Department of physics of optical communications

PHYSICS

Module 1. Electrical current and magnetic field of a current

ELECTROMAGNETISM

PART 3: LABORATORY WORKS

for bachelor training of educational area 0924 –"Telecommunications"

APPROVED
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The following methodical guide is about section "Electromagnetism" of physics course for telecommunications technician. Four laboratory works allow students to learn basics of electrical engineering and measuring technique applied to determine main characteristics of electrical systems and magnetic field. It contains sufficient theoretical information combined with detailed descriptions of applied electromagnetic equipment construction such as Whetstone bridge, tangent-compass, and measuring techniques such as bridge circuit, comparison method.

Recommended for students of TE-group, educational area 0924 – "Telecommunications".

CONFIRMED
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from 27.02.2009

MODULE STRUCTURE

Module № 1. „ Electrical current and magnetic field of a current" – 72 hours total
Lectures – 16 hrs, pract. trainings – 0 hrs, labs – 16 hrs, self-studies – 33 hrs.

LIST OF LABORATORY WORKS

Number of lessons	Denomination of laboratory work, code of the work	Hours
	Module № 1	
1	2-2. The Wheatstone bridge measurement of resistance (application of errors theory).	2
2	The principle of superposition of electric fields. Calculation of resultant electric intensity.	2
3	2-3. Finding of EMF source and potential distributions along closed circuit.	2
4	Calculation of branched circuits.	2
5	2-4. Power of direct current circuit.	2
6	The principle of magnetic fields' superposition. Calculation of resultant magnetic intensity.	2
7	3-1. Investigation of magnetic field by the tangent compass.	2
8	Electromagnetic induction law. Calculation of EMF of the induction.	2

INTRODUCTION

All laboratory works are provided in a frontal way, i.e. all group makes the same laboratory work at the same time.

Appropriate homework must forego to work in a lab. The homework contains self- studying of theory and methodology of work accomplishment, preparation of protocol which includes experimental setup schematic drawing, equipment table drawing, measurement table drawing, writing a list of working formula with decryption of all quantities which are in, writing answers of control questions previously given by an instructor.

Permission to proceed laboratory work will have those of students who have done homework and have positive result on express mini-quiz in a lab.

Content of the reports for all laboratory work has to be the following:
1) Title and number of laboratory work.
2) Goal of the work.
3) Setup schematic drawing.
4) Equipment table.
5) Equations for calculation with decryption of all quantities in.
6) Standart table of measurements for each measured quantity. It has to be checked and verified by an instructor.
7) Experimental data processing (write one if there will be many similar)
8) Standart form result (confidence interval and relative error look at a graph)
9) Conclusion
10) Date, name of a student.

Becides this guide is recommended to use literature from bibliography given at the end of this guide.

LABORATORY WORK № 2-2
MEASUREMENT OF RESISTANCE WITH WHEATSTONE BRIDGE

1 Goal of the work:
1 Study the method of measurements by means of a bridge circuit.
2 Study the method of data processing.
3 Finding the resistance of conductors.

2 Main concepts
A general aim of Works № 2‑2... unite № 2‑4 ‑ research of the direct current circuit. Panel of laboratory table has two current sources and some resistors, from which we can mount different circuits, and measurement devices, with the help of which we can make researches. In this work package measures resistances of resistors by the Wheatstone bridge method (work №2 ‑ 2); measures e.m.f (electromotive force) and internal resistance of current sources, researches the distribution of the potential along the closed circuit, mounted from the researching resistors and the current sources (work №2 ‑ 3); researches the dependence of the power and the sources efficiency from the current and an external resistance (work №2‑4). Data, obtained from one work, uses in others.

In order to complete all these works, you should learn the laws of a direct current.

2.1 Ohm's law

Electric current is an ordered motion of electric charges. This ordered motion is also called *flow*. There are conduction currents, convection currents, displacement currents and currents in vacuum. In this section we will consider only conduction current.

