -2$ and $x_{up} = 2$. Write down measured average value, and root-mean-square deviation, graphs of probability distribution function and probability density function. On the instructions of the teacher repeat measurements for other values x_{min} and x_{max}.

5.3 Research of a Gaussian process

Click in the menu “Choice of process” item “Gaussian process”. Place in proper windows values a and σ, given in the hometask, and choose values x_{min} and x_{max} such that they cover a range of values x from $a - 3\sigma$ up to $a + 3\sigma$. Write down measured average value, and root-mean-square deviation, graphs of probability distribution function, and the probability density function. By the instructions of the teacher repeat measurements for other values of average value a and root-mean-square deviation σ.

5.4 Research of statistical characteristics of a harmonic oscillation

Click in the menu “Choice of process” item “Harmonic oscillation”. Place in proper windows value of amplitude $A = 1$, value of frequency f of the order 10...20

kHz and value of an accidental phase φ. Establish ultimate values of argument analysing of distributions so that they cover a range of values x from $-A$ up to $+A$. Write down measured average value and root-mean-square deviation, graphs of probability distribution function, and the probability density function. By the instructions of the teacher repeat measurements for other values of A, frequency f, phase φ.

6 Description of laboratory model

Laboratory work is performed on a computer in the HP VEE environment using a virtual model. The block diagram of virtual model is given on figure 2. The model enables to investigate characteristics of random process with a uniform probability distribution, Gaussian random process, and harmonic oscillation.

This virtual model realizes two basic functions for each process:

1. Generation of the N samples of researched random process $X(t)$. Samples are displayed as “Realization of a process”;

2. Calculations on the basis of the generated samples of values and displaying it:

a) probability distribution function;

b) probability density function;

c) average value of process;

d) root-mean-square deviation of process.

For every researched random process different methods of generation of samples, different parameters of processes are used.

The generation of samples of process with a uniform distribution is performed by the built-in function “randomize”. The values of $x_{\min}$ and $x_{\max}$ are preset in the model.

The generation of samples of Gaussian process is performed by nonlinear transformation of two arrays of samples $u(i)$ and $v(i)$ of random process with a uniform distribution on an interval (0, 1).

Transformation is given by

$$X(i) = a + \sigma \cdot \sqrt{-2\ln(u(i))} \cdot \cos(2\pi v(i)),\ \ i = \overline{1, N}, \tag{9}$$

here i is the number of the sample in an array; a and σ are the average value and root-mean-square deviation of researched random process, which a researcher sets on a model.

A built-in functional generator performs the generating of samples of harmonic oscillation. A researcher sets the amplitude, the frequency, and the initial phase of oscillation.

The calculation of values of probability distribution function and probability density function is made in the range of argument values from lower-range value x_{low} and to upper-range value x_{up}. An interval (x_{low}, x_{up}) is divided on M of identical subintervals with the length $\Delta x = (x_{\text{up}} - x_{\text{low}})/M$; the quantity of samples k_j, that get on the j-th subinterval is calculated (j takes on values from 1 to M). Hit frequency of sample values on the j-th subinterval $q_j = k_j/N$. In the case of sufficiently large values M and N (in the model M = 200, N = 10000) values of frequency q_j give the probability of getting hit of the sample values on the j-th subinterval. Values hit probability on the j-

th subinterval is $q_j = p(x_j)\Delta x$, where $x_j = j\Delta x$ (according to line 1 in the table 1). Therefore

$$p(x_j) = \frac{k_j}{N\Delta x} = \frac{k_j M}{N(x_{\text{up}} - x_{\text{low}})}, \quad j = \overline{1, M}$$

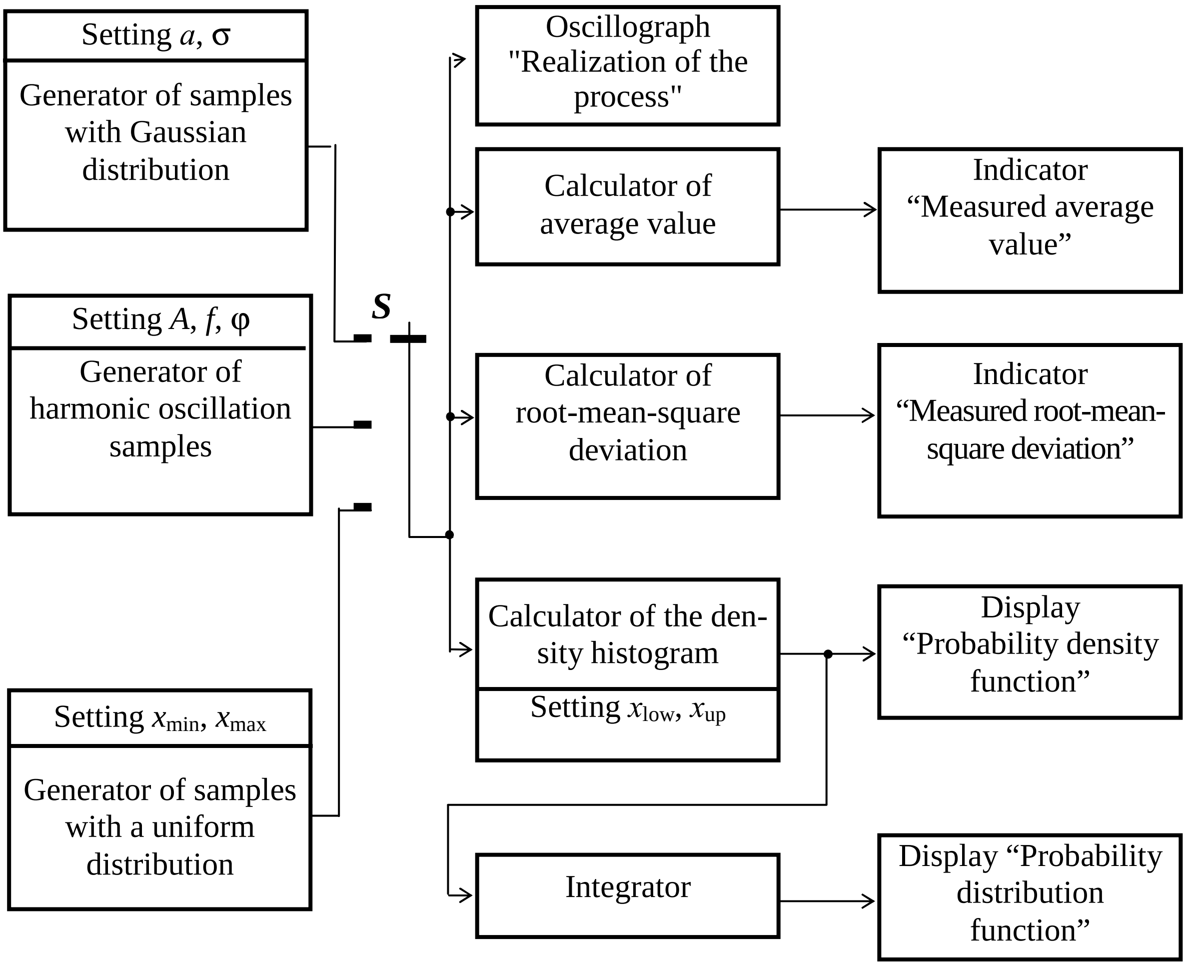


Figure 2 – Virtual model block diagram

Arrays of values $p(x_j)$ and x_j are displayed as "Probability density function".

Using property of probability distribution function $F(x)$ (line 5 table 1), the array of values is calculated as:

$$F(x_j) = \Delta x \sum_{k=1}^{j} p(x_k), \quad j = \overline{1, M}.$$

Arrays of values $F(x_j)$ and x_j are displayed as "Probability distribution function".

The average value of the researched process is calculated by the formula

$$\overline{X(i)} = \frac{1}{N}\sum_{i=1}^{N} X(i),$$

where $X(i)$, $i=\overline{1,N}$ is i-th sample of the researched process. The value $\overline{X(i)}$ is displayed. This display is called “Measured average value”.

Root-mean-square deviation of the researched process is calculated as

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X(i)-\overline{X(i)})^2}\ .$$

The value σ is displayed as “Measured root-mean-square deviation”.

7 Requirements to a report

7.1 Title of the laboratory work.

7.2 Objectives of the laboratory work.

7.3 Results of the homework performing.

7.4 Block diagram of researches.

7.5 Results of the execution of items 5.2–5.5 of laboratory task (graphs, oscillograms, numerical values, etc.).

7.6 Conclusions on every item of the laboratory task, with analysis of the got results:

- coincidence of form of functions $p(x)$ and $F(x)$ each of researched process to theoretical one;
- implementation of properties $p(x)$ and $F(x)$,
- coincidence of measured average value and root-mean-square deviation with calculated, on the given parameters of the researched process (x_{min} and x_{max}, A);
- dependence of functions $p(x)$ and $F(x)$ on frequency and initial phase of harmonic oscillation.

7.7 Signature of student about the laboratory work performing, teachers signature for the laboratory work defense with mark and date.

Literature

1. **Баскаков С.И.** Радиотехнические цепы и сигналы: Учебник для вузов.– М.: Радио и связь, 1988 (1983).

2 **Теория** передачи сигналов: Учебник для вузов / А.Г. Зюко и др. – М.: Радио и связь, 1986.

LW 1.3 Researching of correlation characteristics of random processes and deterministic signals

1. Objectives

1.1 Studying the method of experimental investigation of correlation characteristics of random processes and deterministic signals.

1.2 Research of the connection between correlation functions and spectra of random processes and deterministic signals.

2. Main principles

2.1 The correlation function (CF) of the random process $X(t)$ is the mathematical expectation of the process values product, which they take on in the time moments t_1 and t_2:

$$K_X(t_1, t_2) = \overline{X(t_1) \cdot X(t_2)}\,. \tag{1}$$

CF values $K_X(t_1, t_2)$ determine the quantity of statistical dependence between the values of process in the time moments t_1 and t_2. For the stationary processes, the values of the CF do not depend on choice and t_2. They depend on the distance between them $\tau = t_2 - t_1$. CF is denoted as $K_X(\tau)$. Further we will consider only stationary processes and suppose that they are ergodic. For the ergodic processes CF is determined as:

$$K_X(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t+\tau)\, dt\,, \tag{2}$$

where $x(t)$ is realization of the process $X(t)$.

2.2 Regardless of the form of the CF of different processes, correlation function has such properties as:

– $K_X(0) = P_X$, where P_X is average power of process;

– $K_X(0) \geq K_X(\tau)$ – if $\tau = 0$ the value of the function $K_X(\tau)$is maximal;

– $K_X(\tau) = K_X(-\tau)$ – $K_X(\tau)$ is an even function;

– $K_X(\infty) \to \overline{X(t)}^2$, where $\overline{X(t)}$ is the average value of the process.

2.3 The less value of $K_X(\tau)$ in comparison with $K_X(0)$, the less statistical dependence between the values of process, that are distant on τ from one another. If $K_X(\tau) = 0$, the values of process $X(t)$, that are distant on such time interval as τ, are uncorrelated. It is easier to compare the values $K_X(\tau)$ and $K_X(0)$, if to pass to the normalized correlation function

$$R_X(\tau) = \frac{K_X(\tau)}{K_X(0)} \tag{3}$$

$R_X(0) = 1$ and $-1 \leq R_X(\tau) \leq 1$.

2.4 Often, for a description of correlation properties of random processes instead of the CF a correlation time τ_c is used. The correlation time is used for "rough" description of correlation properties of process. Values of process, distant from one another on $\tau > \tau_c$, are uncorrelated. Values of process, that are distant from one another on $\tau \le \tau_c$, are correlated. Different methods of determination of correlation time are used:

1) Correlation time τ_c is the base of rectangle in high $K_X(0)$, the area of this rectangle is equal to the area under the curve of the CF module (figure 1, *a*):

$$\tau_c = \frac{1}{K_X(0)} \int_0^{\infty} |K_X(\tau)| \, d\tau \tag{4}$$

2) Such values of τ_c, should be such that under the $\tau > \tau_c$ values of CF do not exceed some given level (figure 1, *b*).

