UKRAINIAN STATE COMMITTEE OF COMMUNICATIONS AND INFORMATIZATION

ODESSA NATIONAL ACADEMY OF TELECOMMUNICATIONS after A. S. POPOV

Department of physics of optical communications

PHYSICS

Module 2. Electromagnetic oscillations and waves

OSCILLATIONS AND WAVES

PART 3: LABORATORY WORKS

for bachelor training of educational area 0924 –“Telecommunications”

APPROVED
by the Faculty Council
Protocol № 5
from 24.12.2009

ODESSA 2010

UDK 538 (075.8)

Publication plan 2010.

Writers: assoc. prof. Gorbachov V.E, instructor Kardashev K.D.

The following methodical guide is about section “Electromagnetic oscillations and waves” of physics course for telecommunications technician. Four laboratory works allow students to learn basics of electrical engineering and measuring technique applied to determine main characteristics of oscillations systems and waves. It contains sufficient theoretical information combined with detailed descriptions of applied electromagnetic equipment construction and measuring techniques.

Recommended for students of TE-group, educational area 0924 – “Telecommunications”.

CONFIRMED
at the Department session
Protocol № 4
from 10.11.2009

MODULE STRUCTURE

Module № 2. „ Electromagnetic oscillations and waves" – 72 hours total.

Lectures – 16 hrs, practical trainings – 0 hrs, labs – 16 hrs, self-studies – 33 hrs.

LIST OF LABORATORY WORKS

Number of lessons	Denomination of laboratory work, code of the work	Hours
Module № 1		
1	4-1. Investigation of harmonic oscillations of mechanical systems.	2
2	Calculation of parameters of harmonic electrical oscillations.	2
3	4-3. Investigation of damped oscillations of mathematical pendulum.	2
4	Calculation of parameters of damping electrical oscillations.	2
5	4-6. Investigation of driven oscillations in oscillation circuit.	2
6	Calculation of current and voltage in RLC-circuit.	2
7	5-1. Finding of frequency of oscillations of vibrator by Meldje method.	2
8	Calculation of parameters of electromagnetic waves.	2

INTRODUCTION

All laboratory works are provided in a frontal way, i.e. all group makes the same laboratory work at the same time.

Appropriate homework must forego to work in a lab. The homework contains self-studying of theory and methodology of work accomplishment, preparation of protocol which includes experimental facility’s schematic drawing, equipment table's drawing, measurement table's drawing, a list of working formulae with description of all quantities which are in, list of control questions' answering.

The allowance to performing of laboratory work will be had only those of students who have fulfilled homework and have positive result on express mini-quiz in a lab.

Content of the reports for all laboratory works has to be the following:

1) ***Title*** and number of laboratory work.
2) ***Goal*** of the work.
3) Laboratory research facility's ***scheme***.
4) ***Equipment*** table.
5) ***Equations*** for calculation with decryption of all quantities in.
6) Standard ***table of measurements*** for each measured quantity. It has to be checked and verified by an instructor.
7) Experimental ***data processing*** (write one for many similar)
8) Standard form of ***result*** (confidence interval and relative error or a graphic result)
9) ***Conclusion***
10) ***Date, name*** of a student.

Besides this guide is recommended to use literature from bibliography given at the end of this guide.

WORK 4-1
EXPLORING of HARMONIC OSCILLATIONS

1 Goal of the work:

1. Studying physical pendulum undamped oscillations description method.
2. Studying a moment of inertia and equivalent length of physical pendulum determination method.

2 Main concepts

Oscillations (including vibrations and variations) are processes changes of state that are repeated more or less regularly with time. There are many kinds of oscillations, not only mechanical. Any physical quantity makes oscillations if it repetitively varies in opposite directions near some of its value.

A motion of a mechanical system near its equilibrium position, during which the system passes through an equilibrium position over and over in opposite directions is being termed ***mechanical oscillations***.

A state of a system, left by its own, in which it can stay indefinitely long without any motion, is called ***equilibrium state***.

Free oscillations **(*simple harmonic and dumped oscillations*)** are ones that occur in absence of externally applied variable action on the system. ***Forced oscillations*** are ones set up in system as a result of variable external effects (periodic external force in mechanical systems or generator's alternating emf in electromagnetic systems).

Simplest model of oscillatory system is called a ***harmonic oscillator***. When the oscillatory system not looses energy, it is being described by homogeneous second order differential equation that is termed ***equation of harmonic oscillator in differential view***:

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0 \qquad \text{or} \qquad \ddot{x} + \omega_0^2 x = 0. \qquad (1)$$

A motion of such system is termed simple harmonic and the oscillating physical quantity x varies under sine or cosine law. Thus general solution for this differential equation (1) will be ***equation of simple harmonic oscillations***:

$$x(t)=A\cdot\cos(\omega_0 t+\varphi_0) \qquad \text{or} \qquad x(t)=A\cdot\sin(\omega_0 t+\varphi_0+\pi/2), \qquad (2)$$

where $x(t)=f(t)$ – value of oscillating quantity at the time t is called in mechanical systems as ***displacement*** (relatively to equilibrium position).

Constant A, which is equal to the largest absolute value of oscillating physical quantity, is called the ***amplitude of oscillations***. Amplitude measurement unit is a same, as measurement unit of a quantity x.

Expression

$$\omega_0 t+\varphi_0 = \Phi(t), \qquad (3)$$

which defines the magnitude of quantity x at given moment of time, is called the ***phase of oscillation***.

SI measurement unit for phase is radian - [*rad*].

At initial moment of time (t =0) phase Φ is equal to ***initial phase*** φ_0:

$$\Phi(t=0)=\varphi_0,$$

When we deviate the pendulum at the first time to the left, then

$$x=x(t=0)=-A \qquad \text{and} \qquad \varphi_0=\pi, \tag{4}$$

but when we deviate the pendulum at the first time to the right, then

$$x_0=x(t=0)=+A \qquad \text{and} \qquad \varphi_0=0. \tag{4a}$$

Differentiation of (3) gives:

$$\omega_0=\frac{d\varphi}{dt}, \tag{5}$$

hence, ***cyclic frequency*** is the time rate of change (velocity of variation) of a phase that measurement unit is radians per second - [*rad/s*].

Parameters of oscillations:

The period of the oscillations T is the smallest interval of time after which repeat all values of physical quantities characterized oscillatory motion. One full oscillation completes by the time of period T.

Frequency is defined as a number of oscillations per unit time:

$$\nu=\frac{1}{T} \tag{6}$$

SI measurement unit for frequency is Hertz – [*Hz*].

One full oscillation's ***phase cycle*** corresponds to 2π radians (as period of harmonic function). Then ***cyclic frequency*** (5) of oscillations is the number of ***phase cycles*** per seconds:

$$\omega=2\pi\nu=\frac{2\pi}{T} \tag{7}$$

SI measurement unit for cyclic frequency is radians per second – [*rad/s*].

Then the period of the oscillations

$$T=\frac{2\pi}{\omega} \tag{8}$$

SI measurement unit for period is second – [*s*].

2.1 Harmonic oscillations of physical pendulum

Harmonic oscillations are the type of ***free oscillations***, which the system performs under activity only of restoring force. The ***restoring force*** is a force acted to the centre of gravity of a system and always directed to equilibrium position. The restoring force as well is termed as ***quasi-elastic force***, because for its describing introduce the elasticity equation:

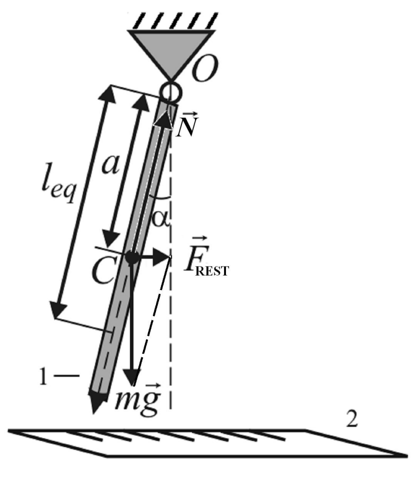


Figure 1 – Restoring force of physical pendulum

$$F_{\text{REST}} = -k_{qe} \cdot x, \tag{9}$$

where x – displacement from equilibrium position; k_{qe} – quasi-elastic coefficient.

Negative sign on the right hand side of the equation (9) means that the restoring force always acts in the opposite direction of the displacement. The role of restoring force is fulfilled usually by the total of gravity force and supporting or spring force.

Natural system is an ideal physical model of system with no any energy losses. In the real-world systems there are always act different kind of damping forces, which increase period and decrease amplitude of oscillations (see Laboratory work № 4-3). Regime of oscillations without of losing energy is termed ***eigenmodes*** and frequency of such oscillations – ***eigenfrequency***. Parameters of eigenmodes of oscillations are being identified by 0 index.

Let's consider natural mechanical oscillations of physical pendulum (Fig.1). ***Physical pendulum*** is a rigid body suspended from a fixed point in it, free to pivot about some horizontal axis through that point under the force of gravity.

The oscillations description method is in building and solving of differential equations for appropriate motions. If the pendulum shifted from its equilibrium position then there will appear a moment of restoring force (Fig.1):

$$M = F_{\text{REST}} \cdot a = -mga \cdot \sin\alpha, \tag{10}$$

here m – mass of a pendulum, a – distance from the pivot point to center of gravity C of the pendulum, α – small enough angle of deflection from equilibrium.

The moment equation gives:

$$M = J\varepsilon, \tag{11}$$

here $\varepsilon = \dfrac{d^2\alpha}{dt^2}$ – angular acceleration, so

$$-\ mga\ \sin\alpha = J\varepsilon,$$

where J – moment of inertia of the physical pendulum.

Moment of inertia J is a physical ***analog of the mass*** of a rotating rigid body. For example, moment of inertia of the rotating point mass m apart on distance l from an axis is

$$J = m \cdot l^2. \tag{12}$$

Then SI measurement units for moment of inertia is kilogram-square meter – [$kg \cdot m^2$]. We note that such oscillating point mass m on weightless suspension of length l is being termed as ***simple (mathematic) pendulum***.

As α is small we can replace $\sin\alpha$ by α in moment equation(10):

$$-mga\alpha = J\frac{d^2\alpha}{dt^2},$$

from here we obtain ***differential equation of oscillations of physical pendulum***:

$$\frac{d^2\alpha}{dt^2} + \frac{mga}{J}\alpha = 0 \qquad \text{or} \qquad \frac{d^2\alpha}{dt^2} + \omega_0^2 \cdot \alpha = 0 \tag{13}$$

Thus, oscillatory motion of physical pendulum is being described by the homogeneous second order differential equation.

General solution for this differential equation will be ***equation of oscillations of physical pendulum***:

$$\alpha = \alpha_m \cos(\omega_0 t+\varphi_0) \tag{14}$$

Coefficient near α in (12) is squared cyclic eigenfrequency ω_0, then

$$\omega_0 = \sqrt{\frac{mga}{J}}\,. \tag{15}$$

Taking in to account (8) we'll have a eigenperiod of the physical pendulum

$$T_0 = 2\pi\sqrt{\frac{J}{mga}} \tag{16}$$

In special case (12), for simple (mathematic) pendulum $J = m \cdot l_{eq}^2$ and the eigenperiod will be

$$T_0 = 2\pi\sqrt{\frac{l_{eq}}{g}}\,. \tag{17}$$

Comparison of (16) and (17) shows that it is possible to pick such simple (mathematic) pendulum with length

$$l_{eq} = \frac{J}{ma}, \tag{18}$$

which will have same period as a given physical has. Such length l_{eq} is called ***equivalent length*** of physical pendulum.