Conduction current is an ordered motion of free electrons and ions. In metals the negatively charged free electrons flow, in electrolytes – ions. The flow of positive charges the same as direction of a current is called ***Conventional current.*** That's why the direction of a current in metals is opposite to the direction of electrons motion.

The main characteristics of a current are ***current intensity*** and ***current density.***

Current intensity (or simply **"*current*"**) *is a scalar physical quantity whose magnitude is charge carried through cross-sectional area of a conductor per unit of time.* For *direct current* (DC):

$$I = \frac{q}{t}, \tag{1}$$

where q – is charge, carried through cross-sectional area of a conductor per time t. Same for *alternating current* (AC) is:

$$I = \frac{dq}{dt}, \tag{2}$$

where dq – is an infinitesimal charge, carried through cross-sectional area of a conductor per infinitesimal period of time dt.
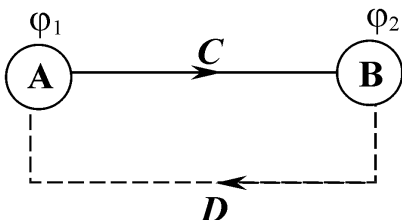
SI unit for current is *ampere* (*A*).

In order to create a current in a conductor we must create an electric field in it, hence, the potential difference must exist on its ends.

As it was experimentally established by Ohm, *the current in a conductor is directly proportional to the potential difference* φ_1-φ_2 *on its ends, and inversely proportional to the resistance of this conductor*:

$$I = \frac{\varphi_1 - \varphi_2}{R}. \tag{3}$$

Let's find out under which condition the direct current will flow through a conductor. If two bodies *A* and *B* (Fig. 1), charged up to potentials φ_1 and φ_2 (let $\varphi_1 > \varphi_2$), connect with the wire *C*, then the forces of electric field will cause the flow of free charges in it. A current appears in the wire *C*, hence the potential of the body *A* will decrease, and of *B* – will increase until they become equal. The field inside *C* will disappear and the current will stop. Hence, charged bodies can give a current only for a short period of time only if we don't take special action to maintain electric field in the wire. For direct current flowing through the wire *C*, we should create such conditions, when potential of body *A* doesn't decrease, and of body *B* doesn't increase. In order to do this we must return the charges, moved from the body *A* to *B* back on the body *A* along the path *D* (see Fig. 1). But we have to move charges against the force of electrostatic field on *D*. Such work can be done only by forces of non-electrostatic origin, for example, mechanical, magnetic, chemical etc. These forces are called ***extraneous***.

Figure 1 – Direct current appearance mechanism

Quantity, equal to work, done by extraneous forces required to move positive unit charge inside the source against the forces of electrostatic field, is called an ***electromotive force (EMF) of a source***.

$$\varepsilon = \frac{A_{\text{EXT}}}{q}. \tag{4}$$

Quantity, equal to work, done by forces of electrostatic field required to move the positive unit charge from one point of the field to another, is called ***potential difference***, *or* ***voltage*** *between these points of the field*:

$$\varphi_1 - \varphi_2 = \frac{A_{\text{FIELD}}}{q}. \tag{5}$$

Let's consider the circuit, which consists of two EMF sources, switched one towards another, r_1 and r_2 are their internal resistance. Also the circuit has resistors with resistances R_1, R_2, R_3, R_4 (Fig. 2).

Let $\varepsilon_1 > \varepsilon_2$, then current in the circuit will flow clockwise. Let's derive Ohm's law for the circuits' part *ABC*, which contains EMF.
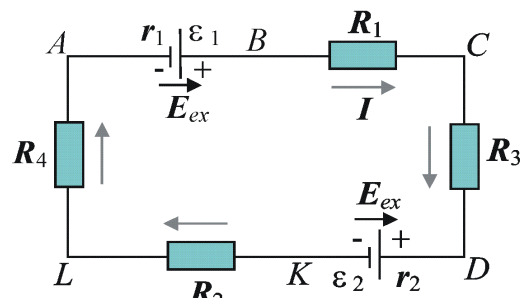
Figure 2 – Derivation of Ohm's law