3) If the CF has an oscillating character, a value of τ under which CF first time takes on a zero value, may be taken as the correlation time τ_c, (figure 1, *c*).

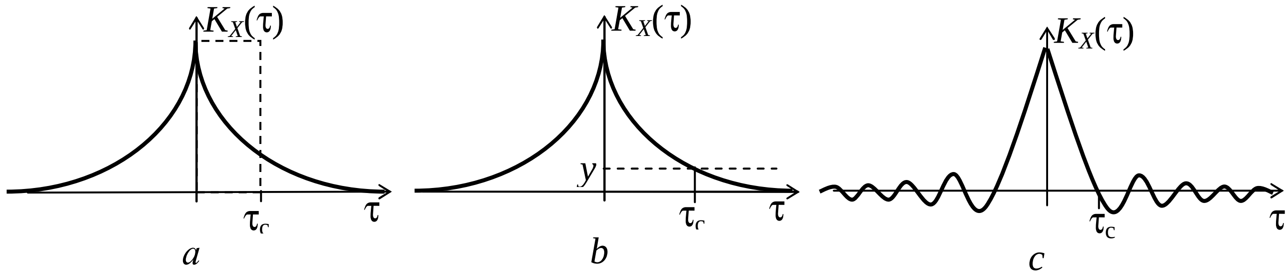


Figure 1 – Determination of correlation time

2.5 According to (2) it is impossible to measure CF precisely, because the realization of process of infinite duration is needed. It is possible to measure CF in case of realization of the random process of finite duration. It is obvious that the longer the realization of the process T_{real}, the more precisely measured CF of realization represents CF of process. The device for measuring CF of realization is named a correlation meter (figure 2). Here delay time τ defines the argument of the measured value of the CF. If correlation meter, shown on a figure 2, performed on a processor or on a computer, it is possible to get the array of the $K_X(kT_s)$ values, where T_s is sampling interval of the process realization *x(t)*; the values of argument taken from the interval $-T_{real} \le kT_s \le T_{real}$. The got arrays of values kT_s and $K_X(kT_s)$ are displayed.

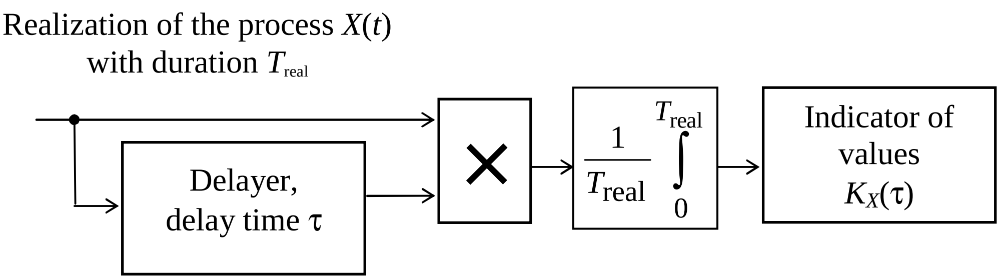


Figure 2 – Block diagram of correlation meter

2.6 The power spectral density $G_X(f)$, which determines the distribution of power of the process on frequencies, is a main spectral description of random processes. Quantitatively the function $G_X(f)$ determines power of process in bandwidth 1 Hz near frequency f. Khinchin-Wiener theorem states that the functions $K_X(\tau)$ and $G_X(\omega)$ are connected by the Fourier transform

$$\left.\begin{aligned} G_X(\omega) &= 2\int_0^\infty K_X(\tau)\cdot\cos(\omega\tau)d\tau; \\ K_X(\tau) &= \frac{1}{\pi}\int_0^\infty G_X(\omega)\cdot\cos(\omega\tau)d\omega. \end{aligned}\right\} \tag{5}$$

If the $G_X(f)$ function is known, it is possible to define average power of a process

$$P_X = \int_0^\infty G_X(f)\,df \tag{6}$$

In particular, if a process is a quasi-white noise with the power spectral density N_0 in band $(0, F_{max})$, so

$$P_X = N_0 \cdot F_{max}. \tag{7}$$

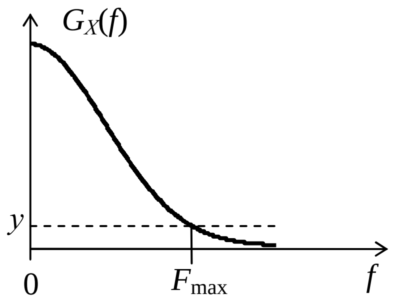


Figure 3 - Determination of bandwidth

2.7 It is often enough to know the bandwidth of the process F_{max}. The bandwidth of random process is determined by the function $G_X(f)$ by such methods as the bandwidth of the deterministic signal. On figure 3 it is shown, how to determine a bandwidth under given level y, i.e. F_{max} is the bandwidth, beyond which the power spectral density of process does not exceed the value y.

As $K_X(\tau)$ and $G_X(f)$ functions are bound by the Fourier transform, there is connection between the bandwidth F_{max} and correlation time τ_c of the process:

$$\tau_к \cdot F_{max} = 0{,}5. \tag{8}$$

In expression (8), equal sign “=” means that the product of correlation time and bandwidth of process is a value of magnitude 0,5 order.

2.8 A correlation function is a description of a deterministic signal, but it does not have such interpretation, as for a random process. CF of a nonperiodic deterministic signal is determined as

$$K_s(\tau) = \int_0^{T_s} s(t)s(t+\tau)dt, \tag{9}$$

where T_s is duration of signal $s(t)$.

To measure the CF of a deterministic signal is possible with the correlation meter, the block diagram of which is given on figure 2. According to this diagram integration is performed on the interval $(0, T_s)$ and a factor before this integral is missed.

Let $s(t)$ be rectangular video pulse of the amplitude A and duration T_p

$$s(t) = \begin{cases} A, & 0 \le t < T_\text{p}, \\ 0, & t < 0, \quad t \ge T_\text{p}. \end{cases} \tag{10}$$

After substitution of the expression (10) in expression (9) we will get

$$K_s(\tau) = \begin{cases} A^2 T_\text{p}\left(1 - |\tau|/T_\text{p}\right), & |\tau| \le T_\text{p}, \\ 0, & |\tau| > T_\text{p}. \end{cases} \tag{11}$$

The CF of a rectangular video pulse is shown on figure 4, *a*.

It follows from the expression (9) that $K_s(0) = E_s$ is the energy of the signal $s(t)$. Fourier transform from $K_s(t)$ gives the square of amplitude spectrum (energy spectral density) of the signal $s(t)$. The Fourier transform from expression (11) gives the square of known expression for the amplitude spectrum of rectangular video pulse

$$S^2(f) = \left(A T_\text{p} \frac{\sin\left(\pi f T_\text{p}\right)}{\pi f T_\text{p}} \right)^2, \quad -\infty < f < \infty. \tag{12}$$

2.9 Let consider rectangular radio pulse, duration T_p

$$s(t) = \begin{cases} A\sin\left(2\pi f_0 t + \varphi_0\right), & 0 \le t < T_\text{p}, \\ 0, & t < 0, \; t \ge T_\text{p}, \end{cases} \tag{13}$$

where A, f_0 and φ_0 are amplitude, frequency and initial phase of oscillation accordingly.

After substitution (13) in (9) we will get

$$K_s(\tau) = \begin{cases} 0{,}5 A^2 T_\text{p}\left(1 - |\tau|/T_\text{p}\right)\cos 2\pi f_0, & |\tau| \le T_\text{p}, \\ 0, & |\tau| > T_\text{p}. \end{cases} \tag{14}$$

From expression (14) follows, that CF of rectangular radio pulse is cosine curve with a zero initial phase and does not depend on the phase of rectangular radio pulse. Therefore, if the initial phase of rectangular radio pulse φ_0 is random, CF of rectangular radio pulse is determined by formula (14). CF envelope of rectangular radio pulse coincides with CF of rectangular video pulse. The graph of CF of rectangular radio pulse, built on a formula (14) for $f_0 = 4/T_\text{p}$, is given on figure 4, *b*.

Fourier transform from expression (14) gives the square of amplitude spectrum of signal (12)

$$S^2(f) = 0{,}25\left(A T_\text{p} \frac{\sin\left(\pi(f - f_0)T_\text{p}\right)}{\pi(f - f_0)T_\text{p}} \right)^2, \quad -\infty < f < \infty. \tag{15}$$

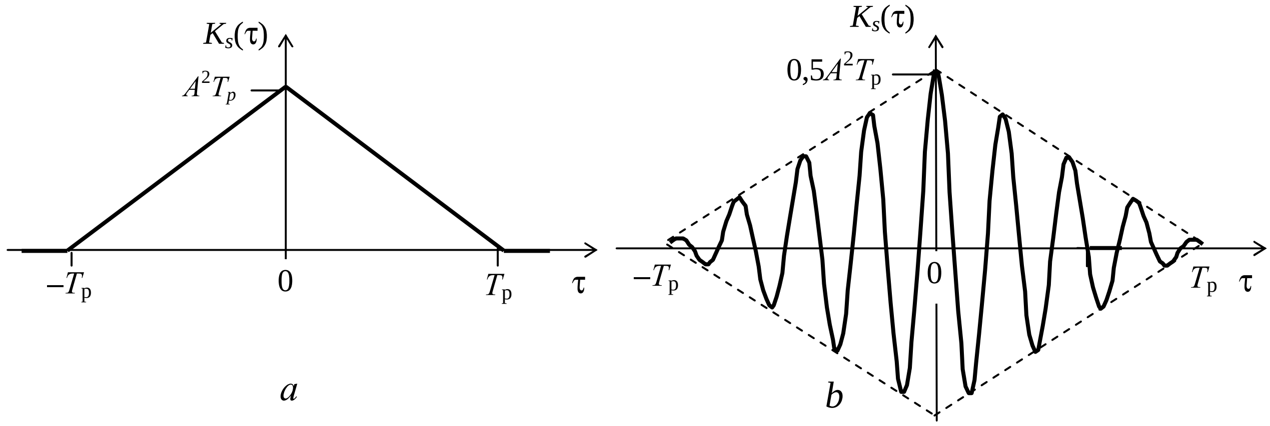


Figure 4 – Correlation functions of: *a* – rectangular video pulse, *b* – rectangular radio pulse

3. Questions

3.1 Give the definition of the CF of random process.
3.2 How to determine CF of the process?
3.3 Enumerate main properties of the CF of random process.
3.4 What random process parameters are possible to define according to CF?
3.5 What does Wiener-Khinchin theorem state?
3.6 Enumerate methods of correlation time determination.
3.7 How are bandwidth and correlation time of random process connected?
3.8 What form has the CF of rectangular video pulse?
3.9 What form has the CF of rectangular radio pulse?
3.10 Why does the initial phase of rectangular radio pulse not influence on its CF?

4. Home task

4.1 Study the chapter "Correlation theory of random processes" from the compendium of lectures and literature [1, p. 73...79, 149...164; 2, p. 67...72].

4.2 Build a block diagram of the correlation meter for the research of correlation functions of random processes and deterministic signals.

4.3 Perform calculations and build graphs for the CF of rectangular video pulse and rectangular radio pulse for such input data: T_p = 2 ms, frequency of oscillation of radio pulse signal $f_0 = 500(N + 1)$ Hz, where N is the number of workplace. Perform calculations and build graphs of spectra for the given pulses using expressions (12) and (15).