Moment of inertia of physical pendulum we can determine using formula (14):

$$J = \frac{T_0^2 mga}{4\pi^2}\,. \tag{19}$$

<u>3 Description of laboratory research facility and methodology of measurements</u>

Devices and outfits: console with a pendulum, millimetre scaled ruler, stopwatch.

In a given work we observe natural oscillations of physical pendulum. Such pendulum consists from a metallic rod, suspended by one of its ends and free to rotate in a vertical plane. Mass of a pendulum is indicated on a metallic rod in the form of numerical imprint. Length of a pendulum is being measured by a ruler once, but with pinpoint accuracy (important!). Instrumental accuracy – 0,001m.

Use small, about 3^{O} – 5^{O}, deflections of a pendulum from its equilibrium state to make it oscillate. Only in this case oscillations will be harmonic.

For small angle deflection α it is possible to calculate value of amplitude via an approximate equation:

$$A = l\sin\alpha \approx l\alpha = \frac{l\pi \cdot \alpha^{o}}{180^{o}}, \tag{20}$$

here l – length of a pendulum.

Direct measurements of period. The period is a small quantity, then for increasing an accuracy of its determination it is possible to measure time of 10 oscillations.

When pendulum appears in one of its edge positions start stopwatch and count 10 complete oscillations (cycles). Use 0.01 precision when calculating average value of period.

From average value of period it is possible to calculate a cyclic frequency of oscillations. From initial position according (4) and (4a) it is possible to calculate initial phase of oscillations and write equation of oscillations of pendulum with numerical coefficients A,ω_0, φ_0.

Indirect measurement of moment of inertia. A task for students will be determination of moment of inertia of a given physical pendulum using its oscillations. Center of gravity of uniform rod is on the half of its length, so

$$a = l/2 \,. \tag{21}$$

Substituting (21) in (19) it is possible to obtain an ***experimental value*** of moment of inertia:

$$J_{\text{EXP}} = \frac{T_0^2 mgl}{8\pi^2}. \tag{22}$$

Free fall acceleration is 9.81 m/s^2.

From theoretical mechanics it is possible to calculate a ***theoretical value*** of moment of inertia:

$$J_{\text{THEOR}} = \frac{ml^3}{3}. \tag{23}$$

<u>4 Data processing</u>

For representation of the result of **<u>direct measurements</u>** of quantity x it is necessary:

1) Obtain the sequence of measured values x_1, x_2, x_3, ..., x_n and write result of these n measurements in a second column of Table of measurements. In a first column of Table of measurements write an ordinal number of measurement.
2) Calculate the ***average value*** of measurand:

$$< x >= \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \,. \tag{24}$$

3) Find an ***abmodality*** each measurement and write result in a third column of Table of measurements:

$$\Delta x_1 =< x > - x_1; \quad \Delta x_2 =< x > - x_2; \quad ... ; \quad \Delta x_n =< x > - x_n \,. \tag{25}$$

4) Square each abmodality in a fourth column of Table of measurements and summarize them:

$$\sum_{i=1}^{n}(\Delta x_i)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + \ldots + (\Delta x_n)^2 . \tag{26}$$

5) Find a ***statistical absolute error*** Δx_{ST} of measurements from Student's equation:

$$\Delta x_{\text{ST}} = t_{\alpha,n} \cdot \sqrt{\frac{\sum_{i=1}^{n}(\Delta x_i)^2}{n(n-1)}} \quad . \tag{27}$$

where α – confidence probability; n – number of measurements; $t_{\alpha;n}$ – Student's coefficient.

6) If it is not identified an absolute instrumental error Δx_{DEV} on measuring tool, it is necessary to find a ***device absolute error*** of measurements from accuracy class β of electrical measuring instrument:

$$\Delta x_{\text{DEV}} = \frac{\beta \cdot x_{\max}}{100}, \tag{28}$$

where $x_{\max}$ – grid limit.

7) Find a ***total absolute error*** of measurements

$$\Delta x = \sqrt{(\Delta x_{\text{ST}})^2 + (\Delta x_{\text{DEV}})^2} \tag{29}$$

7) Calculate ***relative error*** of measurements:

$$\delta = \frac{\Delta x}{< x >} \; . \tag{30}$$

8) ***Final result*** should be represented by a ***confidence interval*** and ***relative error***:

$$x = (< x > \pm \Delta x)_6 = (\ \ldots\ \pm \ldots\ )_{0.95}; \qquad \delta_x\,\% = \frac{\Delta x}{< x >} \cdot 100\% = \ldots\ \%. \tag{31}$$

For representation of the result of **indirect measuring** of quantity y it is necessary:

1) Calculate the ***average value*** of measurand $<y>$ by formula from average values of known quantities $<a>$, $<b>$, $<c>$, for example:

$$< y >= \frac{8 < a >^4 \cdot \sqrt[3]{< b >^2}}{7 < c >^5} . \tag{32}$$

2) Calculate ***relative error*** of measurand δ_y from relative errors of known quantities δ_a, δ_b, δ_c by formula that should be gained accordingly to this example:

$$\delta_y = \sqrt{(4\delta_a)^2 + \left(\frac{2}{3}\delta_b\right)^2 + (5\delta_c)^2} = \sqrt{\left(4\frac{\Delta a}{< a >}\right)^2 + \left(\frac{2}{3}\frac{\Delta b}{< b >}\right)^2 + \left(5\frac{\Delta c}{< c >}\right)^2}, \tag{33}$$

where Δa, Δb, Δc – absolute errors of known quantities; $<a>$, $<b>$, $<c>$ – its average values.

3) Find an ***absolute error*** of measurand

$$\Delta y = <y> \cdot \delta_y . \tag{34}$$

4) ***Final result*** should be represented by a ***confidence interval*** and ***relative error***:

$$y = (\langle y \rangle \pm \Delta y)_6 = (\; ... \; \pm \; ... \;)_{0.95}; \qquad \delta_y\,\% = \delta_y \cdot 100\% = \; ... \; \%. \qquad (35)$$

5 Work execution order and experimental data analysis

1. Make a direct measurements of ***length*** of pendulum l, with ruler accuracy. Write ***mass*** of a pendulum from the numerical imprint on metallic rod, with accuracy $0{,}001 kg$.
2. Make a direct measurements of ***time*** of 10 complete oscillations with help of stopwatch and write obtained data in a table of measurements. Repeat it direct measurement five times.
3. Determine ***average value*** of a period (24), abmodality (25), and sum of squares of abmodalities (26), ***absolute*** (29) and ***relative*** (30) ***errors*** of a period. Represent a final result of direct measurements of period in (31) view.
4. For 3^0 - 4^0 angular deflection from equilibrium state determine ***amplitude*** of oscillations A in metres according (20). Determine ***cyclic frequency*** ω_0 according to (7), ***initial phase*** φ_0 according to (4) and (4a). Write ***equation of oscillations*** of linear displacement of pendulum in (2) view in metres with numerical value of coefficients of A, ω_0 and φ_0.
5. Calculate (make an indirect measurement) the ***moment of inertia*** of a pendulum using two methods: experimentally based J_{EXP} – using formula (22), and theoretical based: using formula (23). Compare these two quantities. Calculate relative (33) and absolute (34) errors of experimental moment of inertia. Represent a final result of indirect measurement of experimental moment of inertia in (35) view.
6. Calculate average value of ***equivalent length*** l_{eq} of physical pendulum (18).

6 Control questions

1. Which motion we call periodic?
2. What are physical and simple mathematic pendulums?
3. What is period, frequency, cyclic frequency? How do they related?
4. What are harmonic oscillations? Under which condition free oscillations of any mechanical system became natural (harmonic)?
4. What is restoring (quasi-elastic) force? How does it appear in a case of physical pendulum?
5. Write an equation of restoring (quasi-elastic) force and its moment dependence from angle of deflection from equilibrium state.
6. What is necessary to do, when oscillations description method is used?
7. Formulate a differential equation of physical pendulum oscillations.
8. Set up a formula of period of physical pendulum.
9. Obtain an expression of moment of inertia of physical pendulum.
10. What is equivalent length of physical pendulum?
11. How free fall acceleration and mass affects on a period of physical pendulum?

7 Content of the report

Laboratory work № 4-1

I. Home work

(answer on control questions from p.11).

...

II. Laboratory work № 4-1 implementation protocol.

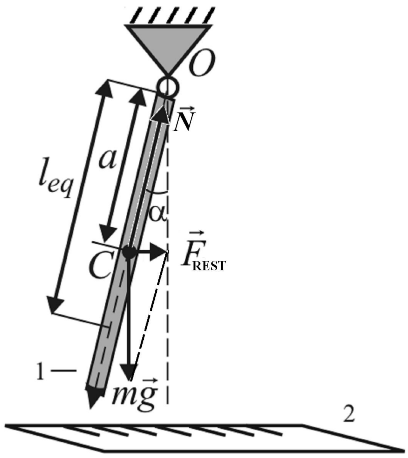
1) Topic: **EXPLORING of HARMONIC OSCILLATIONS.**

2) Goal:

1. Studying physical pendulum undamped oscillations description method.

2. Studying moment of inertia and equivalent length of physical pendulum determination method.

3) Scheme of laboratory research facility:



1 – physical pendulum;
2 – ruler;
O – pivot point;
C – center of gravity;
l_{eq} – equivalent length;
α – angle of deflection;
a – distance between pivot point (axis of rotation) and center of gravity;
$\vec{F}_{REST}$ *– quasi-elastic force;*
$\vec{N}$ *– supporting force;*
$m\vec{g}$ *–gravity force.*

4) Table of measuring instruments:

№	Name	Type	Serial №	Grid limit	Grid unit	Absolute error
1.	Stopwatch	УХЛ-42		99,99 s	0,01 s	0,01 s
2.	Ruler	У		1000 mm	1mm	1mm

5) Equations for calculation:

1. Statistical absolute error for direct measurements of period:

$$\Delta T_{ST} = t_{\alpha,n} \cdot \sqrt{\frac{\Sigma(\Delta T_i)^2}{n(n-1)}},$$

where $\alpha = 0{,}95$ – *confidence probability;* $n = 5$ – *number of measurements;* $t_{0,95\,;\,5} = 2{,}77$ – *Student's coefficient.*

Total absolute error of period

$$\Delta T = \sqrt{(\Delta T_{ST})^2 + (\Delta T_{DEV})^2},$$

where $\Delta T_{DEV} = 0{,}01s$ – *absolute instrumental error of stopwatch (see Table of measuring instruments).*

2. Amplitude of oscillations

$$A = l\sin\alpha \approx l\alpha = \frac{l\pi \cdot \alpha^{\circ}}{180^{\circ}},$$

here l *– length of a pendulum;* $\alpha = 5^{\circ}$ *– angle of deflection.*

Cyclic eigenfrequency of oscillations

$$\omega_0 = \frac{2\pi}{<T>},$$

where $<T>$ *– average value of period of oscillations.*

Initial phase of oscillations:

$\varphi_0 = \pi$, *when initial deflection to the left and* $x=x(t=0)= -A$;

$\varphi_0 = 0$, *when initial deflection to the right and* $x=x(t=0)= +A$.