Resistance of the part is R_1+r_1. This part of the circuit has electric forces and extraneous forces. If the wires are immovable, the only result of a current flowing will be emitting of a heat. By the time t on the part ABC will be emitted quantity of heat Q, which, according to the Joule-Lenz law, is

$$Q = I^2 \cdot (R_1 + r_1)t. \tag{6}$$

This quantity of the heat equals to algebraic sum of works of electrostatic and extraneous forces

$$Q = A_{EXT} + A_{ES}. \tag{7}$$

By the time t through the wire flows the charge

$$q = I \cdot t.$$

Work of electrostatic forces

$$A_e = (\varphi_A - \varphi_C)q.$$

Work of extraneous forces

$$A_{EXT} = \varepsilon_1 \, q.$$

In the part ABC forces of electrostatic field moves the positive charge in the direction of the current, from A to C. Extraneous forces move the positive charge in the source from negative pole to positive, against the field. As it is seen from Fig. 11, electric and extraneous forces move the charge in the same direction. That's why both works in equation (7) should be taken with a plus sign:

$$I^2 (R_1+r_1)t = (\varphi_A - \varphi_C) \, q + \varepsilon_1 q.$$

Reducing both parts of the equation on $q = It$, we obtain:

$$I_2 (R_1+r_1) = \varphi_A - \varphi_C + \varepsilon_1. \tag{8}$$

*A product of a current on a resistance of circuits' part is called a **voltage drop** in this part.* From (8) it follows that *voltage drop on a part of a circuit equal to algebraic sum of works, done by both electrostatic and extraneous forces, required to move unit positive charge along this part.*

In the part DKL extraneous and electrostatic forces are opposite to each other (in the source ε_2 extraneous forces move the positive charge in the direction, which is opposite to the current). Extraneous forces create negative work. Equation (8) will look like

$$I (R_2+r_2) = (\varphi_D - \varphi_L) - \varepsilon_2. \tag{9}$$

In more common case this equation will look like

$$I (R +r) = \varphi_1 - \varphi_2 \pm \varepsilon. \tag{10}$$

The current

$$I = \frac{\varphi_1 - \varphi_2 \pm \varepsilon}{R + r}. \tag{11}$$

This equation represents Ohm's law for the part of circuit, which includes EMF. Such part is called **non-uniform**.

In order to write equations (10) and (11) correctly, we should keep up the rule of signs.

1. First of all, we should choose the direction of bypassing the part of a circuit, for example, from A to C (look at Fig. 2).

2. If the direction of bypassing coincides with the direction of the current, in the left part of equations current will be written with the plus sign. In the opposite case – with the minus sign.

3. In the right part of equations in the potential difference part all terms should be written in the bypass direction. For example, if bypassing from *A* to *C* then it will be $\varphi_A - \varphi_C$, but not $\varphi_C - \varphi_A$.

4. The magnitude of EMF should be taken with the plus sign, if we mentally move from negative pole of the source to the positive with chosen direction of bypassing. In the opposite case EMF will be with the minus sign.

Analysis of equation (10) gives us such consequences::

1. On the part, which doesn't include EMF, voltage drop equals to potential difference on its ends, for example, on the part *CD*.

$$IR_3 = \varphi_C - \varphi_D. \tag{12}$$

The part of a circuit, that doesn't include EMF, is called **uniform**. For it we have

$$I = \frac{\varphi_C - \varphi_D}{R_3}.$$

2. *Potential difference on the poles of source isn't equal to its EMF if there is a current in it.*

For the part *AB*:

$$\varphi_B - \varphi_A = \varepsilon_1 - Ir_1. \tag{13}$$

Same for *KD*:

$$\varphi_D - \varphi_K = \varepsilon_2 - Ir_2. \tag{14}$$

If the current in a source flows from negative pole to the positive (coincides with EMF), as it on *AB*, then the potential difference on poles of the source is less than its EMF on the magnitude of voltage drop in the source. If the current flows in the source from positive pole to negative (against EMF), as it on *KD*, then potential difference is greater than EMF on the magnitude of voltage drop in the source.