4.4 Prepare for discussion on key questions.

5 Laboratory task

5.1 Acquaintance with a virtual model on a workplace

Start the program **1.3**, using the icon **TT(English)** on the desktop. It is necessary to study the structure of a virtual model using its description in part 6 of this LW and master entering of parameters. Coordinate the plan of fulfilling of the laboratory task with the teacher.

5.2 Research of correlation and spectral characteristics of realization of noise

Set in generator of the quasi-white noise F_{max} = 1000 Hz. After program execution, analyse the experimental data and write down it. Check up implementation of properties of correlation function, determine maximal frequency on a spectrum, and determine correlation time on a correlation function, find their product, compare it with the theoretical value (8). Give the visual estimate of the average value of the power spectral density N_0 on an interval (0, F_{max}). Multiply the value of power spectral density N_0 on F_{max} and compare the product with the value of the measured average power of realization – expression (7).

By instructions of the teacher repeat measurements for other values F_{max}.

5.3 Research of correlation and spectral characteristics of rectangular video pulse

Set in the generator of rectangular video pulse A = 2 V, T_p = 0,5 ms. After program execution, complete the $K_s(\tau)$ and $S^2(f)$ graphs. Analyse the experimental data and write down it. Compare the experimental dependence $S^2(f)$ with the theoretical (12); compare the experimental dependence $K_s(\tau)$ with the theoretical one(11); compare measured value of pulse energy with the value of $K_s(0)$.

By instructions of the teacher repeat research for other values A and T_p.

5.4 Research of correlation and spectral characteristics of rectangular radio pulse

Set in the generator of rectangular radio pulse A = 2 V, f_0 = 1000 Hz. After program execution, complete the $K_s(\tau)$ and $S^2(f)$ graphs. Analyse the experimental data and write down it. Compare the experimental dependence $S^2(f)$ with theoretical (15), compare the experimental dependence $K_s(\tau)$ with theoretical one(14), and compare the measured value of pulse energy with the value of $K_s(0)$. Write down the value of the initial phase. Launch the program and make sure, that a correlation function does not depend on an initial phase.

By instructions of the teacher repeat research for other values A and T_p.

6 Description of laboratory model

Laboratory work is performed on a computer in the HP VEE environment using of virtual model. The block diagram of virtual model is given on figure 5. A model contains the following generators:

– generator of noise, which produce the realization of quasi-white noise with the band (0, F_{max}), with duration 20 ms, in the form of 5000 samples; it is possible to set the F_{max} value 1000, 2000 and 3000 Hz;

– generator of single rectangular video pulse allows to set pulse duration 0,5, 1 and 1,5 ms and arbitrary amplitude of pulse;

– generator of rectangular radio pulse, with duration 2 ms, allows to set arbitrary amplitude of pulse and frequency of oscillation f_0 1000, 2000 and 3000 Hz. The initial phase of oscillation is a random value, this value is displayed on the indicator φ.

The switch S allows to choose the researched process.

If noise is chosen for research, on displays are represented:

– noise realization;

– value of the measured average power of realization;

– correlation function of realization, calculated using the algorithm which is given on figure 2;

– power spectral density of noise realization, got as Fourier transform from the correlation function of realization. The program generates samples of quasi-white noise. However, because of few samples, the spectrum is far from white in the band (0, F_{max}).

If a rectangular video pulse or rectangular radio pulse is chosen, on displays are represented:

– pulse oscillogramms;

– measured pulse energy value;

– correlation function of pulse, calculated by formula (9);

– square of amplitude spectrum of pulse, got as Fourier transform from the correlation function of pulse.

In all cases for the calculation of CF the built-in function “Xcorrelate” is used.

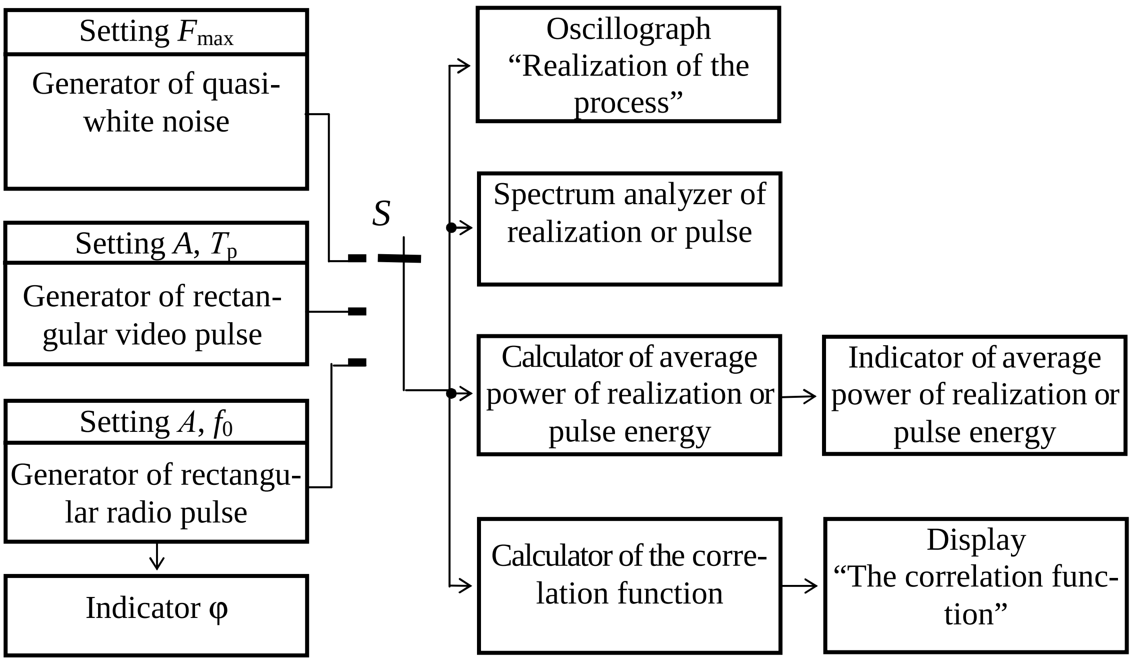


Figure 5 - Block diagram of virtual model

7 Requirements to the report

7.1 Title of laboratory work.

7.2 Objectives of laboratory work.

7.3 Results of homework performing.

7.4 Block diagram of researches, list of devices which are used in LW.

7.5 Results of implementation of items 5.2-5.4 of laboratory task (graphs, oscillogramms, numerical values, etc.).

7.6 Conclusions on every item of laboratory task, with analysis of the got results (review of implementation of correlation functions properties, coincidence of experimental and theoretical data, etc.).

7.7 Signature of student about the laboratory work implementation, signature of teacher about the laboratory work defence with mark, date.

Literature

1. **Баскаков С.И.** Радиотехнические цепы и сигналы: Учебник для вузов.– М.: Радио и связь, 1988 (1983).
2. **Гоноровский И.С.** Радиотехнические цепы и сигналы: Учебник для вузов. – М.: Радио и связь, 1986 (1977).

LW 1.4 Researching of AM, DSB-SC and SSB modulated signals

1. Work objectives

1.1 Research of time and spectral characteristics of analog modulated signals.
1.2 Research of relation between characteristics of modulated and modulating signal.

2 Main principles

2.1 Carrier, in the case of amplitude (AM), double-sideband-suppressed-carrier (DSB-SC) and single-sideband (SSB) modulations is harmonic oscillation $u_{car}(t) = A_0 \cos(2\pi f_0 t + \varphi_0)$. Modulating signal is a telecommunication baseband continuous signal $b(t)$ with such characteristics:

– signal spectrum maximum frequency is F_{max};
– signal is normalized such as module maximum values $|b(t)|_{max} = 1$;
– signal average value $\overline{b(t)} = 0$.

2.2 In case of AM the carrier amplitude changes are proportional to instant values of a modulating signal, i.e. amplitude of the modulated signal $A(t) = A_0 + \Delta A b(t)$, where ΔA – factor of proportionality which is chosen in such way that amplitude $A(t)$ does not accept negative values. As $|b(t)|_{max} = 1$, so ΔA defines the greatest carrier amplitude change on the module. In order the amplitude $A(t)$ does not accept negative values, it is necessary to provide $\Delta A \le A_0$. Frequency and initial phase of a carrier are invariable. It is convenient to pass to a relative maximum change of amplitude – the amplitude modulation factor $m_{AM} = \Delta A/A_0$. It is clear, that $0 < m_{AM} \le 1$.

Analytical expression of AM signal in case of any modulating signal looks like

$$s_{AM}(t) = A_0[1 + m_{AM} b(t)]\cos(2\pi f_0 t + \varphi_0). \qquad (1)$$

We see, that parameters of AM signal are m_{AM}, A_0, f_0 and φ_0. Time diagram of AM signal is shown on figure 1. It is interesting that envelope of the modulated signal repeats the form of a modulating signal – amplitude of AM signal $A(t)$ is envelope of high-frequency oscillations $\cos(2\pi f_0 t + \varphi_0)$ (on figure 1 envelope is represented by a dashed-line curve).

2.3 On figure 2 any amplitude spectrum of a modulating signal and amplitude spectrum of AM signal corresponding to it are shown. Amplitude spectrum of AM signal consists of carrier frequency harmonic oscillation, of upper sideband of frequencies (USB) and lower sideband of frequencies (LSB). Thus USB is a copy of a spectrum of the modulating signal, which is shifted on frequency on f_0. LSB is mirror reflection of USB relatively to carrier frequency f_0.

Figure 2 shows the important result: the AM signal spectrum width F_{AM} is equal to the doubled value of modulating signal spectrum maximum frequency, i.e. $F_{AM} = 2F_{max}$.

2.4 Calculations show, if modulating signals are telecommunication baseband signals than sidebands power makes some percent out of modulated signal power. Therefore it is expedient to generate a signal with a spectrum, which consists only of two frequencies sidebands (carrier frequency oscillation is absent), – such signal is the signal of double-sideband-suppressed-carrier modulation.

Such kind of modulation when the modulated signal is a product of a modulating signal and a carrier is called double-sideband-suppressed-carrier. Analytical expression signal DSB-SC looks like

$$s_{\text{DSB-SC}}(t) = A_0 b(t)\cos(2\pi f_0 t + \varphi_0)\,. \qquad (2)$$

Time diagrams of the modulating and modulated signals are shown on figure 3. As the modulating signal influences on amplitude of a carrier, DSB-SC considered as version of AM. From figure 3 it is clear, that envelope of signal DSB-SC $A(t) = A_0\,|\,b(t)\,|$ (shown by a dashed line) does not repeat modulating signal.

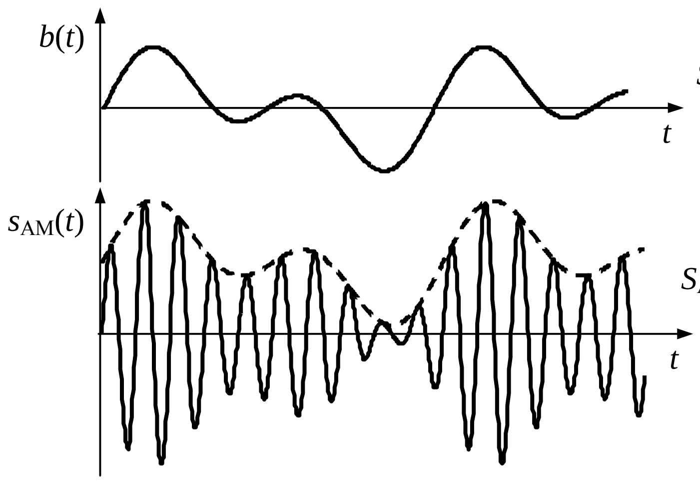


Figure 1 – Modulating $b(t)$ and modulated $s_{\text{AM}}(t)$ signals

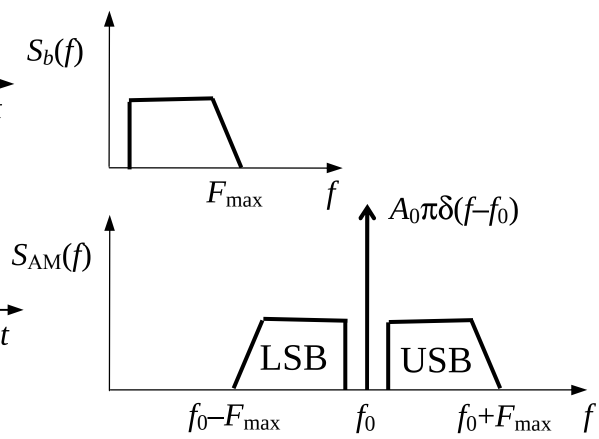


Figure 2 – Modulating and AM signals spectrum

From comparison of the mathematical expressions describing AM signal (1) and DSB-SC signal (2) we see, that spectrum DSB-SC signal differs from spectrum AM signal with the absence of carrier frequency oscillation. On figure 4 any amplitude spectrum of a modulating signal and DSB-SC signal amplitude spectrum corresponding to it, which consists of USB and LSB, are shown. From figure 4 follows, that DSB-SC signal spectrum width $F_{\text{DSB-SC}}$ is the same, as AM signal spectrum width: $F_{\text{DSB-SC}} = 2F_{\max}$.