Equation of oscillations of physical pendulum:

$$x(t)=A\cos(\omega_0 t+\varphi_0);$$

where x *– linear displacement of pendulum;* t *– time.*

3. Experimentally determined by indirect measurement an average value of moment of inertia:

$$< J_{\text{EXP}} > = \frac{< T >^2 mgl}{8\pi^2},$$

where m *– mass of a pendulum*; $g = 9.81$ m/s^2 *acceleration due to gravity*; l *– length of a pendulum.*

Absolute error for indirect measurement of moment of inertia:

$$\Delta J_{\text{EXP}} = < J_{\text{EXP}} > \cdot \delta_J,$$

where δ_J *– relative error for indirect measurement of moment of inertia*:

$$\delta_J = \sqrt{\left(2\frac{\Delta T}{< T >}\right)^2 + \left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta l}{l}\right)^2},$$

here $\frac{\Delta m}{m}$, $\frac{\Delta l}{l}$, $\frac{\Delta T}{< T >}$ *– relative errors for mass, length and period of the pendulum;* Δm, Δl, ΔT *– absolute errors for mass, length and period of the pendulum.*

4. Theoretically determined a value of moment of inertia:

$$J_{\text{THEOR}} = \frac{ml^2}{3}.$$

5. Average value of equivalent length of the pendulum:

$$l_{eq} = \frac{2 < J >}{ml}.$$

6) ***Table of measurements***

$m =$ … kg; Δm =0,001 kg; $l =$ … m; $\Delta l = 0{,}001$ m;

№	t_i, s	T_i, s	ΔT_i, s	$(\Delta T_i)^2$, s^2
1.				
2.				
3.				
4.				
5.				
	average value $<T>=$	…	$\Sigma(\Delta T_i)^2 =$	…

7) ***Data processing:***

…

8) ***Final results:***

1. T=($<T>$ ± $\Delta T)_\alpha$ = (... ± …)$_{0.95}$ s, $\delta_{T\%} = \dfrac{\Delta T}{< T >} \cdot 100\% =$ … %.
2. $x(t)$ = … ·cos(...t +...) m;
3. $J_{\text{EXP}} = (< J > \pm \Delta J)_\alpha$ = (... ± …)$_{0.95}$ $kg{\cdot}m^2$, $\delta_{J\%} = \delta_J \cdot 100\%$= … %;
4. J_{THEOR} = … $kg{\cdot}m^2$;
5. $l_{\,eq}$ = … m.

9) ***Conclusion***:

(Compare moment of inertia defined experimentally by formula (22) with that of defined by theoretical calculation by formula (23)).

10) Work done by: ***Work checked by:***

WORK 4-3
DETERMINATION of DAMPED OSCILLATIONS PARAMETERS

1 Goal of the work:

Studying key parameters and method of description for damped oscillations of mechanical systems.

2 Main concepts

Real-world oscillatory systems experience different kind of resistances. They loose its energy and with no external energy supply they stop after some finite interval of time. ***Damped oscillations*** are such type of ***free oscillations,*** which energy decreases with time.

Mechanical energy of oscillatory system gradually decreases transforming into heat. This process called ***energy dissipation***, and such system – ***dissipative system.***

Besides the restoring (quasi-elastic) force (9) that acts in natural oscillatory systems, in the free ***linear oscillatory systems*** a drag force acts:

$$F_{\mathrm{DRAG}} = -\, r \cdot \upsilon, \tag{36}$$

where υ – velocity of pendulum's moving; r – drag coefficient. "Minus" on the right hand side of (36) means that the drag force always acts in the opposite direction of the velocity.

Thus, for two forces (9) and (36) Newton's second law for linear damped oscillations will be

$$ma = -kx - r\upsilon.$$

In scalars, substituting acceleration of motion $a = d^2x/dt^2$ and velocity of motion $\upsilon = dx/dt$ and obtain

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + kx = 0,$$

where m – mass of oscillator (a body or a system of oscillating bodies).

Now let's rearrange this equation and obtain an ***equation of damped oscillator in differential view***:

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0, \tag{37}$$

here we introduce designations of variables:

$$2\beta = \frac{r}{m}, \qquad \omega_0^2 = \frac{k}{m}.$$

Solution for this differential equation will be the dependence of displacement x from time t, which is termed ***equations of damped oscillations*** :

$$x(t)=A_0 \cdot e^{-\beta t} \cdot \cos(\omega t+\varphi_{01}) \quad \text{or} \quad x(t)= A_0 \cdot e^{-\beta t} \cdot \sin(\omega t+\varphi_{02}), \tag{38}$$

The basic parameters of damping oscillations are:
damping coefficient (damping factor)

$$\beta = \frac{r}{2m} \tag{39}$$

and cyclic frequency of damped oscillations

$$\omega = \sqrt{\omega_0^2 - \beta^2} \tag{40}$$

here ω_0 – cyclic eigenfrequency (resonant cyclic frequency); $A_0 e^{-\beta t}$ – exponentially decaying amplitude; A_0 – initial amplitude, determines by energy of the system at instant $t = 0$.

On the Fig. 2 represented plots of amplitude versus time (dashed line) and displacement

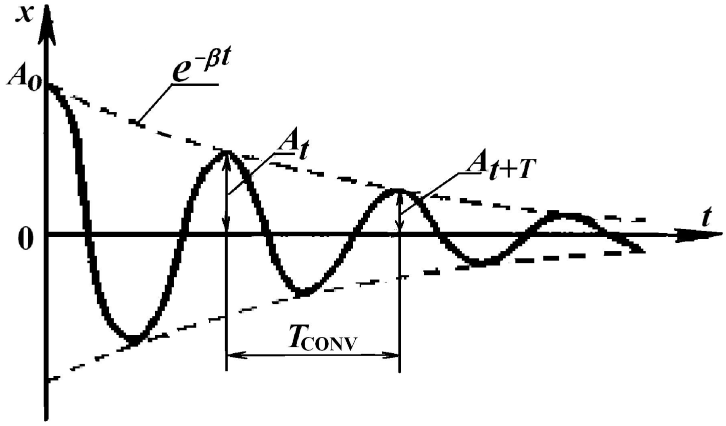


Figure 2 – The underdamping oscillations.

versus time (solid line) dependencies.

Damped oscillations are non-periodic: values of oscillating physical quantities (such as displacement, velocity, acceleration) never repeat in damped oscillations process. That is why we can't use concepts of period and frequency in the way that it been done for periodic (undamped) oscillations.

Conventional period of damped oscillations is such interval of time between to serial states of oscillating system at which oscillating physical quantities vary in the same direction, decreasing or increasing their magnitudes.

Knowing that $\omega = \sqrt{\omega_0^2 - \beta^2}$ and $\omega = 2\pi/T$, obtain

$$T_C = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}. \quad (41)$$

The conventional period (41) greater than the eigenperiod of $T_0 = 2\pi/\omega_0$, when no any damping forces are present.

There are different modes of oscillating systems with respect to value of damping coefficient (see Fig.3):

a) $\beta = 0$, so $r = 0$, $T = 2\pi/\omega_0 = T_0$ – harmonic oscillations;

b) $\beta < \omega_0$, $\omega_0^2 - \beta^2 > 0$, $T = 2\pi / \sqrt{\omega_0^2 - \beta^2} > T_0$ – almost periodic ***underdamping mode***;

c) $\beta = \omega_0$, $\omega_0^2 - \beta^2 = 0$, $T \to \infty$ – aperiodic ***critical mode***;

d) $\beta > \omega_0$, $\omega_0^2 - \beta^2 < 0$, T is imaginary – ***overdamping mode***.

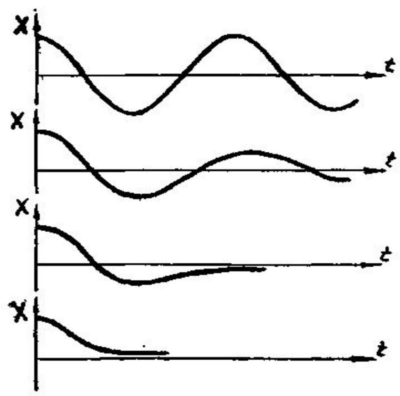


Figure 3 – Different modes of damping: harmonic; underdamping; critical; overdamping.

Decay decrement is a ratio of two serial amplitudes of the same sign A_t and A_{t+T} separated in time from each other by the period T:

$$D = \frac{A_t}{A_{t+T}} = \frac{A_0 e^{-\beta t}}{A_0 e^{-\beta(t+T)}} = e^{\beta T} = const.$$

Logarithmic decay decrement is a natural logarithm of this ratio:

$$\delta = \ln \frac{A_t}{A_{t+T}} = \beta T. \quad (42)$$

In practical calculations, for small damping, it is usually suggested that $\delta = \beta T_0$, where T_0 – period of undamped oscillations of the system.

Denoting a τ – ***relaxation time*** as interval of time over which the amplitude of oscillations decreases in e times ($e = 2.7183$ – is the base of natural logarithm), we can write:

$$\frac{A_t}{A_{t+\tau}} = e^{\beta\tau} = e.$$

Hence, $\beta\tau = 1$, or

$$\tau = \frac{1}{\beta}. \quad (43)$$

Thus, for meaning (physical sense) the damping coefficient β it is inversely proportional time over which amplitude of oscillations decreases in e = 2.7 times.

By the time τ system makes $N_e = \tau/T$ oscillations (relaxation number), so, taking into account (42) and (43) we'll have

$$\delta = \beta T = \frac{1}{\tau} \cdot \frac{\tau}{N_e} = \frac{1}{N_e}. \tag{44}$$

So, for meaning (physical sense) the logarithmic decay decrement it is inversely proportional to number of oscillations producing by the time over which amplitude of oscillations decreases in e = 2.7 times.

Quality factor of a system is a ratio of coefficients from equation of damped oscillator (37):

$$Q = \frac{\omega_0}{2\beta}.$$

Taking into account previous equations we obtain

$$Q = \frac{\pi}{\delta} = \pi N_e. \tag{45}$$

Thus, for meaning (physical sense) the quality factor it is proportional to the number of oscillations N_e done by a system by the time τ over which amplitude of oscillations decreases in e = 2.7 times.

3 Description of laboratory research facility and methodology of measurements

***Devices and outfits*:** physical pendulum with damper, stopwatch, ruler.

As oscillating system in this work we have same physical pendulum **1** as in work 4-1 with plate **3**, connected to pendulum with help of moving connector **2** (Fig. 4). The connector make possible fixation of the plate in any place of pendulum, at any angle to plane of oscillations. By varying angle between plane of oscillations and the plate we can obtain different magnitudes of friction between oscillating system (pendulum + plate) and air. Free end of the pendulum has an arrow, using which we can fix an amplitude of oscillation by the ruler **4**.