3. Potential difference on the poles of the source equals to its EMF, if the current in the source equals zero, for example, in the open circuit.

4. If the circuit consists of one source with EMF ε with internal resistance *r* and external resistance *R*, then for ***total circuit*** (closed) equation (10) looks as

$$\varepsilon = IR + Ir. \tag{15}$$

EMF equals to the sum of voltage drops on an external and internal parts of the circuit. Equation (11) for such circuit looks as

$$I = \frac{\varepsilon}{R + r}. \tag{16}$$

2.2 Distribution of potential along the closed circuit

By the given value of EMF of the sources and resistances of all parts of the circuit, we can build a graph of potential distribution in the circuit.

Let's consider unbranched circuit with two sources, connected towards one to another (Fig. 3). Let $\varepsilon_2 > \varepsilon_1$.

With this condition current in the circuit flows counterclockwise. Let's build the graph of potential distribution along the circuit *ABCDA,* we choose the bypass direction − clockwise, against the current. On axis of ordinates we put potentials of marked points *A*, *B*, *C*, *D*, and on axis of abscises − resistance of the parts, located between these points. Current can be calculated with the formula

$$I = \frac{\varepsilon_2 - \varepsilon_1}{R_1 + R_2 + r_1 + r_2}.$$ (17)

Let the potential of point A be zero. On the graph (Fig. 4) this point will be in the origin of coordinates. We can calculate potentials of the points B, C and D using Ohm's law for the part of the circuit (formula (10)). For simplification we will take resistances of the sources equal zero. We can define potential of the point B from Ohm's law for part AB. As the direction of bypassing is opposite to that of for current, then voltage drop will be with minus sign:

$$-Ir_1 = \varphi_A - \varphi_B + \varepsilon_1,$$

here ε_1 has been taken with plus sign, because we pass the source from (−) to (+), in the direction of potential increasing. Potential of the point B equals

$$\varphi_B = \varphi_A + \varepsilon_1 + Ir_1,$$

but as $r_1 = 0$, then

$$\varphi_B = \varphi_A + \varepsilon_1.$$

Figure 3 – Calculation of potential distribution along closed circuit

Potential of the point B is higher than potential of point A on the value of EMF. In order to plot point B on the graph, we should put value ε_1 on axis of ordinates, along the axis of abscises we don't have to put anything, because $r_1 = 0$.

For defining of the potential of the point C let's write Ohm's law for the part of the circuit BC (bypass from B to C):

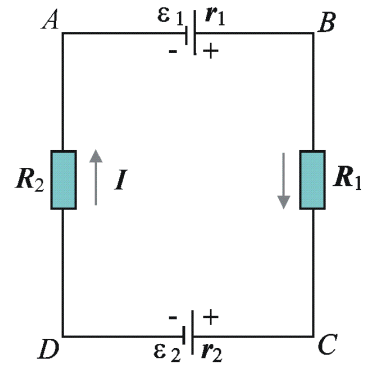$$-IR_1 = \varphi_B - \varphi_C;$$
$$\varphi_C = \varphi_B + IR_1.$$

Potential of point C is higher than potential of point B on the quantity IR_1.
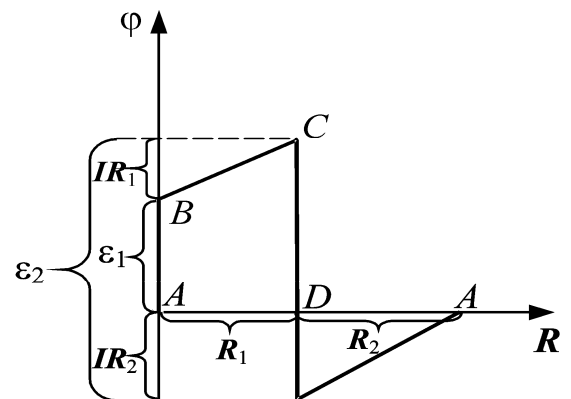
It's evident from the Fig. 4. Current on the uniform part of the circuit always flows from the point with greater potential to the point with smaller potential. If we know the direction of a current, we can immediately define where potential is higher, and where it is lower. On the part BC current flows from C to B, so $\varphi_C > \varphi_B$.