2.5 Such kind of modulation whereby the modulated signal spectrum coincide with modulating signal spectrum shifted on carrier frequency or the modulated signal spectrum is an inversion of the shifted spectrum according to carrier frequency, is called single-sideband modulation. The SSB signal spectrum contains one sideband – upper or lower. The SSB signal can be written as

$$s_{\text{SSB}}(t) = A_0 b(t)\cos(\omega_0 t + \varphi_0) \mp A_0 \tilde{b}(t)\sin(\omega_0 t + \varphi_0)\,, \qquad (3)$$

where the sign "–" concerns the description of a signal with the upper sideband of frequencies, and a sign "+" – with the lower sideband; $\tilde{b}(t)$ – conjugated on Hilbert signal with a signal $b(t)$. The physical sense of Hilbert transform is simple enough:

signal $\tilde{b}(t)$ differs from signal $b(t)$ so that phases of all its components are shifted on a $\pi/2$ angle.

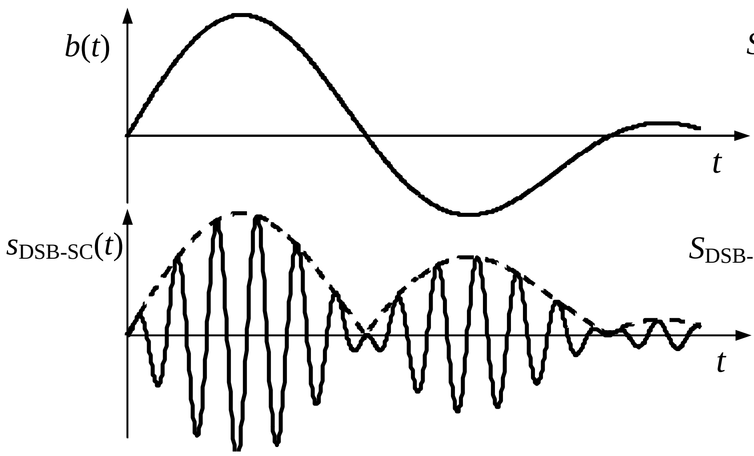


Figure 3 – Modulating $b(t)$ and modulated $s_{DSB\text{-}SC}(t)$ signals

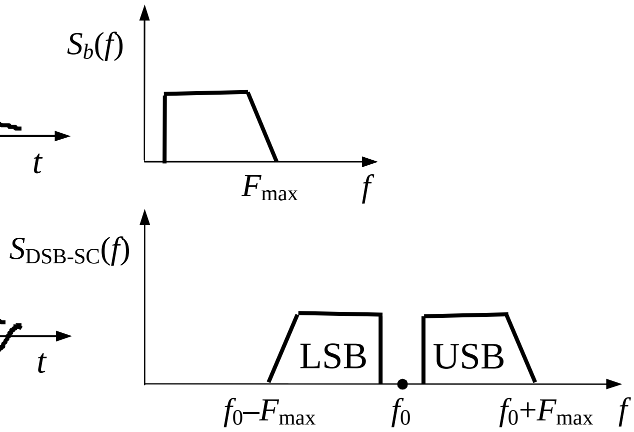


Figure 4 – Modulating and DSB-SC signals spectrum

Time modulating signal diagrams $b(t)$, conjugated on Hilbert $\tilde{b}(t)$ and SSB signal are shown on figure 5. From figure 5 it is clear, that envelope SSB signal $A(t) = A_0\sqrt{b^2 + \tilde{b}^2}$ (it is shown by a dashed line) does not repeat modulating signal.

On figure 6 any amplitude spectrum of a modulating signal and an amplitude spectrum corresponding to it SSB USB signal are shown. From figure 6 follows, that the width of SSB signal spectrum F_{SSB} is twice less than width of AM and DSB-SC signals spectrum: $F_{SSB} = F_{max}$.

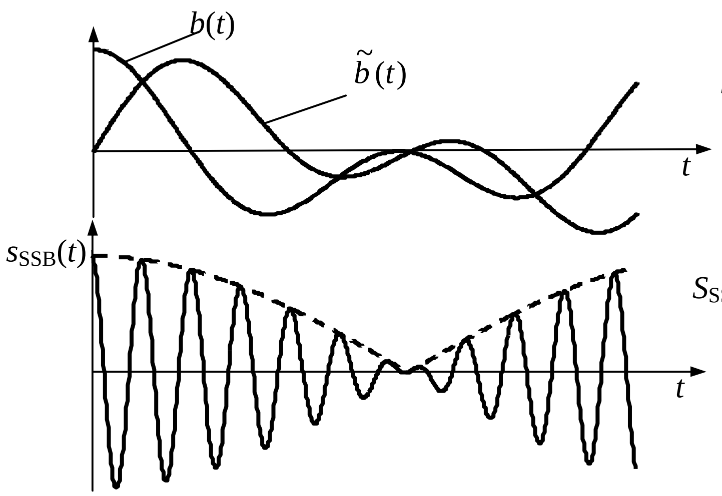


Figure 5 – Modulating $b(t)$ and modulated $s_{SSB}(t)$ signals

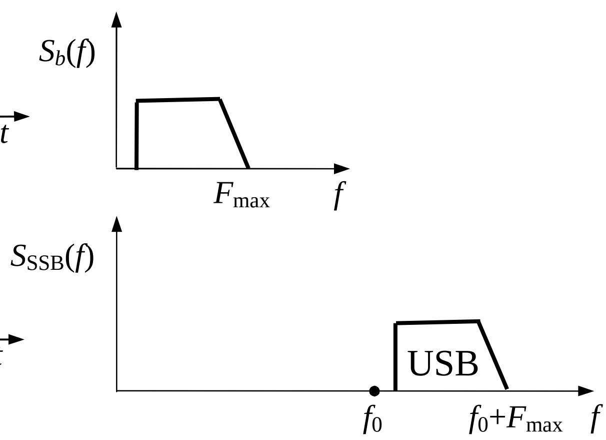


Figure 6 – Modulating and the SSB signals spectrum

2.6 Mathematical models of AM, DSB-SC and SSB signals as (1...3) are used for designing signal forming schemes and signals detecting.

3 Questions

3.1 What is the purpose of modulation usage in telecommunication systems?

3.2 Give the definition of amplitude, double-sideband-suppressed-carrier and single-sideband modulations.

3.3 What is the amplitude modulation factor? What values can it accept?

3.4 What is Hilbert transform? What is its physical essence?

3.5 Draw time diagrams AM, DSB-SC and SSB signals if a modulating signal is harmonic oscillation.

3.6 Represent AM, DSB-SC and SSB signals spectra if a modulating signal is harmonic oscillation .

3.7 Represent AM, DSB-SC and SSB signals spectra at a set modulating signal spectrum.

3.8 Explain, why SSB signal envelope on figure 5 has such kind?

4 Home task

4.1 Study section “Amplitude modulation and its versions” on the compendium of lectures and the literature [1, pp. 53-60; 2, pp. 88-96] and on the description of a laboratory model in section 6 of these instructions.

4.2 Carrier oscillation of frequency f_0 are modulated by a baseband signal $b(t) = A_1\sin(2\pi F_1 t) + A_2\sin(2\pi F_2 t) + A_3\sin(2\pi F_3 t)$. Represent baseband signal spectrum and AM, DSB-SC and SSB signals spectra (put $m_{AM} = 1$). Initial data of the task according to your laboratory place number are given in table 1.

4.3 Be ready to discuss the questions.

Table 1 – Initial data for the home task

Workplace number	A_1, V	F_1, Hz	A_2, V	F_2, Hz	A_3, V	F_3, Hz	f_0, Hz
1	0,3	50	0,4	100	0,3	250	800
2	0,3	100	0,3	200	0,4	300	900
3	0,4	50	0,3	200	0,3	250	1000
4	0,3	100	0,4	150	0,3	250	1100
5	0,3	50	0,3	250	0,4	300	1200
6	0,4	100	0,3	250	0,3	300	1000
7	0,3	50	0,4	100	0,3	150	800
8	0,3	100	0,3	200	0,4	300	900

5 Laboratory task

5.1 Acutance with a virtual model on a workplace.

Start the program **1.4**, using the icon **TT(English)** on the desktop. Study scheme model, using the description in section 6 of this LW. Specify with the teacher the laboratory task performance plan.

5.2 Carry out researches of the modulated signals in time and frequency domain. For this purpose:

– set values A_1, F_1, A_2, F_2, A_3, F_3, factor m_{AM} and frequency f_0 the same, as in the homework;

– set the AM modulation kind and run the program;

– draw in the report the signals oscillogram and the spectrogram on the modulator input and output;

– set sequentially DSB-SC, USB SSB, LSB SSB modulation kinds, run the program and draw in the report the signals on the modulator output spectrogram;

– compare calculated in a homework and the obtained on model spectrograms, compare results write down into report conclusions;

– make the conclusions concerning correspondence of modulating signal forms and envelope of the modulated signal for modulation of different kinds.

5.3 Carry out modulated signals spectrum researches in case of changing carrier frequency. For this purpose at first increase on 200 Hz, and then reduce on 200 Hz carrier frequency, draw in the report obtained signals spectrogram on the modulator output. Put in the report conclusions the changes in spectrograms in comparison with received in item 5.2.

5.4 Carry out research of AM signal spectrum dependence on modulation factor. For this purpose:

– set parameters A_1, F_1, A_2, F_2, A_3, F_3 and frequency f_0 the same, as in the home task;

– set a kind of AM modulation and factor m_{AM} =0,7;

– compare the obtained oscillograms and spectrograms on the output of the modulator with obtained in item 5.2, results of comparison put in report conclusions.

5.5 Carry out research of the SSB signal in case of a single-tone modulating signal. For this purpose:

– set values A_1 = 1V, F_1 = 100 Hz, $A_2 = A_3 = 0$, frequency f_0 the same, as in the home task;

– set a kind of SSB USB, and then SSB LSB modulation;

– draw in the report $b(t)$, $\tilde{b}(t)$ and $s_{SSB}(t)$ signals oscillogram and spectrogram;

– make conclusions concerning correspondence of $b(t)$, $\tilde{b}(t)$, $s_{SSB}(t)$ signals and envelope of the modulated signal $A(t) = A_0\sqrt{b^2+\tilde{b}^2}$.

6 Laboratory model description

Laboratory work is carried out on the computer in the environment of HP VEE with usage of the virtual model which block diagram is on figure 7.