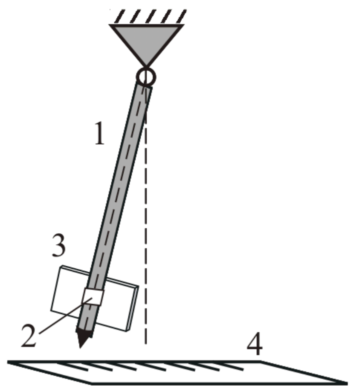


Figure 4 – Physical pendulum with damper.
1 – physical pendulum;
2 – moving connector;
3 – plate;
4 – ruler.

When a pendulum is in a static position put a ruler (or any millimetre scale) with zero mark straight under pendulum arrow. Distance between arrow and scale has to be about 1-2 mm.

In following work we measure time of relaxation τ and number of oscillations N_e during which amplitude decreases in e = 2.7 times.

4 Data processing (see laboratory work № 4-1).

5 Work execution order and experimental data analysis

1. Set plate in a random position on a pendulum.
2. Make a direct measurements of a ***time interval*** τ and ***number of oscillations*** N_e during which amplitude decreases in e = 2.7 times, also a ***conventional period*** T of damped oscillations.

For this purpose it is necessary to shift a pendulum on 60 mm from equilibrium position then free. Start stopwatch when amplitude become 54 *mm* and count number of oscillations until amplitude decreases to 20 *mm* then stop counting and stopwatch. Write obtained values of ***relaxation time*** τ and ***number of oscillations*** N_e in a Table of measurements.

By ratio τ and N_e obtain a ***conventional period*** T of oscillations. Write obtained value in a Table of measurements.

Repeat these direct measurements five times.

3. Determine ***average value*** of relaxation time, number of oscillations and conventional period (24), abmodality of a conventional period (25), and sum of squares of abmodalities of conventional period (26), ***absolute*** (29) and ***relative*** (30) ***errors*** of a conventional period.

Represent a final result of direct measurements of conventional period in (31) view.

4. Determine ***cyclic frequency*** ω according to (7), ***initial phase*** φ_0 according to (4) and (4a), ***damping coefficient*** from (43). In final results write ***equation of damped oscillations*** of linear displacement of pendulum in (38) view in metres with numerical value of coefficients of A_0, ω and φ_0.
5. Calculate a logarithmic decrement according to (44) and quality factor according to (45). Write their values in final results.

6 Control questions

1. Which mechanical systems we call dissipative?
2. What is damped oscillations? Are they periodic? Why?
3. What is conventional period of linear damped oscillations?
4. What is the description method (differential equation) for damped oscillator?
5. Write down and explain Hook's law for quasi-elastic force.
6. Write down an expression for drag (resistance) force.
7. Write down equation of linear damped oscillations and explain meaning of physical quantities in it.
8. Explain the meaning (physical sense) of damping factor. What is time of relaxation? How does magnitude of damping factor affect conventional period?
9. What is a decay decrement? What does it mean (physical sense)?
10. What is a logarithmic decay decrement? Explain it meaning (physical sense).
11. What is quality factor? Explain it meaning (physical sense).
12. How do quality factor, damping factor and logarithmic decay decrement related?

7 Content of the report

Laboratory work № 4-3

I. Homework

(answer on a control question from p. 18).

...

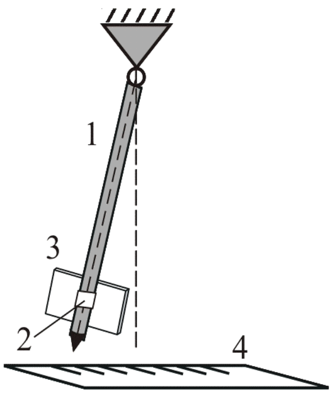
II. Laboratory work № 4-3 implementation protocol.

1) Topic:

DETERMINATION of DAMPED OSCILLATIONS PARAMETERS.

2) Goal: *Studying key parameters and method of description for damped oscillations of mechanical systems*

3) Scheme of laboratory's research facility:



1 – physical pendulum;
2 – moving connector;
3 – plate;
4 – ruler.

4) Table of measuring instruments:

№	Name	Type	Serial №	Grid limit	Grid unit	Accuracy class
1.	Stopwatch	УХЛ-42		99,99	0,01 sec	0,01 sec
2.	Ruler			1000 mm	1mm	1mm

5) Equations for calculation:

1. Statistical absolute error for conditional period:

$$\Delta T = t_{\alpha,n} \cdot \sqrt{\frac{\Sigma(\Delta T_i)^2}{n(n-1)}},$$

where α=0,95 – *confidence probability*; n=5 – *number of measurements;* $t_{0,95\,;\,5}$= 2,77 – *Student's coefficient.*

2. Average value of damping coefficient:

$$\langle\beta\rangle = 1/\langle\tau\rangle,$$

where $\langle\tau\rangle$ – *average value of relaxation time.*

3. Average value of logarithmic decay decrement:

$$\langle\delta\rangle = \langle\beta\rangle \cdot \langle T\rangle,$$

where $\langle T\rangle$ – *average value of conventional period.*

4. Cyclic frequency of damped oscillations:

$$\langle\omega\rangle = 2\pi/\langle T\rangle.$$

5. Quality factor:

$$\langle Q\rangle = \pi \cdot \langle N_e\rangle,$$

where $<N_e>$ - average value of number of oscillations at which amplitude decreases in e times.

6. Equation of physical pendulum damped oscillations:

$$x(t) = A_0 \cdot e^{-<\beta>\cdot t} \cdot \cos <\omega>t,$$

where x – displacement of pendulum; A_0 – initial amplitude.

6) Table of measurements

№	τ_i, s	N_{ei}	T_i, s	ΔT_i, c	$(\Delta T_i)^2$, c^2
1.					
2.					
3.					
4.					
5.					
	$<\tau>=$ ...	$<N_e>=$ …	$<T>$ = ...	$\Sigma(\Delta T_i)^2=$	…

7) Quantities calculation:

…

8) Final results:

1. T=($<T>\pm\Delta T)_\alpha$ = (... ± …)$_{0.95}$ s, $\delta_{T\%} = \dfrac{\Delta T}{T_{ave}} \cdot 100\%$ = … %;

2. $x(t) = \ldots \cdot e^{-\ldots t} \cdot \cos(\ldots t + \ldots)$ m;

3. $<\delta>$ =…;

4. $<Q>$=… .

9) Conclusion:

(Compare obtained value of conventional period with eigenperiod defined in previous laboratory work № 4-1).

10) Work done by: ***Work checked by:***

WORK 4-6
EXPLORING of FORCED OSCILLATIONS in SERIES *RLC*-CIRCUIT

1 Goal of the work:

1. Studying of dependencies of current and voltage on capacitor in the RLC-circuit from ratio of driving frequency and circuit eigenfrequency.
2. Studying resonance phenomena in AC circuit.

2 Main concepts

Forced (or **driven**) **oscillations** are the continuous oscillations of oscillatory system, when the system experiences action of external periodical force.

In order to continue the oscillations beyond the time allowed by the damping mechanism, it is necessary to replenish the oscillator's energy by a **driving mechanism**. The efficiency with which the driving mechanism supplies energy to the oscillator depends on the frequency of the driver as compared to the natural frequency of the oscillator.

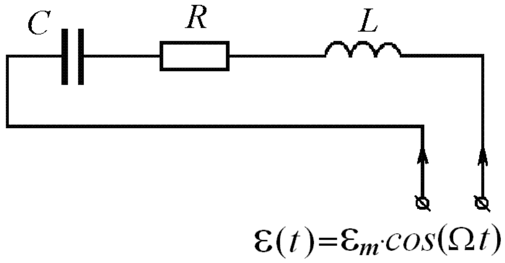


Figure 5 – Series RLC-circuit.

If we connect in series (Fig. 5) capacitor, resistor, inductor and external source of periodical alternating EMF (generator), the oscillations, which will appear in such circuit, will be forced (or driven).

Let's consider oscillations of current and voltages in series RLC-circuit driven by external EMF varying under harmonic law:

$$\varepsilon = \varepsilon_m \cdot \cos(\Omega t + \varphi_0), \qquad (46)$$

here ε_m – external EMF amplitude, φ_0 – external EMF's initial phase, Ω – external EMF's cyclic frequency (driving cyclic frequency).

Now let's apply second Kirchhoff's rule to the given RLC-circuit:

$$u_R + u_C = \varepsilon_{\text{BACK}} + \varepsilon_m \cos(\Omega t + \varphi_0), \qquad (47)$$

here $u_R = iR$ – the voltage on resistor, $u_C = q/C$ – voltage on capacitor. Produced by inductor the back EMF $\varepsilon_{\text{BACK}} = -L(di/dt)$ we obtain from self-induction law. Taking into account all mentioned above, let's rearrange equation (47) as

$$L\frac{di}{dt} + iR + \frac{q}{C} = \varepsilon_m \cos(\Omega t + \varphi_0). \qquad (48)$$

Differentiation of (28) and division it on L gives

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = -\frac{\varepsilon_m}{L}\Omega \cdot \sin(\Omega t + \varphi_0), \qquad (49)$$

here $i = \frac{dq}{dt}$. Introducing notations $\beta = \frac{R}{2L}$ and $\omega_0^2 = \frac{1}{LC}$ we'll have

$$\frac{d^2 i}{dt^2} + 2\beta\frac{di}{dt} + \omega_0^2 i = -\frac{\varepsilon_m}{L}\Omega \cdot \sin(\Omega t + \varphi_0). \qquad (50)$$

This second order linear non-homogeneous differential equation is differential equation of driven oscillations. Its solution

$$i(t) = I_0 e^{-\beta t}\cos(\omega t + \varphi_0) + I_m \cos(\Omega t + \varphi_0 + \Delta\Phi). \qquad (51)$$

First term in (51) represents natural damped oscillations of current in the circuit with frequency ω (40), which quite fast decay. So further we'll deal only with second term of (51):

$$i(t) = I_m \cdot \cos(\Omega t + \Delta\Phi) \qquad (52)$$

called **the steady state solution** of driven oscillations differential equation. As it seen from (46) and (52) current varies under the same law with the same frequency Ω, as the driving EMF do, but with the phase difference $\Delta\Phi$.

Let's choose a current's initial phase equal to zero $\varphi_0=0$, and denote its phase lead as $\Delta\Phi$ (or phase lag of external EMF $-\Delta\Phi$) we'll have

$$i(t) = I_m \cdot \cos\Omega t\ ; \qquad (53)$$

$$\varepsilon(t) = \varepsilon_m \cdot \cos(\Omega t - \Delta\Phi), \qquad (54)$$

where I_m is the amplitude of current, which has to be determined.

In order to determine current amplitude and the phase lead $\Delta\Phi$, let's substitute instantaneous value of current (53) in (47) and consider each term of obtained equation:

1. **Voltage on inductor u_L**. With the assumption that inductor has no active resistance, the voltage u_L is equal to self-induction (back) EMF with opposite sign:

$$u_L = L\frac{di}{dt} = -I_m\Omega L\cdot \sin\Omega t = U_{mL}\cdot\cos(\Omega t + \frac{\pi}{2}), \qquad (55)$$

here $U_{mL}=I_m\Omega L=I_m\cdot X_L$ – amplitude of voltage on inductor and
$X_L=\Omega L$ – inductive *reactance*.