For building corresponding point C along the axis of abscises on the graph, we should put R_1, and along the axis of ordinates add IR_1 to the potential of point B. On the graph the part BC will be straight, as along on the part of circuit BC in the direction of bypassing potential grows linearly according to the law $\varphi_C = \varphi_B + IR$. The tangent of an angle of inclination of the section BD to the axis of abscises numerically equals to quantity of a current on this part. It follows from Ohm's law

$$I = (\varphi_C - \varphi_B) / R_1.$$

Figure 4 – distribution of potential along closed circuit at Fig.3 with no internal resistance

For defining the potential of point D let's write Ohm's law for the part of circuit *CD*.

$$-Ir_2 = \varphi_C - \varphi_D - \varepsilon_2;$$
$$\varphi_D = \varphi_C - \varepsilon_2 + Ir_2.$$

In consideration of $r_2 = 0$, we get

$$\varphi_D = \varphi_C - \varepsilon_2.$$

Potential of the point B is lower than potential of the point C on the quantity ε_2. On the graph correspondent point D will lie lower than point C but on one vertical with it.

Let's compare potential of point D with potential of point A. In order to do this we have to write Ohm's law for the segment *DA*:

$$-IR_2 = \varphi_D - \varphi_A;$$
$$\varphi_D = \varphi_A - IR_2.$$

Potential of the point D is lower than potential of point A on the quantity IR_2, so potential of the point D is negative and correspondent point D lies lower than axis of abscises on the graph.

For finishing of plotting the graph we must add resistance R_2 along axis of abscises and put the point A with zero potential again. Points D and A have to be joined with a straight segment, as the part of circuit *DA* is uniform. On the graph, segments *BC* and *DA* must be parallel, because current on the parts *BC* and *DA* is the same.

Figure 5 – distribution of potential along closed circuit at Fig.3 including both internal and external resistances

Let's consider the general case, when we can't neglect the internal resistances of sources. For potentials of the points A, B, C, D we'll have

$$\varphi_A = 0; \qquad \varphi_B = \varphi_A + \varepsilon_1 + Ir_1; \qquad \varphi_C = \varphi_B + IR_1; \qquad \varphi_D = \varphi_C - \varepsilon_2 + Ir_2.$$

For same ε_1, ε_2, R_1, R_2, current in the circuit will be lower.

Potentials of points B, C and D will change. If in the source potential changes uniformly along the internal resistance, the graph will be similar to that of shown on the Fig. 14. Segments *BC* and *DA* here also will be parallel, but its inclination angle will become smaller. If on the parts *AB* and *CD* potential changes non-uniformly along resistances r_1 and r_2, then the shape of segments *AB* and *CD* becomes more complex.

2.3 Kirchhoff's rules

For a solution the branched circuits use two laws describing the flow of currents in electric circuits.

2.3.1 First Kirchhoff's rule

In the complex electric circuit at one point can converge more than two wires (points B and D on the Fig. 6). Such points are called **nodes**. If currents in branches are constant, quantity of the charge, which flow into the point B by specific time, must be equal to the quantity of charge, which flow out it for the same time. That's why sum of currents, which flow into the node, equal to the sum of currents, which flow out it. For the node B

$$I_1 + I_3 = I_2, \qquad \text{or} \qquad I_1 - I_2 + I_3 = 0.$$

Kirchhoff's first rule states that algebraic *sum of currents, which converge in the node, equals zero*. This rule is also called , Kirchhoff's point rule, Kirchhoff's junction rule (or nodal rule) and Kirchhoff's current law. In general it looks like:

$$\sum_{i=1}^{n} I_i = 0. \tag{18}$$

2.3.2 Kirchhoff's second rule

This rule is also called Kirchhoff's second law, Kirchhoff's voltage law, and Kirchhoff's **closed loop** rule.