Virtual model consists of the modulating continuous signal generator $b(t) = A_1\sin(2\pi F_1 t) + A2\sin(2\pi F_2 t) + A3\sin(2\pi F_3 t)$ and the modulator (the carrier generator is a part of the modulator). Harmonic oscillation frequencies and amplitudes values A_1, F_1, A_2, F_2, A_3, F_3, f_0, factor m_{AM} and carrier frequency is possible to change.

The virtual model scheme gives the chance to set modulation kinds: AM, DSB-SC, SSB USB and SSB LSB. Time and spectral diagrams of signals can be observed in two points of the virtual model scheme: on the modulator input and output. In a case of SSB oscillograph on modulator input displays not only modulating signal $b(t)$ but a signal $\tilde{b}(t)$ too. Together with modulated signal oscillogram, the schedule of signal envelope is drawn by dotted line.

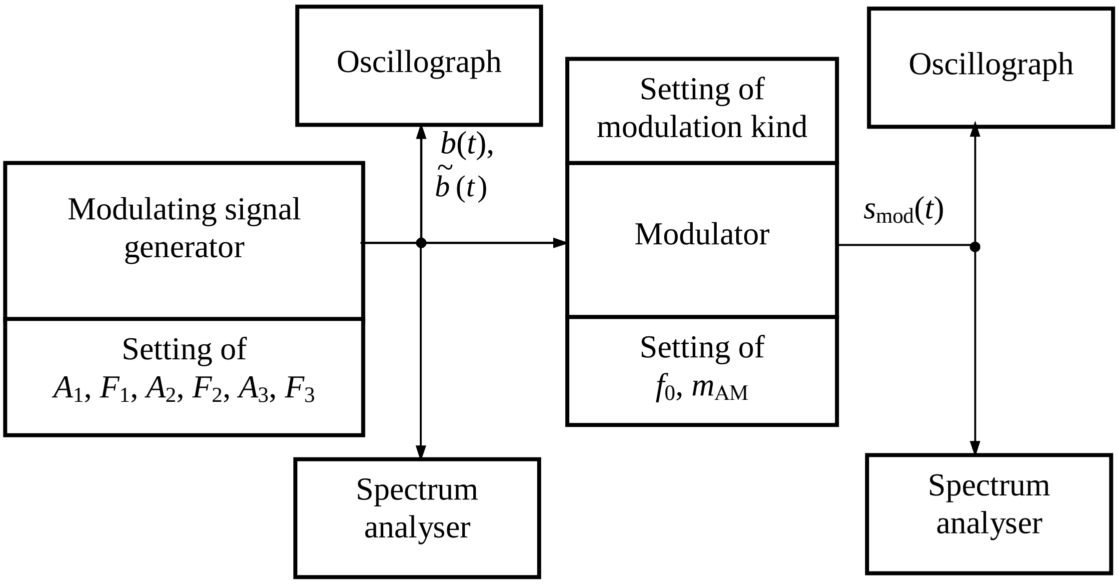


Figure 7 – Virtual model block diagram

7 Requirements to the report

7.1 Laboratory work title.
7.2 Work purpose.
7.3 Results of home task performance.
7.4 Block diagrams of researches and results of performance of item 5.2... 5.5 in the laboratory tasks (oscillograms and spectrograms, each of it should have the caption).
7.5 Conclusions on each item of the task in which you have to give the analysis of the obtained results (coincidence of theoretical and experimental data, displayed signals properties, etc.).
7.6 Date, the student signature, the teacher visa with mark .

Literature

1. Панфілов І. П., Дирда В. Ю., Капацін А. В. Теорія електричного зв’язку: Підручник для студентів вузів 1-го та 2-го рівнів акредитації. – К.: Техніка, 1998.

2. **Баскаков С. И.** Радиотехнические цепы и сигналы. Учебник для вузов.– М.: Радио и связь, 1988 (1983).

LW 1.5 Research of digital modulated signals

1. Objectives

1.1 Study of transmission methods of digital signals with modulated MASK, MPSK and BFSK signals.

1.2 Research of time and spectral characteristics of MASK and MPSK signals for M = 2 and 4 and BFSK signal.

2. Main principles

2.1 A baseband digital signal $b_d(t)$ is a sequence of binary symbols (bits) 1 and 0, that follow in clock interval T_b. In digital devices the rectangular pulse of high level corresponds to symbol 1, and the pulse of low level corresponds to symbol 0.

2.2 A digital modulation signal $s(t)$ is a sequence of radio pulses, that reflect a baseband signal and follow in clock interval T:

$$s(t) = \sum_{k=-\infty}^{\infty} s_i^{(k)}(t - kT), \qquad (1)$$

where $s_i(t)$, $i = 0, \ldots, M - 1$, are the elementary signals (radio pulses);

M is a number of elementary signals;

$s_i^{(k)}(t - kT)$ is the i-y radio pulse, that is transmitted on k-y time interval;

T is a clock interval.

2.3 The general mathematical expression for radio pulse is:

$$s_i(t) = a_i A(t)\cos(2\pi f_i t + \varphi_i), \quad i = 0, 1, \ldots M - 1, \qquad (2)$$

where a_i, f_i, φ_i – the parameters which are defined by a form of digital modulation;

$A(t)$ – a function, that determines the form of pulse.

Radio pulses can differ in amplitudes, phases or frequencies. There are different types of digital modulation, for example:

– MASK is M-ary amplitude modulation (pulses differ in parameter a_i);
– MPSK is M-ary phase modulation;
– MAPSK is M-ary amplitude-phase modulation;
– MQAM is M-ary quadrature-amplitude modulation;
– MFSK is M-ary frequency modulation.

If M = 2, there is the binary $s(t)$ signal: radio pulse $s_0(t)$ is used for transmission 0, and radio pulse $s_1(t)$ – for transmission 1. If $M > 2$, the multi-level signal $s(t)$ takes place. As a rule, M = 4, 8, …, 2^n, where n is an integer. Here every radio pulse $s_i(t)$ is used for transmission of $n = \log_2 M$ bits of baseband digital signal $b_d(t)$. Mapping code sets the concrete bit sequence, that each radio pulse keeps. In the case of binary signals the clock interval $T = T_b$, but in the case of multi-level signals, the clock interval increased: $T = T_b \log_2 M$.

In the case of MASK and BPSK signals, elementary signals can be written as:

$$s_i(t) = a_i A(t)\cos(2\pi f_0 t), \quad i = 0, 1, \ldots, M - 1, \qquad (3)$$

where a_i is a number which represents n bits, that the $s_i(t)$ signal keeps;

f_0 is the carrier frequency.

In the case of MPSK ($M \geq 4$) and MAPSK, it is convenient to describe the elementary $s_i(t)$ signals with cosine and sine components:

$$s_i(t) = a_i A(t) \cos 2\pi f_0 t + b_i A(t) \sin 2\pi f_0 t, \quad i = 0, 1, ..., M-1, \tag{4}$$

where a_i, b_i are coefficients, representing a sequence of n bits, that is transfered by the elementary signal $s_i(t)$.

The following record is equivalent to expression (4):

$$s_i(t) = A_i A(t) \cos(2\pi f_0 t - \varphi_i), \quad i = 0, 1, ..., M-1, \tag{5}$$

$A_i = \sqrt{a_i^2 + b_i^2}$, $\varphi_i = \mathrm{arctg}(b_i / a_i)$, i.e. expression (4) maps radio pulse.

2.4 It is accepted to represent the elementary signals $s_i(t)$ as signal points in a certain space. Diagrams on which elementary signals are represented as signal points are called signal constellations. The purpose of such representation is to reflect the difference of signals.

As it follows from the expression (3), elementary signals, in the case of MASK and BPSK signals, differ only in the coefficients a_i. Therefore, the signal points of MASK and BPSK signals are located on a numerical axis, and the MASK and BPSK signals are named one-dimensional (figure1). On this figure the mapping codes are also reflected (the index i corresponds to a binary number, which is formed by transmitting bits):

– BASK signal: the transmission of 0 corresponds to a_0= 0, and the transmission of 1 corresponds to $a_1 = a$.

– BPSK signal: 0 $\rightarrow a_0 = -a$; 1 $\rightarrow$ $a_1 = a$.

– QASK signal: 00 $\rightarrow$ $a_0 = -a$; 01 $\rightarrow$ $a_1 = -3a$; 10 $\rightarrow$ $a_2 = a$; 11 $\rightarrow$ $a_3 = 3a$.

The number a determines the energies of elementary signals.

MAPSK and MPSK ($M \geq 4$) signals are two-dimensional since elementary signals in expression (4) are described by two coefficients. Functions $A(t) \sin 2\pi f_0 t$ and $A(t) \cos 2\pi f_0 t$, that are presented in expression (4), are orthogonal, and they form two-dimensional space. Signal constellations of two-dimensional signals are reflected on a plane. For example, the signal constellation of QPSK signal is shown on figure 2. Here x symbolizes $\cos 2\pi f_0 t$ oscillation, and y symbolizes $\sin 2\pi f_0 t$ oscillation. It is taken into account that for the MPSK signals expression (5) can be rewritten in that form:

$$s_i(t) = aA(t) \cos(2\pi f_0 t - \varphi_i), \quad i = 0, 1, ..., M-1. \tag{6}$$

The mapping code of QPSK signal on figure 2, is:

00 $\rightarrow$ φ_0 = 135° ($a_0 = -a; b_0 = a$);

01 $\rightarrow$ φ_1 = 45° ($a_1 = a; b_1 = a$);

10 $\rightarrow$ φ_2 = 225° ($a_2 = -a; b_2 = -a$);

11 $\rightarrow$ φ_3 = 315° ($a_3 = a; b_3 = -a$).

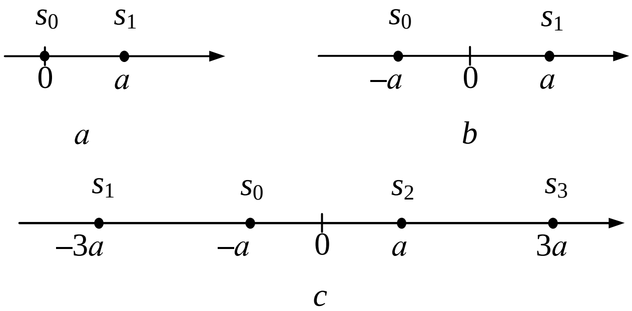


Figure 1 – Signal constellations of signals: *a* – BASK; *b* – BPSK; *c* – QASK

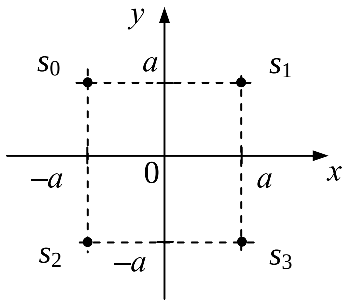


Figure 2 – Signal constellation of QPSK signal

2.5 As it follows from expression (2), elementary pulses are the signals of analogue double-sideband-suppressed-carrier (DSB-SC) and, therefore, the spectrum of radio pulse $s_i(t)$ consists of two side bands, concentrated near the carrier frequency f_0. Spectral properties of $s_i(t)$ radio pulse are determined by the $A(t)$ function.

If $A(t)$ function is a rectangular pulse of T duration, a radio pulse spectrum is wide. But it is important for the transmission of digital signals to form a compact spectrum. In order, that the spectrum of $s_i(t)$ radio pulse will be compact, and inter-symbol interference would be absent, a function $A(t)$ must be Nyquist pulse. Then side bands will be the copies of Nyquist spectrum (figure 3), and the width of spectrum of MASK and BPSK signals is determined by expression:

$$F = 2 f_N (1+\alpha) = \frac{1+\alpha}{T} = \frac{1+\alpha}{T_b \log_2 M}, \tag{7}$$

where $f_N = 0{,}5/T$ is the Nyquist frequency;

α is a roll-off factor ($0 \le \alpha \le 1$).