2. **Voltage on resistor u_R**.

$$u_R = iR = U_{mR}\cdot\cos\Omega t\ , \qquad (56)$$

here $I_mR=U_{mR}$ – amplitude of voltage on resistor and
R – *resistance* of resistor.

3. **Voltage on capacitor u_C**. From definition of current $i = \frac{dq}{dt}$, then $dq=idt$ and

$$q = \int idt = \int I_m\cos\Omega t = \frac{I_m}{\Omega}\sin\Omega t = \frac{I_m}{\Omega}\cos(\Omega t - \frac{\pi}{2}). \qquad (57)$$

Thus from definition of electrocapacity

$$u_C = \frac{q}{C} = \frac{I_m}{\Omega C}\cos(\Omega t - \frac{\pi}{2}). = U_{mC}\cos(\Omega t - \frac{\pi}{2}), \qquad (58)$$

where $U_{mC} = \frac{I_m}{\Omega C} = I_m\cdot X_C$ – amplitude of voltage on capacitor and
$X_C = 1/\Omega C$ – capacitive *reactance*. Both X_L and X_C measured in Ohms.
Substituting (55), (56) and (58) in (47) we'll have

$$U_{mL}\cdot\cos(\Omega t + \frac{\pi}{2}) + U_{mR}\cdot\cos\Omega t + U_{mC}\cdot\cos(\Omega t - \frac{\pi}{2}) = \varepsilon_m\cdot\cos(\Omega t - \Delta\Phi). \qquad (59)$$

As seen in (59) the external EMF is equal to the sum of three harmonic oscillations with same frequency but different initial phases. In order to sum these oscillations let's use ***vector diagram method***. In this method each oscillation is being graphically represented as a vector that revolved around some axis with angular velocity equal to the driving cyclic frequency Ω. Length of each vector equals to amplitude of the individual oscillation. The angles between these vectors equal to phase difference that individual oscillations have with respect to each other. Vector representation of (59) shown on a Fig.6.

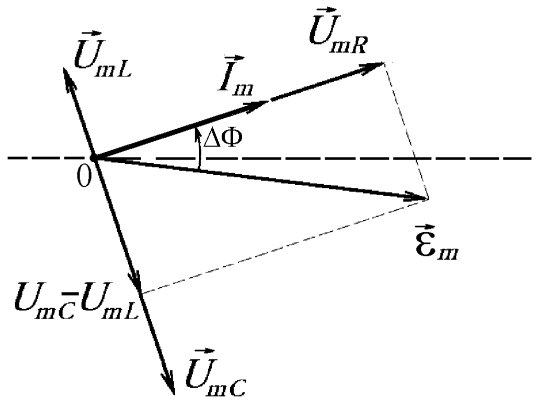


Figure 6 – Vector diagram of voltages on RLC-circuit's elements at low frequency $\Omega<\omega_0$ (current has a phase lead relative to an external EMF).

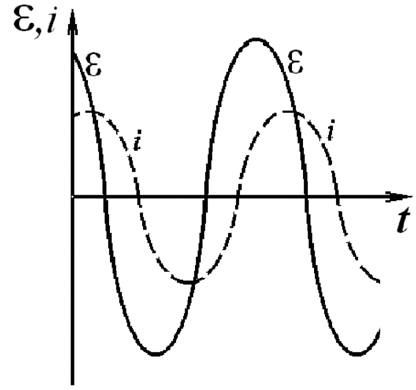


Figure 7 – Instantaneous values of current and EMF in RLC-circuit at low frequency $\Omega<\omega_0$ (current has a phase lead relative to an external EMF).

Simple geometry gives:

$$\varepsilon_m^2 = U_{mR}^2 + \left(U_{mC} - U_{mL}\right)^2 \quad \text{or} \quad \varepsilon_m^2 = I_m^2 R^2 + \left(I_m X_C - I_m X_L\right)^2, \quad \text{so}$$

$$I_m = \frac{\varepsilon_m}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{\varepsilon_m}{Z}. \tag{60}$$

This equation is analogue of Ohm's law for DC homogeneous circuit unit, if to introduce the ***impedance***:

$$Z = \sqrt{R^2 + (X_C - X_L)^2}, \tag{61}$$

here $X_C - X_L$ is the ***reactance*** of the circuit, R – resistance. Phase lead of current relative to an EMF (see Fig.6):

$$tg\Delta\Phi = \frac{U_{mC} - U_{mL}}{U_{mR}} = \frac{X_C - X_L}{R} \tag{62}$$

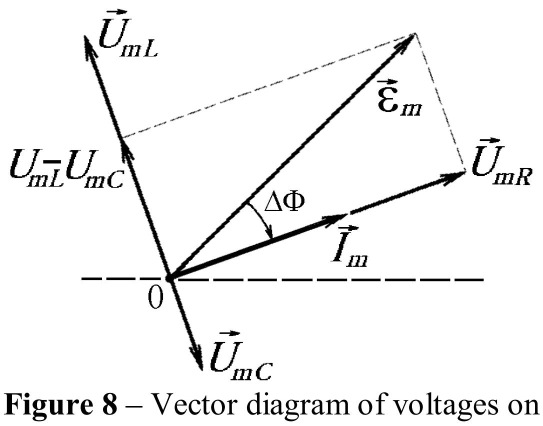


Figure 8 – Vector diagram of voltages on RLC-circuit's elements at high frequency $\Omega>\omega_0$ (current has a phase lag ΔΦ relative to an external EMF).

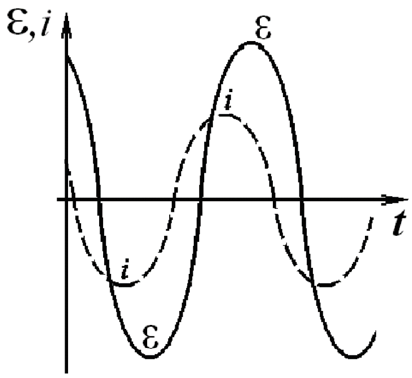


Figure 9 – Instantaneous values of current and EMF in RLC-circuit at high frequency $\Omega>\omega_0$ (current has a phase lag relative to an external EMF).

Let's analyze obtained equations. Obviously that change of driving frequency Ω will lead to change of current amplitude I_m. On Fig. 10 represented plot of I_m versus Ω. From equation (60) follows that:

a) If $\Omega = 0$, then $X_C \to \infty$ and $I_m = 0$. Increasing of Ω leads to increasing of a current;

At low frequency $\Omega \ll \omega_0$ current is limited by capacitance reactance $X_C \gg X_L$:

$$I_m = \frac{\varepsilon_m}{\sqrt{R^2 + X_C^2}} \text{ and } \Delta\Phi = arctg\frac{X_C}{R} > 0 \text{ (see Fig.6 and 7).}$$

b) At high frequency $\Omega \gg \omega_0$ current is limited by inductive reactance $X_C \ll X_L$:

$$I_m = \frac{\varepsilon_m}{\sqrt{R^2 + X_L^2}} \text{ and } \Delta\Phi = arctg\frac{-X_L}{R} < 0 \text{ (see Fig.8 and 9).}$$

Increasing of driving frequency Ω leads to further decreasing of a current.

c) When the $U_{mC} = U_{mL}$ (***voltage resonance***), then current's amplitude has a maximum:

$$I_m = \frac{\varepsilon_m}{R} \text{ and } \Delta\Phi = 0 \text{ (see Fig.10 and 11).}$$

The resonance frequency we find from resonance condition for reactance $X_C = X_L$:

$$\Omega_{\text{RES}}L - \frac{1}{\Omega_{\text{RES}}C} = 0, \text{ so } \Omega_{\text{RES}}^2 = \frac{1}{LC} \text{ or } \Omega_{\text{RES}} = \frac{1}{\sqrt{LC}}.$$

So we see that $\Omega_{\text{RES}} = \omega_0$.

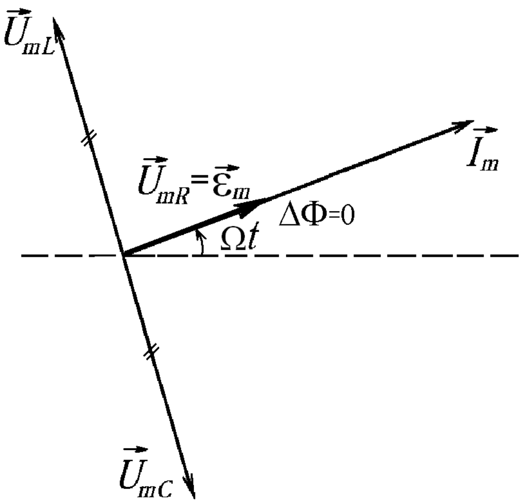


Figure 10 – Vector diagram of voltages on RLC-circuit's elements at resonance $\Omega = \omega_0$ (current synphase to an external EMF).

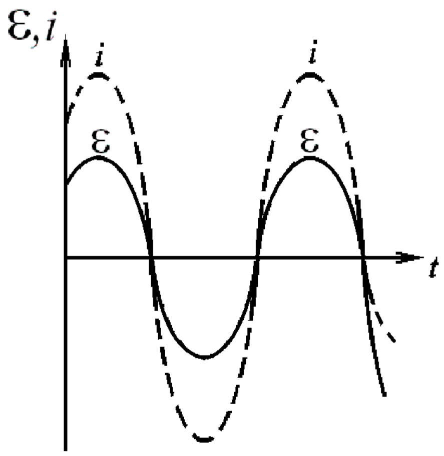


Figure 11 – Instantaneous values of current and EMF in RLC-circuit at resonance $\Omega = \omega_0$ (current synphase to an external EMF).

Resonance is a fast increasing of amplitude of oscillations when the driving mechanism's frequency approach to eigenfrequency. Then circuit's impedance equal to it active resistance and phase difference $\Delta\Phi$ between driving EMF and current equal zero. Obviously that maximal value of current depends only on active resistance of the circuit. On a Fig. 12 represented current's amplitude in the circuit for different values of its resistance (quality factor).

At resonance, forces of electric field, created by EMF source, tend to accelerate motion of charges. The amplitude would be increasing to infinity during the time of period if there was no active resistance in the circuit. In the real-world circuit increasing of a current leads to increasing of energy losses. Amplitude of current will approach its steady-state value when these losses will be equal to work done by the source's electric field forces. Phase difference $\Delta\Phi$ between current and EMF isn't equal zero when driving frequency doesn't equal to eigenfrequency. In this case, at one part of the period source's electric field accelerates charges and decelerates at another one. That is why current's amplitude is lower than it is at resonance and, for the period, at increasing of phase difference $\Delta\Phi$ deceleration time is greater than acceleration time.

At resonance the circuit consume a minimum energy from the source. Stored energy of electric field $W_C = CU_{mC}^2/2$ completely transforms into energy of magnetic field $W_L = LI_m^2/2$ and vice versa, as in the case of harmonic oscillations. Source's energy spent only to compensate energy losses in the circuit. Instantaneous value of a power loss can be determined as:

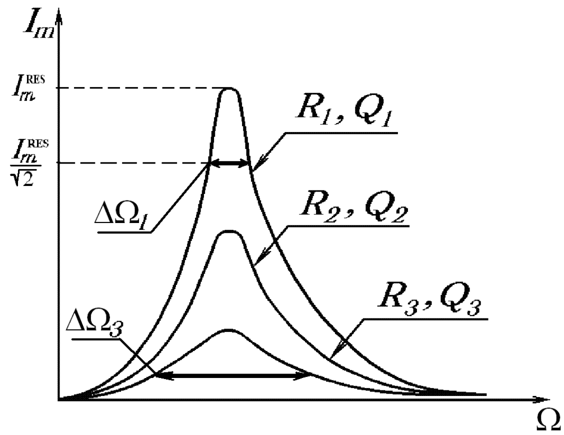


Figure 12 – Resonant curves: amplitude of current versus driving frequency plot for different values of resistance and quality factor: $R_1 < R_2 < R_3$; $Q_1 > Q_2 > Q_3$.

$$P(t) = i(t)\cdot u(t) = I_m\cos\Omega t \cdot RI_m\cos\Omega t = I_m^2 R\cdot\cos^2\Omega t.$$

Circuit's energy loss during the period

$$\Delta W = \int_0^T P dt = \int_0^T I_m^2 R\cos^2(\Omega t)dt = \frac{1}{2}I_m^2 RT,$$

$$\text{as } \int_0^T \cos^2(\Omega t)dt = \frac{T}{2}.$$

Resonant properties of RLC-circuit can be characterized by the ***quality factor*** :

$$Q = \frac{\omega_0}{2\beta}$$

One of *physical senses of a quality factor* is the ratio of energy stored in the circuit to energy dissipated at resonance:

$$Q = 2\pi\frac{W_C}{\Delta W} = 2\pi\frac{W_L}{\Delta W} = 2\pi\frac{\frac{1}{2}LI_m^2}{\frac{1}{2}I_m^2 RT} = 2\pi\frac{L}{R2\pi\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

From voltage resonance condition we have equality of U_{mL} and U_{mC} magnitudes, but, they are in antiphase, so at any instant their sum equals zero:

$$U_{mL} = U_{mC} = I_m\cdot\frac{1}{\Omega_{\text{RES}}C} = \frac{\varepsilon_m}{R}\frac{\sqrt{LC}}{C} = \varepsilon_m\frac{1}{R}\sqrt{\frac{L}{C}} = \varepsilon_m\cdot Q.$$

Thus we get another *physical sense of quality factor*, according which quality shows in how many times the inductance voltage amplitude or capacitance voltage amplitude at resonance greater than driving EMF amplitude:

$$Q = \frac{U_{mC}^{\text{RES}}}{\varepsilon_m} = \frac{U_{mL}^{\text{RES}}}{\varepsilon_m}.$$

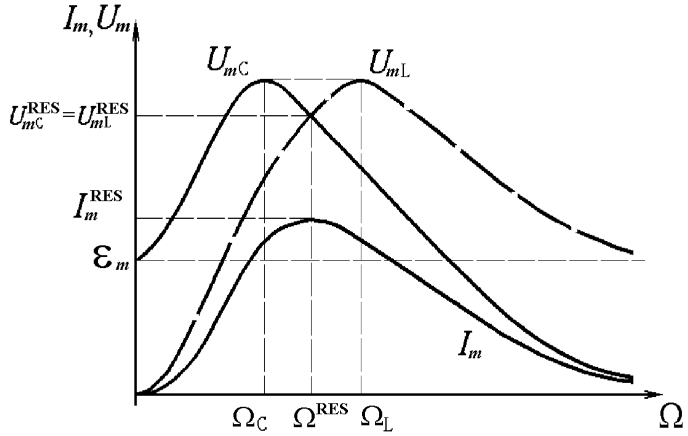


Figure 13 – Amplitude – frequency characteristics: capacitance, inductance voltages and current's amplitudes versus driving frequency Ω.

For small active resistance, i.e. if $R << \sqrt{\frac{L}{C}}$, $Q>>1$, hence $U_{mL}=U_{mC}>>\varepsilon_m$.

Than higher is quality than clearly and sharply resonance is. This is another *physical sense of quality factor* :

$$Q = \frac{2\pi\Omega_{RES}}{\Delta\Omega},$$

here ΔΩ – full width of resonance at half energy maximum (FWHM).

On the Fig. 12 represented current's amplitude-frequency characteristics for different values of quality factor. It is visible, that higher quality factor has a narrow FWHM.

As it was mentioned above amplitudes of inductance and capacitance depend on the frequency of driving EMF. Expression for amplitude of voltage on inductor can be written as:

$$U_{mL}=I_m\Omega L=\frac{\varepsilon_m \Omega L}{\sqrt{R^2+\left(\frac{1}{\Omega C}-\Omega L\right)^2}}. \tag{63}$$

Voltage on inductor approaches maximum at frequency Ω_L that is greater then circuit's resonant frequency $\Omega_{\text{RES}}=\omega_0$. In order to find Ω_L it is necessary to investigate expression (63) of the maximum, i.e. solve the equation $\frac{dU_{mL}}{d\Omega}=0$.

Amplitude of voltage on capacitor equals

$$U_{mC}=\frac{I_m}{\Omega C}=\frac{\varepsilon_m}{\Omega C\sqrt{R^2+\left(\frac{1}{\Omega C}-\Omega L\right)^2}}. \tag{64}$$

It approaches maximum at frequency Ω_C that is smaller than $\Omega_{\text{RES}}=\omega_0$.

As it seen from Fig. 13 at $\Omega_{\text{RES}}=\omega_0$ amplitudes U_{mC}^{RES} and U_{mL}^{RES} are numerically equal but <u>not maximal </u>!! Difference of frequencies $\Omega_L - \Omega_C$ is smaller than greater quality of the circuit is. At high quality factor $\Omega_C \approx \Omega_{\text{RES}}=\omega_0 \approx \Omega_L$.

3 Description of laboratory research facility and methodology of measurements

Devices and outfits: Set of inductances, set of capacities, set of resistances, voltmeter, sound generator Г3-36.

In a given work we study capacitor, inductor and resistor voltages dependencies on external driving EMF frequency **Ω**. Magnitudes of *R, C, L* and amplitude of driving EMF E_m remain constant during the work.

Facility's scheme represented on Fig.13. There we have capacitor *C* (one value from the set of capacities), inductor *L* (one value from the set of inductances) and resistor *R* (one value set on resistance box). The real inductance *L* has its own active resistance R_L. Such inductance is equivalent to series *RL* circuit. As driving EMF source we have Г3-36 generator (or any similar). It output voltage (i.e. EMF ε) can be measured with help of built-in voltmeter. Note that voltmeter indicates effective voltage magnitude, not amplitude. **Effective** current (voltage) magnitude is such magnitude of direct current which produces same heat in same circuit that alternating current does by the time of a period. Effective current is also called **root mean square** (RMS) value of alternating current. For harmonic alternating current relations between effective and amplitude values are so:

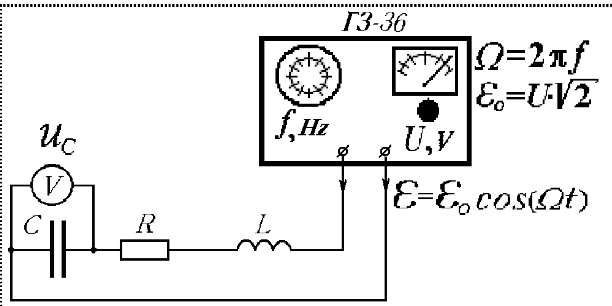


Figure 14 – Laboratory research facility's scheme.

$$I = \frac{I_m}{\sqrt{2}}, \quad \varepsilon = \frac{\varepsilon_m}{\sqrt{2}}$$

Obviously that effective current and EMF related as

$$I = \frac{\varepsilon}{\sqrt{R^2 + (X_C - X_L)^2}}$$

If we'll change the frequency of generator then phase lag between current and EMF also will be changed as the reactance changes. As it seen from (63), (64) and Fig.13, at low frequencies $u_L < u_C$ and $u_L > u_C$ at high. Obviously that there will be some frequencies $\Omega_C = \Omega_L$ at which $u_C = u_L$ and get maximum.

Using indexes of voltmeter, switched to capacitor or inductance coil we can find frequency Ω_C or Ω_L, which, in case of high enough quality, will be equal to resonance frequency.

So, in this work we can plot:

a) Effective voltage on capacitor U_C versus driving EMF frequency $\Omega = 2\pi\nu$, where ν determined with Г3-36 limb;

b) Effective current $I = 2\pi\nu C U_C$ in a circuit versus Ω;

c) Effective voltage on inductor $U_L = I\Omega L$, or substituting *I* and Ω, $U_L = 4\pi^2\nu^2 C U_C L$. Inductance *L* can be determined from the match of driving and natural frequencies of the circuit at resonance, i.e.:

$$L = \frac{1}{\Omega_{\mathrm{RES}}^2 C}$$

4 Data processing (see laboratory work № 4-1).

5 Work execution order and experimental data analysis

1. Turn on ГЗ-36. Wait 5 minutes till it warms up.
2. Using potentiometer "Рег.выхода" set generator output voltage 1.5 – 3 *V*. This voltage should remain unchangeable during the work.
3. Set fixed value of resistance R on the set of resistances. Fix value of capacity C.
4. Set 20 *Hz* on generator and fix voltage on a voltmeter (is equal to voltage on capacitor). By changing the frequency ν_{GEN} and look after voltmeter data let's pass all frequency range from 20 to 200 *Hz*.

Change in 10 times the frequency band by «Множитель» switcher if there no resonance voltmeter data increasing will be found and so on until the frequency range at which resonance frequency ν_{RES} situation will be determined. At this frequency ν_{RES} the voltmeter reading will be maximal $U_C = U_C^{RES}$.

Note: generator scale gives frequencies ν_{GEN} in *Hz*, whereas $\Omega = 2\pi\nu$ in *rad/s*.

5. In established frequency range do 5-7 measurements of effective values U_C for $\nu_{GEN} < \nu_{RES}$ and 5-7 measurements – for $\nu_{GEN} > \nu_{RES}$.
6. By obtained data of ν_{GEN} calculate values of cyclic frequency $\Omega_{GEN} = 2\pi\nu_{GEN}$. By obtained data of effective values U_C for given capacity C calculate values of amplitude of current from definition of amplitude of voltage on capacitor $U_{mC} = I_m \cdot X_C$. and relation between the amplitude of voltage and its effective value $U_{mC} = U_C\sqrt{2}$. Read calculated data to the measurement table.
7. By resonance frequency Ω_{RES} calculate inductance of coil from $\Omega_{RES} = 1/\sqrt{LC}$.
8. Plot a graphs $U_C = f(\Omega)$ and $I_m = f(\Omega)$ as a final result of the work.
9. Calculate quality of circuit $Q = U_C^{RES} / \varepsilon_{GEN}$ and resistance from $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$.
10. Write values Q, Ω_{RES}, L and R in final results.