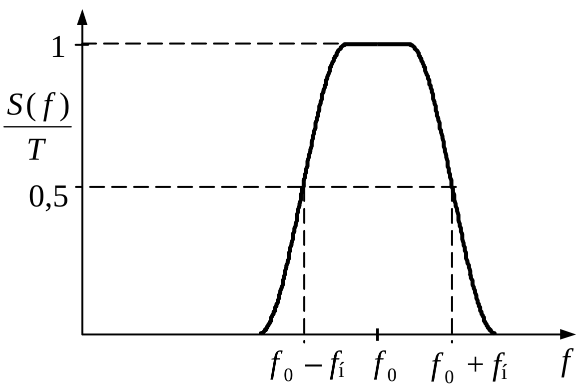


Figure 3 – Spectrum of elementary signal *M*-ASK and PM-*M* (α = 0,6)

Signals, that are introduced by expression (4), are the sum of two DSB-SC signals with the identical amplitude spectra that are determined by the spectrum of $A(t)$ signal. The amplitude spectra of DSB-SC signals add, and the spectrum of their sum has the shape, shown on figure 3, if $A(t)$ is the Nyquist pulse. Therefore, the bandwidth of elementary signals of MPSK and MAPSK is described by expression (7).

An important conclusion follows from expression (7) – increasing the number of signal positions courses decreasing the bandwidth of elementary signals (2).

2.6 Process of forming one-dimensional and two-dimensional signals on the basis of expressions (3) and (4) is following: the mapper puts in accordance $n = \log_2 M$ of input bits to the two rectangular pulses with amplitudes a_i and b_i (in the case of one-dimensional signals only one pulse with amplitude a_i takes place; b_i =); rectangular pulses are filtered by shaping low-pass filters (LPF) to get Nyquist pulses; the pulses $a_i A(t)$ and $b_i A(t)$ enter DSB-SC modulator input; the got DSB-SC signals are summing up.

2.7 The BFSK signal is formed on the basis of radio pulses, that are differ in frequencies:

$$s_0(t) = aA(t)\cos\left(2\pi\left(f_0 - \Delta f/2\right)t\right),$$
$$s_1(t) = aA(t)\cos\left(2\pi\left(f_0 + \Delta f/2\right)t\right), \qquad (8)$$

where Δf is the frequency deviation;

a is the coefficient, that determines the energy of signals.

If the $A(t)$ function is rectangular pulse, it is necessary to provide forming of signal without the “break” of phase in the BFSK modulator. It is possible, if frequency separation equals $\Delta f = k/(2T)$, k = 1, 2, 3, ...; $T = T_b$. If k = 1, so $\Delta f = 0{,}5/T$, then modulation is called “minimum shift keying” (MSK). In the case of MSK the normalized spectrum of signal is described by expression:

$$S(f) = \frac{\sqrt{1+\cos\left(4\pi\left(f - f_0\right)T\right)}}{\sqrt{2}\left(1 - \left(4\left(f - f_0\right)T\right)^2\right)}. \qquad (9)$$

The diagram of dependence (9) is shown on figure 4. With increasing of the difference $|f - f_0|$, the spectrum decreases with the speed equals $1/f^2$. If to define the bandwidth F_{MSK} on the first zeros of dependence (9), we have

$$F_{MSK} = 1{,}5/T. \qquad (10)$$

In order to get the BFSK signal with a narrow spectrum and without intersymbol interference, it is necessary, function $A(t)$ to be the Nyquist pulse. In this case it is possible to consider, that the spectrum of signal $s_{BFSK}(t)$ is the sum of spectra of two radio pulses with central frequencies $f_0 - \Delta f/2$ and $f_0 + \Delta f/2$. The normalized spectrum of BFSK signal is shown on figure 5. It is shown, that frequency deviation is minimum, if the spectrums of radio pulses adjoin to each other, and this frequency deviation is equal:

$$\Delta f_{min} = \frac{1+\alpha}{T}. \qquad (11)$$

Then bandwidth of BFSK signal is:

$$F_{BFSK} = \Delta f_{min} + \frac{1+\alpha}{T} = \frac{2(1+\alpha)}{T}, \qquad (12)$$

i.e. two times as large then bandwidth of signals BASK and BPSK.

The forming BFSK signals differs from forming MPSK signals by working of mapper and by that the reference frequencies of generators in DSB-SC modulators differ on the value $\Delta f/2$ from carrier frequency.

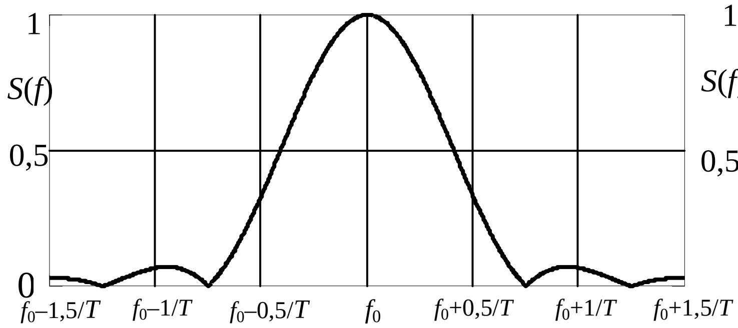


Figure 4 – Spectrum of MSK signal

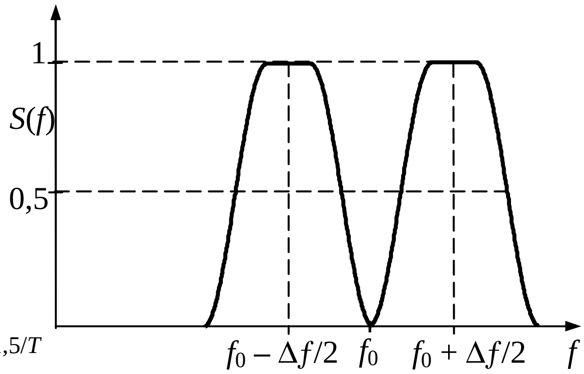


Figure 5 – Spectrum of BFSK signal with $\alpha = 0{,}6$, $\Delta f = 2(1+\alpha)f_N$

3. Questions

3.1 What is the aim of using the modulation in the telecommunication systems?

3.2 Give the definition of digital signal.

3.3 Give the definitions of digital modulation signals: MASK; MPSK; MFSK.

3.4 Why are the radio-frequency pulses with rectangular envelope not used for transmitting digital signals through communication channels? What form must pulse envelope have?

3.5 What are the forms of the spectra of MASK; MPSK; MFSK signals?

3.6 What are the multi-level signals for transmitting digital signals through communication channels used for?

3.7 What signals of digital modulations are one-dimensional, and what signals are two-dimensional?

4. Home task

4.1 Study the section "Digital types of modulation" with the compendium of lectures and main positions of this work. while studing this theme you must use the literature [1, p. 196…204, 231…234].

4.2 Given clock period: T= 50 ms. It is necessary to build the time diagrams of elementary radio pulses of frequency f_0 = 40 Hz for two cases: with rectangular envelope and with Nyquist pulse envelope.

Note. It is necessary to take into account that elementary radio pulse is the product of rectangular pulse of duration T or Nyquist pulse, and harmonic wave. In a case of Nyquist pulse it is possible to take a function

$$A(t) = \frac{\sin(\pi t/T)}{\pi t/T}.$$

Draw diagram of this function on interval (–4T, 4T).

4.3 Be ready to answer questions.

5 Laboratory task

5.1 Acquaintance with a virtual model on a workplace.

Start the program **1.5**, using the icon **TT(English)** on the desktop. Study scheme model, using the description in section 6 of this LW. Specify with the teacher the laboratory task performance plan.

5.2 Preparation of a virtual model.

It is necessary to set a digital signal. For that give a decimal number 128 + 10*N* (*N* is a number of laboratory stand) by binary number. The roll-off factor is equal $\alpha = 1 - 0{,}1\ N$.

5.3 Research of the form and spectrum of BASK and QASK signals as the functions of envelope form.

For this purpose it is necessary to set: type of modulation – BASK; envelope form is a rectangular pulse. You should fix in protocol one under one the time diagrams of the following signals: the digital signal; the output signal of mapper; the modulated signal. Also fix the spectral diagram of the modulated signal. After that it is necessary to set the second envelope form – the Nyquist pulse. Fix in protocol the time and spectral diagrams of the modulated signal.

The same research performs for the QASK signal.

In conclusions, on the basis of comparison of spectral diagrams, you should indicate the appropriateness of using the radio-frequency pulses with Nyquist pulse envelope and the appropriateness of using the multi-positional signals for decreasing the occupied frequency band.

5.4 Research of the form and spectrum of BPSK and QPSK signals as the functions of envelope form.

Repeat the researches, performed in it. 5.3, for the BPSK and QPSK signals. Compare the spectra of MASK and MPSK signals.

5.5 Research of the form and spectrum of BFSK signal as the functions of envelope form.

Repeat researches, completed in it. 5.3 and 5.4, for the MSK and BFSK signals. Compare the spectra of BASK, MSK and BFSK signals.

6 Description of laboratory model

The laboratory work is performed on a computer program in the HP VEE environment with using the virtual model. The structure scheme of model is shown on figure 6.

A model is universal modulator of digital modulated signals. It includes the digital signal generator with duration equals $8T_b$, signal symbols can be changed. Given bit duration is: T_b = 50 ms. Modulator consists of the followings blocks: mapper; shaping filters; carrier generators; two multipliers and adder. The setting of modulation type affects on the mapping code of an encoder and carrier generators and permits to set the followings types of modulation: BASK, QASK, BPSK, QPSK, and QFSK. The signals from two encoder outputs enter the filter inputs, shaping the radio pulse envelopes in the form of the Nyquist pulse. The scheme contains a switch, allowing to exclude the shaping filters from the scheme, so radio pulse have the rectangular envelope. The formed pulses are multiplied with carriers. Given carrier frequency f_0 is equal 40 Hz. In the case of BFSK the frequency deviation Δf is set in accordance with a formula (9), and in the case of MMS the frequency deviation $\Delta f = 0{,}5/T$. Model has oscillographs and spectrum analyser.

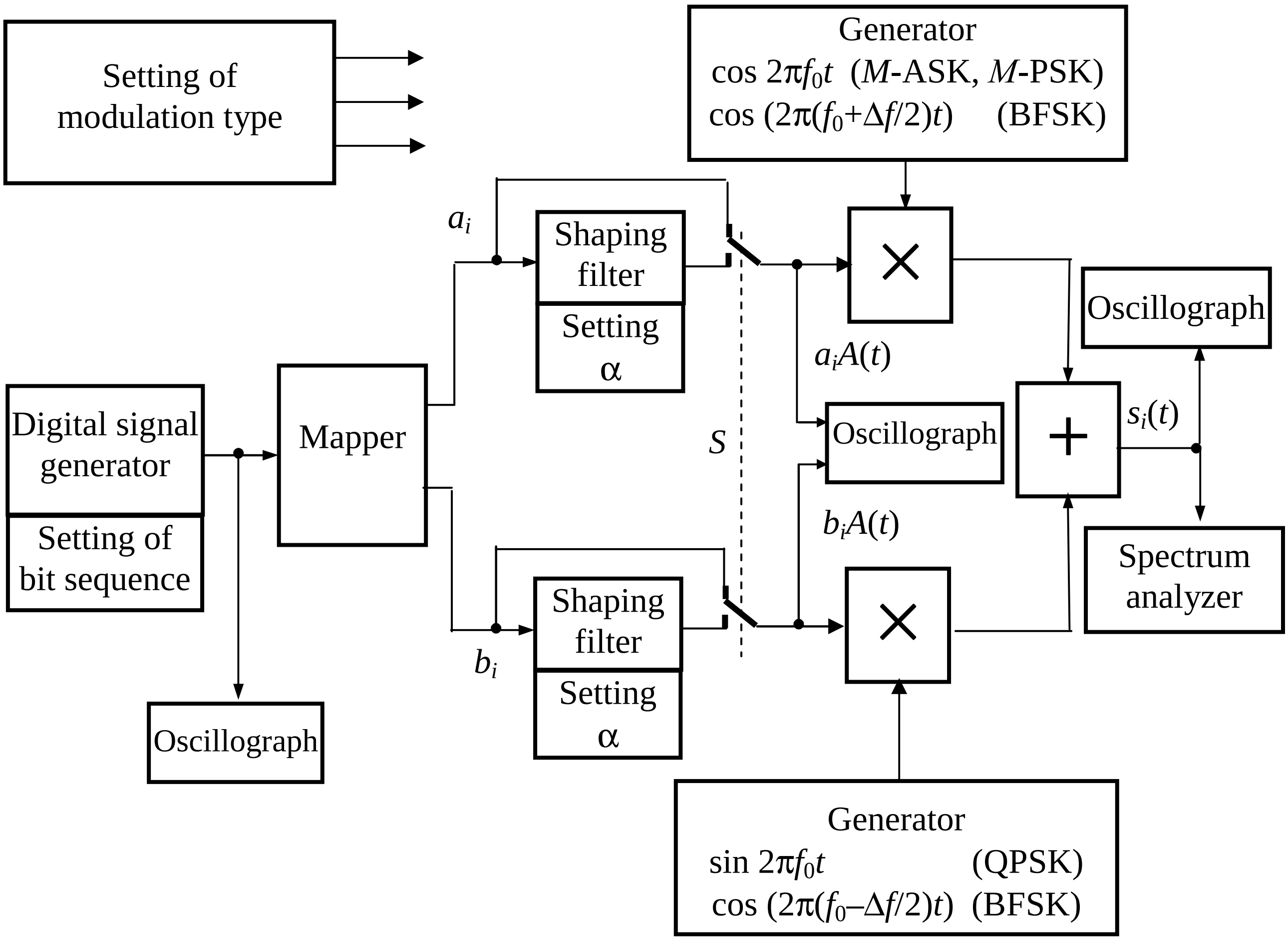


Figure 6 – Virtual model block diagram

7 Requirements to the report

7.1 Title of laboratory work.
7.2 Objectives of work.
7.3 Results of the home task processing.
7.4 The structure schemes of the every laboratory task processing.
7.5 The results of performing of LW items (oscillograms and spectrograms, with captions).
7.6 The conclusions on every item of task, in which it is necessary to make the analyses of the got results (coincidence of theoretical and experimental data, showing properties of signals, etc.).
7.7 The date, signature of student, visa of the teacher with mark.

Literature

1. **Скляр Б.** Цифровая связь. Теоретические основы и практическое применение. 2-е издание.: Пер. с англ. – М.: Издательский дом «Вильямс», 2003. – 1104 с.

7 METHODICAL GUIDELINES FOR FULFILLING INDIVIDUAL TASKS

IT № 1.1 Calculation of random process characteristics

Initial data:

The white Gaussian noise $N(t)$ (Volts) with the one-sided spectral power density N_0 on the input of low-pass filter (LPF) with given amplitude response (AR) $H(f)$, $0 \le f < \infty$ is given.

It is necessary to:

1. Write input data of your variant.
2. Find expression for the noise spectral power density $X(t)$ on the LPF output $G_X(f)$ and build the graph of this function.
3. Define average power of the noise $X(t)$.
4. Define the effective bandwidth Δf_{eff} of noise $X(t)$ and show it on the graph of the $G_X(f)$ function.
5. Find expression for the correlation function of noise $X(t)$ on the LPF output $K_X(\tau)$ and build the graph of this function.
6. Define the correlation time τ_c of noise $X(t)$ and show it on the graph of the $K_X(\tau)$ function.
7. Calculate the product of $\Delta f_{\text{eff}}\tau_c$.
8. Define probability that in the arbitrary time moment noise $X(t)$ will take on the value on the given interval $(x_1,\ x_2)$.
9. Give the list of used literature; there must be references on used literary source with pointing of subsections or numbers of pages in the text of the performed individual task.

Table 1– Given types of filter (the number of variant is determined by the two last number of your student's book number)

№ variant	Type of filter
00...24	Ideal LPF with AR $H(f) = \begin{cases} 1, & 0 \le f \le F_{\text{cut}}, \\ 0, & f > F_{\text{cut}}, \end{cases}$ where F_{cut} is the LPF cut off frequency
25...49	RC-filter with AR $H(f) = \dfrac{1}{\sqrt{1+(2\pi f\tau_{\text{f}})^2}}$, where τ_{f} is the LPF time constant
50…74	Butterworth filter with AR $H(f) = \dfrac{1}{\sqrt{1+(f/F_{\text{cut}})^{2n}}}$, where n is the filter order, let $n = 2$; F_{cut} is the filter cut off frequency
75...99	Gaussian filter with AR $H(f) = \exp(-a^2 f^2)$, where a – the coefficient determining the LPF AR slope

Table 2 – Given numerical values (the number of variant is determined by the last number of your student's book number)

№ variant	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
N_0, 10^{-6} V^2/Hz	0,1	5	2	1	40	10	200	100	5000	1000
F_{cut}, 10^5 Hz	100	4	20	40	1	6	0,3	0,8	0,02	0,1
τ_f, 10^{-6} s	0,04	0,6	0,2	0,06	2	0,4	7	3	100	20
a, 10^{-7} s	0,5	15	3	1,5	60	10	200	75	3000	600
x_1, V	$-\infty$	– 0,5	0	0	1	2	$-\infty$	2	4	0
x_2, V	1	0,5	∞	3	3	∞	0	4	∞	4

Methodical instructions to performance IT № 1.1

Look through [1, p. 133...145; 2, p. 49...60]. Next sequence of the Individual task № 1.1 performance is recommended.

1. Spectral power density of noise $X(t)$ on the LPF output is determined by expression

$$G_X(f) = G_N(f)H^2(f) = N_0H^2(f),$$

it is necessary to build the graph of the $G_X(f)$ function for the interval of frequency values from 0 to the value, at which $G_X(f) \ll G_X(0)$.

2. Average power of noise $X(t)$ is determined by the integral

$$P_X = \int_0^\infty G_X(f)df .$$

3. The effective bandwidth Δf_{eff} of noise $X(t)$ is determined

$$\Delta f_{eff} = \frac{1}{G_X(0)}\int_0^\infty G_X(f)df \quad \text{or} \quad \Delta f_{eff} = \frac{P_X}{G_X(0)},$$

value Δf_{eff} must be shown on the graph of the $G_X(f)$ function.

4. The correlation function of the noise $X(t)$ is determined

$$K_X(\tau) = \int_0^\infty G_X(f)\cos 2\pi f\tau\, df .$$

It is necessary to build the graph of the $K_X(\tau)$ function for the interval of values τ from 0 to the value, at which $|K_X(\tau)| \ll K_X(0)$. It is useful to check implementation of main properties of correlation function:

– $K_X(\tau)$ – is even function;
– $K_X(0) = P_X$, where P_X is average power of process;
– $K_X(0) \geq K_X(\tau)$.

5. Correlation time τ_c of the noise $X(t)$ is possible to define by one of the following methods:

– as a value of τ, when the $K_X(\tau)$ function first time takes on a zero value (it is comfortable in the case of ideal LPF);

– as a value of τ, when function $K_X(\tau) = 0,1K_X(0)$;

– as a result of calculation of integral

$$\tau_c = \frac{1}{K_X(0)}\int_0^\infty |K_X(\tau)| d\tau .$$

The value of τ_c must be shown on the graph of the $K_X(\tau)$ function;

6. Calculate the product $\Delta f_{eff}\tau_c$. The result of this product is a value of order 0,5.

7. To determine probability that in the arbitrary time moment noise $X(t)$ will take on the value on interval (x_1, x_2), it is necessary to use expression

$$P\{x_1 < X(t) \le x_2\} = F(x_2) - F(x_1),$$

where $F(x)$ is probability distribution function of the noise $X(t)$. If Gaussian process acts on the input of linear electric circuit, process on the output also has Gaussian probability distribution. For Gaussian processes the function of probability distribution is written down:

$$F(x) = 1 - Q\left(\frac{x - \overline{X(t)}}{\sigma_X}\right),$$

where $Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^\infty \exp\left(-\frac{t^2}{2}\right)dt$ is Q-function or addition to Gaussian probability distribution function;

$\overline{X(t)}$ – is the average value or the expectation of a noise $X(t)$ (in our task $\overline{X(t)} = 0$);

σ_X –root-mean-square deviation of a random process, it is determined as $\sigma_X = \sqrt{D[X(t)]}$;

$D[X(t)]$ – variance of a noise $X(t)$, as $\overline{X(t)} = 0$, then $D[X(t)] = P_X$.

In the absence Q-function table it is possible to take advantage of approximate formula:

$$Q(z) \cong 0,65 \exp[-0,44(z + 0,75)^2] \text{ when } z > 0;$$

$$Q(z) = 1 - Q(|z|) \text{ when } z < 0; \; Q(0) = 0,5; \; Q(\infty) = 0.$$

For the P_X, $K_X(\tau)$, and τ_c definition you can use following expressions:

$$\int_0^{F_{cut}} \cos 2\pi f\tau \, df = F_{cut}\frac{\sin 2\pi F_{cut}\tau}{2\pi F_{cut}\tau};$$

$$\int_0^\infty e^{-a^2x^2}\cos bx\,dx = \frac{\sqrt{\pi}}{2a}e^{-b^2/4a^2} \text{ when } a > 0;$$

$$\int_0^\infty \frac{\cos mx}{x^4 + 4a^4}dx = \frac{\pi e^{-ma}}{8a^3}(\sin ma + \cos ma);$$

$$\int_0^\infty \frac{\cos ax}{1 + x^2}dx = \frac{\pi}{2}e^{-|a|};$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax}.$$

Literature

1. **Стеклов В. К.,** Беркман Л.Н. Теорія електричного зв’язку: Підручник для студентів ВУЗів. За ред. В.К. Стеклова – К.: Техніка, 2006.

2. **Теория** электрической связи: Учебник для вузов / А.Г. Зюко, Д.Д. Кловский, В.И. Коржик, М.В. Назаров; Под ред. Д.Д. Кловского. – М.: Радио и связь, 1998.

IT № 1.2 Description and calculation of digital modulated signals characteristics

Initial data:

– two types of digital modulation (table 1);
– rate of modulating digital signal (table 2);
– roll-off factor of spectrum of the modulated signal (table 2);

It is necessary:

1. Write down initial data of your variant.
2. Draw on the same figure 2 time diagrams of:

a) realization of digital modulating signal (8–9 binary symbols – two last numbers given by the teacher variant, written in the binary numeration);

b) modulated signals of given modulation types; considering that, radio pulse envelop is rectangular.

3. Build signal constellations of given modulation types, on signal constellation point out a mapping code.
4. Write analytical expressions of channel symbols of given modulation types.
5. Consider that average energy of signals, contained while the transmission of one binary symbol, E_{b} = const; calculate for given modulation types minimal distance between channel symbols, expressed through E_{b}.

6 Calculate and draw the modulated signals amplitude spectrum for given modulation types; calculate bandwidth of signals of given modulation types and show it on the spectrogram.

7. Draw the functional diagram of modulators for given types of digital modulation and explain principles of their operation.
8. Formulate conclusions of the performed task; point out the advantages (or disadvantages) of given multi-level modulation type , in comparison with binary modulation type.
9. Give a list of the used literature; there must be references on used literary source with pointing of subsections or numbers of pages in the text of the performed individual task.