6 Control questions

1. What is forced (driven) oscillations? Draw an oscillating circuit with driving EMF.
2. What is the description method for forced oscillations in RLC-circuit? Derive driven oscillations differential equation for current and write down its steady state solution.
3. Derive equation for current amplitude and phase difference between current, driving EMF, voltages on capacitor and on inductor.
4. How to determine inductive, capacitive and total resistances of contour?
5. Draw voltage vector diagram for series RLC-circuit at low and high frequency.
6. Plot amplitude of voltage on capacitor, amplitude of voltage on inductor and current amplitude versus driving frequency on one graph.
7. Plot current amplitude versus driving frequency for different values of quality.
8. Provide three different derivations of equation of RLC-circuit quality (physical senses). Calculate quality according to graphs mentioned above taking axis scale as following: 1*cm* equals 1*V*.

7 Content of the report

Laboratory work № 4-6

I. Homework

(answer the control questions from p.28).

_…_

II. Laboratory work № 4-6 implementation protocol.

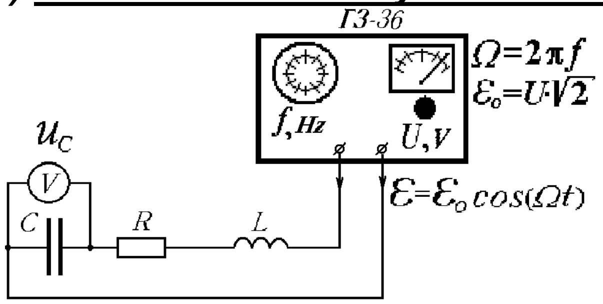
1) Topic:

EXPLORING of FORCED OSCILLATIONS in SERIES RLC-CIRCUIT.

2) Goal: *1. Studying of effective value of voltage on capacitor and effective value of current dependencies in the series RLC-circuit versus ratio of driven frequency to the circuit's eigenfrequency.*

2. Studying resonance phenomena in AC RLC-circuit.

3) Scheme of laboratory research facility:



V – voltmeter;
C – capacitor;
R – resistor;
L – inductor (coil);
***Г3-36** – AC generator;*

4) Table of measuring instruments:

№	Name	Type	Serial №	Grid limit	Grid unit	Accuracy class
1.	Voltmeter	M996		25 V	0.5 V	1.5%
2.	Sound generator	Г3-36		20 - 20000 Hz	100 Hz	

5) Equations for calculation:

1. Generator cyclic frequency:

$$\Omega_{GEN} = 2\pi\nu_{GEN},$$

where ν_{GEN} – generator linear frequency in Hz.

2. Amplitude of current in the circuit:

$$I_m = 2\pi\cdot\nu_{GEN}\cdot C\cdot U_C\sqrt{2},$$

where C – capacity of a circuit (one of the set), U_C – effective voltage on capacitor.

3. Quality factor of the circuit:

$$Q = \frac{U_C^{RES}}{U_{GEN}},$$

where U_C^{RES} – effective voltage on capacitor at resonance; U_{GEN} – effective driving EMF (Г3-36 output voltage, determined by Г3-36 voltmeter scale).

4. Inductance of the circuit:

$$L = \frac{1}{\Omega_{RES}^2 C},$$

where Ω_{RES} – resonance frequency.

5. Resistance of the circuit:

$$R = \frac{1}{Q}\sqrt{\frac{L}{C}}.$$

6) Table of measurements

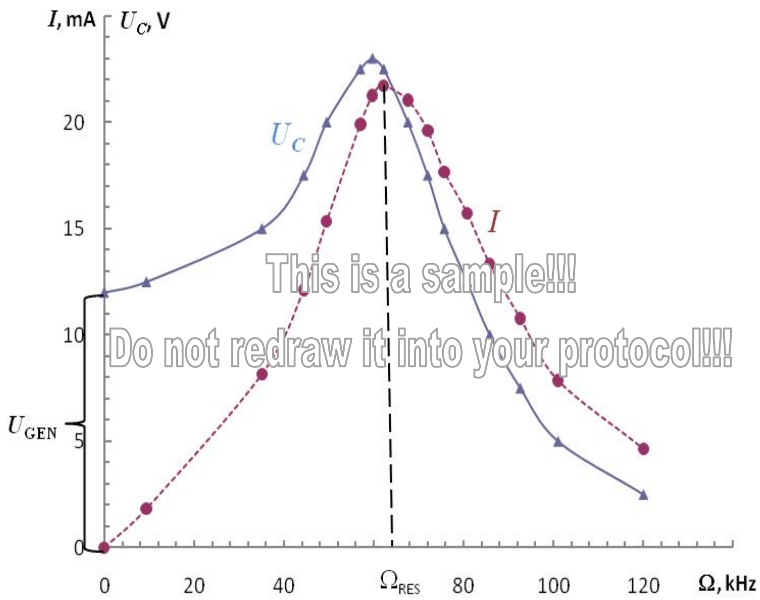
$C = \ldots\ F$; $U_{GEN} = \ \ldots\ V$.

№	ν_{GEN}, kHz	Ω_{GEN}, kHz	U_C, V	I_m, mA

7) Quantities calculation:

$Q = \ldots$;
$L = \ \ldots\ mHn$;
$R = \ \ldots\ Ohm$.

8) Graphs of $U_C(\Omega)$ and $I(\Omega)$ dependencies:



9) Final results :

1. $Q = \ \ldots$;
2. $L = \ldots\ mHn$;
3. $\Omega_{RES} = \ \ldots\ kHz$;
4. $R = \ldots\ Ohm$.

10) Conclusion:

Work done by: **Work checked by:**

WORK 5-1
EXPLORING OF STANDING WAVES VIA MELDE METHOD

1 Goal of the work

1. Studying parametric resonance phenomena.
2. Studying conditions of standing waves generation.
3. Determination of vibrator oscillations frequency.

2 Main concepts

2.1 Waves.

A ***wave*** is a propagation of oscillations in space. Oscillations of elastic medium cause ***elastic waves***, oscillations of electric and magnetic fields – ***electromagnetic waves*.** An oscillating mechanism that is a source of waves is called ***vibrator***.

Elastic wave is the mechanical disturbance (deformation) of a medium propagates through that medium. Propagation of elastic waves is in excitation of oscillations of more and more remote points of a medium. Set of oscillating points of given medium is called the wave field**.** Oscillation of each point is forced by vibrator (or another points). A locus of points having the same phase is called the ***wave surface***. The ***wave front*** is a wave surface whith maximal distance from source at present moment. With respect to the shape of wave front there are ***traveling waves***, ***plane waves***, ***spherical waves***.

In ***longitudinal waves*** the points of medium oscillate at parallel to the direction of propagation. This type of waves related with *compressive deformation* of medium and able to propagate in solids, liquids, and gaseous mediums. In ***transverse waves*** the points of medium oscillate at perpendicular to the direction of propagation. These waves can only occur in media that oppose to *shear deformation*. Only solids have such property. That is why transverse waves propagate only in solids.

The ***phase velocity*** of a wave υ is physical quantity which numerically equal to distance at which any point of wave surface travels by unit of time. Vector of phase velocity $\vec{\upsilon}$ points in the direction of wave propagation, perpendicularly to wave surface. The ***wavelength*** of a sinusoidal wave is a distance between any two next points with the same phase. From another standpoint, ***wavelength*** is a distance which any phase of wave transits for a period.

$$\lambda = \frac{2\pi}{k} = \upsilon T = \frac{\upsilon}{f}, \tag{65}$$

where $T = \dfrac{2\pi}{\omega}$ – period of a wave (smallest time interval after which the value of oscillating quantity is being repeated), f – frequency of a wave (number of oscillations per unit time), $k = \dfrac{2\pi}{\lambda}$ – the wavenumber (number of wavelengths per 2π distance).

Each point of one-dimensional traveling wave oscillates according the ***wave equation*** (equation of traveling wave in differential view)

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{\upsilon^2} \frac{\partial^2 \xi}{\partial t^2}, \tag{66}$$

General solution for this differential equation (66) will be ***equation of traveling wave***

$$\xi\,(t,x) = A\cos(\omega t - kx + \varphi_0) \tag{67}$$

Phase of the wave $(\omega t - kx + \varphi_0)$ is function of position x and time t. For fixed x the displacement $\xi(t,x)$ is harmonic function of time, and for fixed t - a cosinusoid.

2.2 Parametric resonance. Standing waves.

In order to determine vibrator's frequency we use standing waves that appear in tense thread connected to vibrator. Oscillations of any oscillatory system can be made forced by driving any of its parameters. The resonance that will appear in this case is called ***the parametric resonance***.

Let's consider a horizontally tense thread fixed at one end. Another end of the thread connected to vibrator that is able to oscillate along the thread. In such system will appear transverse oscillations caused by periodically varying tension of thread. Let at initial moment of time the vibrator was shifted left and the thread is freely sags in gravity field (Fig. 14, a). When vibrator-rod moves right the tension increases until thread wouldn't come in its horizontal position. In this case all points of thread will move upwards (Fig. 14, b) (velocities of some points shown as arrows). At reverse (right to left) motion of vibrator-rod thread's tension decreases but its points will move upwards by inertia (Fig. 14, c). Over next complete oscillation of vibrator-rod the thread will sag again. Thus, by the time of two complete oscillations of vibrator the thread oscillates once. Hence, vibrator frequency is double thread's frequency. Obviously, that considered above method of excitation of oscillations is parametric – external affection (vibrator's oscillations) causes periodical alternations of one of the parameters of a thread (thread's tension).

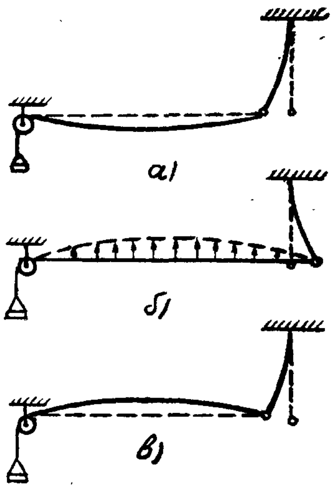


Figure 14 – Standing wave in thread:
a) at initial moment;
b) over half of a period of vibrator.
c) over period of vibrator

Any system which is able to oscillate will perform natural oscillations under the action of random external affection. If the frequency of this parametric affection is double the system's natural frequency the oscillations become forced (holds the conditions of parametric resonance). Same phenomena can be achieved in electric circuit by external alternation of capacitance or inductance. Work of parametric generators and amplifiers is based on parametric resonance phenomenon.

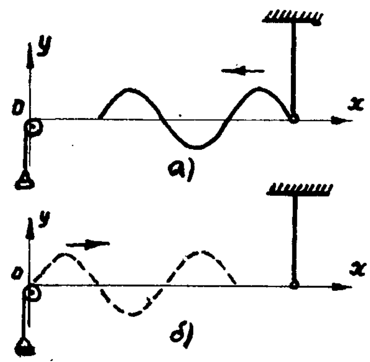


Figure 15 – Traveling elastic wave propagation in a thread:
a) incident wave;
b) reflected wave .