Table 1– Given types of modulation (the number of variant is determined by the last number of your student's book number)

№ var.	0	1	2	3	4	5	6	7	8	9
Digital mod.	BASK, QPSK	BASK, 8PSK	BASK, QASK	BFSK, QPSK	BFSK, 8PSK	BFSK, QASK	BPSK, QPSK	BPSK, QPSK	BPSK, 8PSK	BPSK, QASK

Table 2 – Given *R* and α (the number of variant is determined by the last but one number of your student's book number)

№ var.	0	1	2	3	4	5	6	7	8	9
R, kbits/s	9,6	19,2	24	32	64	128	256	384	512	2048
α	0,20	0,25	0,30	0,35	0,20	0,25	0,30	0,35	0,20	0,25

Methodical instructions of performance IT № 2.2

Data on signals of digital modulation see in methodical instructions to performance of laboratory work 1.5 (p. 18) and [1, p. 196…204, 231…234].

The mapping code should be a Gray code.

The amplitude spectrum of the modulated signal is described by Nyquist spectrum. The baseband Nyquist spectrum is defined by an expression

$$N(f) = \begin{cases} T, & 0 \le |f| \le (1-\alpha) f_{\mathrm{N}}, \\ 0{,}5T\left[1+\sin\left(\dfrac{\pi}{2\alpha}\left(1-\dfrac{|f|}{f_{\mathrm{N}}}\right)\right)\right], & (1-\alpha) f_{\mathrm{N}} < |f| < (1+\alpha) f_{\mathrm{N}}, \\ 0, & |f| \ge (1+\alpha) f_{\mathrm{N}}, \end{cases}$$

where $f_N = 1/T$ is Nyquist frequency;

T is clock period;

α is a roll-off factor of spectrum.

Literature

1. **Скляр Б.** Цифровая связь. Теоретические основы и практическое применение. 2-е издание.: Пер. с англ. – М.: Издательский дом «Вильямс», 2003. – 1104 с.

8 DICTIONARIES

English-Russian dictionary

access system	система доступа
accidental phase	случайная фаза
amplitude modulation	амплитудная модуляция
amplitude modulation factor	коэффициент амплитудной модуляции
analog (амер.), analogue (англ.)	аналоговый, аналог
average power (of a process)	средняя мощность (процесса)
average value	среднее значение
band	полоса частот
bandpass signals	полосовые сигналы
bandwidth (of signal, process)	ширина спектра (сигнала, процесса)
baseband signal	первичный сигнал
bit rate of a signal	скорость цифрового сигнала
block diagram	структурная схема
BPSK (binary phase shift keying)	ФМ-2 (двоичная фазовая модуляция)
broadcasting	вещание
carrier	несущее колебание (несущая)
carrier frequency	частота несущего колебания (несущей)
code block, codeword	кодовая комбинация (блок), кодовое слово
continuous (time-continuous) signal	непрерывный (непрерывный по времени) сигнал
correlation characteristics	корреляционные характеристики
correlation meter	коррелометр
correlation time	интервал корреляции
decomposition factors	коэффициенты разложения
delay, delay time	задержка, время задержки
determined signals	детерминированные сигналы
deterministic signal	детерминированный сигнал
deviation	девиация
digital signal	цифровой сигнал
discrete signal	дискретный сигнал
distortion (of signal)	искажение, изменение формы (сигнала)
double-sideband-suppressed-carrier modulation	балансная модуляция

duplex	двусторонняя передача сообщений
duration (infinite/finite)	длительность (бесконечная/конечная)
duration of pulse	длительность импульса
energy spectral density	спектральная плотность энергии
ensemble	ансамбль, совокупность
envelope	огибающая
ergodic	эргодический
error control code	корректирующий код
even function	четная функция
expectation	математическое ожидание
fluctuation noise	флуктуационная помеха
Fourier series	ряд Фурье
Fourier transformation	преобразование Фурье
full duplex	полнодуплексный
half duplex	полудуплексный
harmonic oscillation, harmonious waveform	гармоническое колебание
Hilbert transform	преобразование Гильберта
initial phase	начальная фаза
inphase or cosine component	синфазная или косинусная составляющая
intersymbol interference	межсимвольная интерференция
joint probability density	совместная плотность вероятности
link	соединение
lower sideband of frequencies	нижняя полоса частот
mapping code	модуляционный код
mapper	кодер модуляционного кода
MAPSK – *M*-ary amplitude-phase modulation;	АФМ-*М*
M-ary amplitude modulation	*М*-ичная амплитудная модуляция
mean-square error	среднеквадратическая ошибка
MFSK – *M*-ary frequency modulation	ЧМ-*М*
middle frequency	средняя частота
modulating signal	модулирующий сигнал
MPSK – *M*-ary phase modulation	ФМ-*М* – *М*-ичная фазовая модуляция
MQAM – *M*-ary quadrature-amplitude modulation	КАМ-*М*

MSK modulation – minimum shift keying modulation	модуляция минимального сдвига
mutual correlation	взаимная корреляция
narrow spectrum	узкий спектр
narrow-band signal	узкополосный сигнал
node	узел сети
Nyquist pulse	импульс Найквиста
one-dimensional	одномерный
one-way message transfer	односторонняя передача сообщения
periodic signal	периодический сигнал
power spectral density function	спектральная плотность мощности
probability density function	плотность вероятности
probability distribution function	функция распределения вероятности
pulse energy	энергия импульса
QPSK (quaternary phase shift keying)	ФМ-4 (четверичная фазовая модуляция)
quadrature or sinus component	квадратурная или синусная составляющая
quadrature splitter	квадратурный расщепитель
random process	случайный процесс
realization of a process	реализация процесса
realizations of random process	реализации случайного процесса
recipient	получатель
rectangular radio pulse	радиоимпульс с П-образной огибающей
rectangular video pulse	П-импульс
reliability	достоверность
Rayleigh probability distribution	Релеевское распределение вероятности
roll-off factor	коэффициент ската
root-mean-square deviation	среднеквадратическое отклонение
sampling	дискретизация
sequence	последовательность
shaping filter	формирующий фильтр
shifted on frequency	сдвинут по частоте
signalling alphabet	сигнальный алфавит
signal constellation	сигнальное созвездие
signal points	точки сигнального созвездия
simplex transfer	только односторонняя передача сообщения

single-sideband modulation (SSB)	однополосная модуляция (ОМ)
spectrum	спектр
spectral spreading	расширение спектра
stationary	стационарный
statistical dependence	статистическая зависимость
stochastic signals	стохастические сигналы
symbol interval	тактовый интервал
symbol rate	символьная скорость, скорость модуляции
terminal equipment	оконечное оборудование
transducer	датчик
two-way message transfer	двусторонняя передача сообщений
ultimate values	крайние значения аргумента
uncorrelated	некоррелированные
uniform distributing	равномерное распределение
upper sideband of frequencies	верхняя боковая полоса частот
variance	дисперсия
waveform	колебание
Wiener-Khinchin theorem	теорема Хинчина-Винера

Russian-English dictionary

амплитудная модуляция	amplitude modulation
аналоговый, аналог	analog (амер.), analogue (англ.)
аналоговый (непрерывный первичный) сигнал	analogue signal
ансамбль, совокупность	ensemble
АФМ-*M* (*M*-ичная амплитудно-фазовая модуляция)	MAPSK – *M*-ary amplitude-phase modulation;
балансная модуляция	double-sideband-suppressed-carrier modulation
верхняя боковая полоса частот	upper sideband of frequencies
вещание	broadcasting
взаимная корреляция	mutual correlation
гармоническое колебание	harmonic oscillation, harmonious waveform
датчик	transducer
двусторонняя передача сообщений	duplex
двусторонняя передача сообщений	two-way message transfer
девиация	deviation
детерминированные сигналы, детерминированный сигнал	determined signals, deterministic signal
дискретизация	sampling
дискретный сигнал	discrete signal
дисперсия	variance
длительность (бесконечная/конечная)	duration (infinite/finite)
длительность импульса	duration of pulse
достоверность	reliability
задержка, время задержки	delay, delay time
импульс Найквиста	Nyquist pulse
интервал корреляции	correlation time
искажение, изменение формы (сигнала)	distortion (of signal)
КАМ-*M* (*M*-ичная квадратурная амплитудная модуляция)	MQAM – *M*-ary quadrature-amplitude modulation
квадратурная или синусная составляющая	quadrature or sinus component
квадратурный расщепитель	quadrature splitter
кодовая комбинация (блок), кодовое слово	code block, codeword

колебание	waveform
корректирующий код	error control code
коррелометр	correlation meter
корреляционные характеристики	correlation characteristics
коэффициент амплитудной модуляции	amplitude modulation factor
коэффициент ската	roll-off factor
коэффициенты разложения	decomposition factors
крайние значения аргумента	ultimate values
математическое ожидание	expectation
межсимвольная интерференция	intersymbol interference
М-ичная амплитудная модуляция	*M*-ary amplitude modulation
модулирующий сигнал	modulating signal
модуляционный код	mapping code
модуляция минимального сдвига	MSK modulation – minimum shift keying modulation
начальная фаза	initial phase
некоррелированные	uncorrelated
непрерывный (непрерывный по времени) сигнал	continuous (time-continuous) signal
несущее колебание (несущая)	carrier
нижняя полоса частот	lower sideband of frequencies
огибающая	envelope
одномерный	one-dimensional
однополосная модуляция (ОМ)	single-sideband modulation (SSB)
односторонняя передача сообщения	one-way message transfer
оконечное оборудование	terminal equipment
первичный сигнал	baseband signal
периодический сигнал	periodic signal
П-импульс	rectangular video pulse
плотность вероятности	probability density function
полнодуплексный	full duplex
полоса частот	band
полосовые сигналы	bandpass signals
полудуплексный	half duplex
получатель	recipient
последовательность	sequence

преобразование Гильберта	Hilbert transform
преобразование Фурье	Fourier transformation
равномерное распределение	uniform distributing
радиоимпульс с П-образной огибающей	rectangular radio pulse
расширение спектра	spectral spreading
реализация процесса	realization of a process
Релеевское распределение вероятности	Rayleigh probability distribution
ряд Фурье	Fourier series
сдвинут по частоте	shifted on frequency
сигнальное созвездие	signal constellation
сигнальный алфавит	signalling alphabet
символьная скорость, скорость модуляции	symbol rate
синфазная или косинусная составляющая	inphase or cosine component
система доступа	access system
скорость цифрового сигнала	bit rate of a signal
случайная фаза	accidental phase
случайный процесс	random process
совместная плотность вероятности	joint probability density
соединение	link
спектр, спектры	spectrum, spectra
спектральная плотность мощности	power spectral density function
спектральная плотность энергии	energy spectral density
среднее значение	average value
среднеквадратическая ошибка	mean-square error
среднеквадратическое отклонение	root-mean-square deviation
средняя мощность (процесса)	average power (of a process)
средняя частота	middle frequency
статистическая зависимость	statistical dependence
стационарный	stationary
стохастические сигналы	stochastic signals
структурная схема	block diagram
тактовый интервал	symbol interval
теорема Хинчина-Винера	Wiener-Khinchin theorem
только односторонняя передача сообщения	simplex transfer

точки сигнального созвездия	signal points
узел сети	node
узкий спектр	narrow spectrum
узкополосный сигнал	narrow-band signal
флуктуационная помеха	fluctuation noise
ФМ-*М* – *М*-ичная фазовая модуляция	MPSK – *M*-ary phase modulation
формирующий фильтр	shaping filter
функция распределения вероятности	probability distribution function
цифровой сигнал	digital signal
частота несущего колебания (несущей)	carrier frequency
четная функция	even function
ЧМ-*М*	MFSK – *M*-ary frequency modulation
ширина спектра (сигнала, процесса)	bandwidth (of signal, process)
энергия импульса	pulse energy
эргодический	ergodic

Education publication

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COMMUNICATION SIGNALS

Education manual