Oscillations of vibrator produce elastic wave, propagating along the thread with frequency ω and amplitude A. All points of thread will perform undamped oscillations if thread is not quite long and dissipations of energy are small.

Let's set O (point of pulley and thread connection) as origin of coordinates. A wave, which has reached this point, will be reflected back. Thus we'll have two waves travelling in opposite directions:

$$\xi_1 = A\cos(\omega t + kx) \tag{68}$$

$$\xi_2 = A\cos(\omega t - kx + \pi) \tag{69}$$

All points of a thread will be behind vibrator's oscillations as much, as smaller will be x in chosen coordinate system (Fig. 15). That is why equation (68) is equation of incident plane travelling wave, and (69) – equation of reflected wave. Minus sign before kx in reflected wave equation means that phase of oscillations of each thread's point in reflected wave as smaller as far this point is from origin of coordinates. Besides, as point O immovable, phase of reflected wave will be deferent at that point from phase of incident wave on π.

The result of superposition of incident and reflected waves will be complex oscillations of all points of thread under the law:

$$\xi = \xi_1 + \xi_2 = 2A\cdot\cos\frac{\omega t + kx + \omega t - kx + \pi}{2}\cdot\cos\frac{\omega t + kx - \omega t + kx - \pi}{2},$$

or

$$\xi = 2A\cdot\cos(kx - \frac{\pi}{2})\cdot\cos(\omega t + \frac{\pi}{2}) \tag{70}$$

Thus, we've obtain equation of steady state standing wave in a thread. A time independent expression $A_{SW}(x)=|2A\cos(kx-\frac{\pi}{2})|$ represents the ***amplitude of standing wave***. As it seen, the amplitude varies under harmonic law with respect to distance x from origin of coordinates. For points at which

$$kx - \frac{\pi}{2} = n\pi, \qquad (n = 0, 1, 2, \ldots) \tag{71}$$

amplitude gets maximal value equal to $A_{SW}(x_{AN})=2A$. These points called ***antinodes*** or *wave crests*. From (71) we can obtaine coordinates of antinodes:

$$x_{AN} = (2n + 1)\frac{\lambda}{4}. \tag{72}$$

Points, at which

$$kx - \frac{\pi}{2} = (n - \frac{1}{2})\pi, \qquad (n = 0, 1, 2, \ldots) \tag{73}$$

do not oscillate and $A_{SW}(x_N)= 0$. These points called ***nodes*** of standing wave. The nodes appear in points at which incident and reflected waves are in antiphase. As it follows from (73), the coordinates of nodes are

$$x_N = 2n\frac{\lambda}{4}. \tag{74}$$

Distance between two nodes or antinodes called ***standing wavelength***. That is why

$$\lambda_{ST} = \frac{1}{2}\lambda.$$

Since there are nodes at both fixed ends of a thread, so, thread's length l must contain integer number of standing wavelengths

$$l = n\,\lambda_{\text{ST}} = n\frac{\lambda}{2}.$$

On a Fig. 16 shown appearance of standing wave by reflection, with phase, changed on π. Solid line represents displacement of thread's points produced by incident wave, dashed line – by reflected wave. The resultant displacement represented by bold line. The figure shows moment at which incident wave come to point O with non-zero phase.

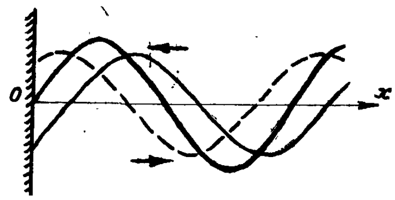


Figure 16 – Standing wave as superposition of incident and reflected waves.

The main feature of standing wave is that it doesn't transport energy. That follows from the fact that position of nodes and antinodes doesn't change with time. Such particularity is the result of the fact that the superposing wave transfer equal quantities of energy in the opposite directions.

3 Description of laboratory research facility and methodology of measurements

There is a vibrator fixed at the edge of laboratory desktop – a rod forced to oscillate by electromagnet. This electromagnet supplied by sinusoidal current, so, the forced oscillations of the rod are driven with frequency of the alternating current. On the opposite edge of the desktop there is a light pulley with a fixed axis of rotation. The vibrator and the pulley joined by a thread. One end of this thread is tied up to vibrator and the other one, thrown through the pulley, is pulled by the load in the plate (see Setup schematic diagram).

In order to determine vibrator frequency we'll need to obtain stable standing waves pattern. Such pattern might appear only when the condition of parametric resonance matched, i.e. when the vibrator frequency is double one of the natural frequencies of the thread. In this case length of thread l will contain integer number of standing wavelengths λ_{st}.

Natural frequency of thread's oscillations depends on its tension. If at some tension of a thread vibrator frequency f_υ is double the lowest frequency f (fundamental harmonic) of the thread, then one standing wave will lay on its length. By decreasing the tension we can obtain conditions at which f_υ is double the second, third,.., n-th harmonic of the thread (frequencies multiple to fundamental harmonic): $f_\upsilon = 2 f_n$. Thus we'll have 2, 3,…, n standing waves, lying on a thread's length l.

$$l = n\,\lambda_{st} \text{ or } l = n\frac{\lambda}{2}. \tag{75}$$

Relation between thread's frequency f and λ is so

$$f = \frac{\upsilon}{\lambda}, \tag{76}$$

here υ – velocity of wave propagation in the thread.

The velocity of wave propagation depends on magnitude of tension *F*, caused by immovable weight, linear density ρ (depends on thread's mass *m*) and length *l*. As $\upsilon = \sqrt{\frac{F}{\rho}}$, then, including (75) and (76) we'll have

$$f = \frac{n}{2l}\sqrt{\frac{F}{\rho}}. \tag{77}$$

Tension *F* is caused by the weight of load, so

$$F = P = Mg, \tag{78}$$

here *M* – mass of the load (including mass of a plate). As the vibrator frequency is double thread's natural frequency, from (78) we'll have

$$f\text{v} = \frac{n}{l}\sqrt{\frac{Mg}{\rho}}. \tag{79}$$

4 Data processing (see laboratory work № 4-1).

5 Work execution order and experimental data analysis

1. Turn on the electromagnet and make rod oscillate.
2. By loading the plate with different weights obtain stable standing wave pattern.
3. Write number of standing waves *n* laying on thread's length *l* and mass of load *M*.
4. Repeat the experiment three-five times, getting stable standing wave pattern with different number of standing wavelength λ_{ST} laying on thread's length *l*.
5. Measure thread's length *l* between vibrator and pulley axle with ruler accuracy.
7. Calculate linear density of thread for each case of stable standing wave pattern. The mass of the thread is on the setup.
8. Calculate thread's linear density absolute and relative error. Write final result as confidence interval with relative error.

6 Control questions

1. What is parametric resonance phenomenon? Explain it using work 5-1. What is a ratio of vibrator frequency to thread's natural frequency in this work?
2. What is a difference between travelling and standing waves? Consider amplitudes and phases of oscillation at different points in both cases.
3. What is a standing wave? What is the condition of standing waves generation? Derive standing wave equation for the case of less dense medium reflection. Analyze this equation.
4. What is a wavelength? What is a standing wave length? What is a relationship between two quantities above?
5. What features of oscillations of a medium with standing waves in it? What is a node of standing wave? What is an antinode of standing wave? Prove node and antinode coordinate calculation equations for the case of reflection from denser medium.

6. Why is different number of standing wave lengths fitting in to thread lengths for different tensions of this thread?
7. How do boundary conditions affect on the position of nodes and antinodes? Obtain standing wave equation for the case of reflection from denser medium. Analyze obtained equation.
8. What is a goal of work 5-1? Describe what steps one needs to complete during the work. Derive an equation for vibrator frequency oscillation calculation.

7 Content of the report

Laboratory work № 5-1

I. Homework

(answer on a control question from p.35).

...

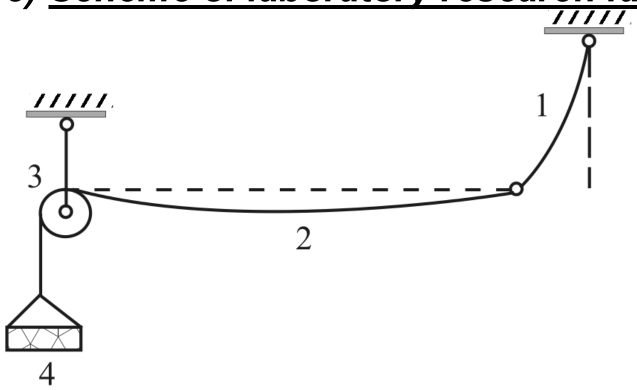
II. Laboratory work № 5-1 implementation protocol.

1) Topic:

EXPLORING OF STANDING WAVES VIA MELDE METHOD.

2) Goal: *Studying parametric resonance, studying conditions of standing waves generation and determining linear density.*

3) Scheme of laboratory research facility:



1 – vibrator;
2 – string;
3 – movable pulley;
4 – plate with weights.

4) Table of measuring instruments:

№	Name	Type	Serial №	Grid limit	Grid unit	Accuracy class
1.	Weights					
2.	Ruler			1000 mm	1 mm	0.5 mm

5) Accessories: *metal cup for weights, thread.*

6) Equations for calculation:

1. When n stable antinodes are being observed in a string, the vibrator frequency can be calculated:

$$f_{\mathrm{V}} = \frac{n}{l}\sqrt{\frac{Mg}{\rho}},$$

where l – thread length between vibrator and pulley axle; M – mass of the load (including mass of plate); ρ – linear density of the thread; $g = 9.81\ m/s^2$ free fall acceleration.

2. Linear density:

$$\rho = \frac{n^2 Mg}{l^2 f_v^2},$$

where $f_V = 100\ Hz$ – *vibrator frequency.*

3. Linear density absolute error:

$$\Delta\rho = t_{\alpha,k}\sqrt{\frac{\sum_1^k (\Delta\rho)^2}{k(k-1)}},$$

where $\alpha=0,95$ – *confidence probability;* $k=5$ – *number of measurements;* $t_{0,95\,;\,5}= 2,77$ – *Student's coefficient.*

7) Table of measurements:

$l = \ldots\ m$

№	M, $кg$	n	ρ_i, $кg/m$	$\Delta\rho_i$, $кg/m$	$(\Delta\rho_i)^2$, $кg^2/m^2$
1					
2					
3					
4					
5					
		average value $\langle\rho\rangle$ =	…	$\sum_1^k(\Delta\rho_i)^2 =$	…

8) Quantities calculation:

…

9) Final results:

$\rho = (\langle\rho\rangle \pm \Delta\rho)_{\alpha} = (\ldots \pm \ldots)_{0.95}\ kg/m$; $\delta_{\rho\%} = \frac{\Delta\rho}{\rho_{ave}} \cdot 100\% = \ldots\ \%$

10) Conclusion:

Work done by: ***Work checked by:***

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Content

Educational methodical issue
Physics
Module 2.
Oscillations and waves.
Part 3: **laboratory work**
Writers: assoc. prof. Gorbachev V.E., instructor Kardashev K.D.