In any closed loop, picked out in the complex branched circuit, algebraic sum of voltage drops on all parts of that loop equal to algebraic sum of EMFs, which acts in this loop.

$$\sum_{i=1}^{n} I_i R_i = \sum_{k=1}^{m} \varepsilon_k. \tag{19}$$

In order to write the second Kirchhoff's rule, we should somehow choose the direction of circuit's bypass. Voltage drop should be taken with a plus sign for those parts, where the direction of bypass coincides with direction of a current, and with minus sign – if bypass opposite to the current. EMF should be taken with the plus sign if bypassing is from the negative pole to positive. In other case EMF should be taken with the minus sign.

Kirchhoff's second rule, for the loop *ABDA*, looks as

Figure 6 – Kirchhoff's rules

$$I_1(R_1+r_1) - I_3 (R_3+r_3) + I_1(R_4+r_4) = -\varepsilon_1 + \varepsilon_3 + \varepsilon_4.$$

3 Description of experimental setup and methodology of measurements

Devices and outfits: Voltage source, microampermeter, two sets of resistances, resistor with known resistance, two resistors with unknown resistance.

Wheatstone bridge is a circuit, shown in Fig. 7, that consists of four parts with resistors R_x, R_1, R_2, R_3. These parts are called the *bridge arms*. Unknown resistor R_x is included in the arm *AB*, which has to be measured in this work. Resistors sets are

switched in the arms *BC* and *CD*, so R_3 and R_2 can be varied. Resistor R_1, of the arm *AD* is constant. So, resistors R_1, R_2, R_3 are known. Diagonal *AC* has a source of current; the diagonal *BD* has microampermeter as zero-indicator (zero is in the middle of the scale).
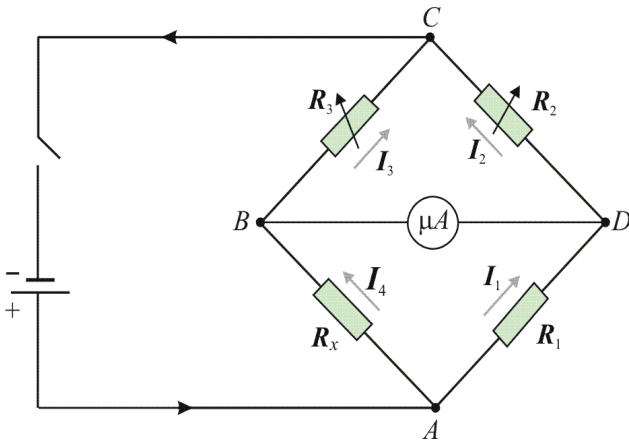

Figure 7 – Wheatstone bridge

When we close the circuit, through the parts *AB*, *AD*, *BC*, *DC* the currents in directions , which are shown in the Fig. 17 will flow. In the diagonal *BD* through microampermeter current will flow from *B* to *D*, if potential of point *B* is greater than potential of the point *D* ($\varphi_B > \varphi_D$), and from *D* to *B*, when $\varphi_D > \varphi_B$. As the points *B* and *D* lie between the points *A* and *C*, their potentials are always between φ_A and φ_C ($\varphi_A > \varphi_C$), as the point *A* switched to the positive pole of the source. So, by changing R_3 and R_2, we can always get the equality of potentials of points *B* and *D* ($\varphi_B = \varphi_D$). In this case, current in diagonal *BD* will be equal zero. The state of electric circuit, when current in microampermeter equals zero is called *balance of the bridge*.

Let's derive the relation between resistances of bridge's arms, when becomes its balance. Ohm's law for the arm *AB*:

$$I_4 = \frac{\varphi_A - \varphi_B}{R_x}, \quad \text{or} \quad I_4 R_x = \varphi_A - \varphi_B. \tag{20}$$

The same for the arm *AD*

$$I_1 = \frac{\varphi_A - \varphi_D}{R_1}, \quad \text{or} \quad I_1 R_1 = \varphi_A - \varphi_D. \tag{21}$$

As at the moment of balance $\varphi_B = \varphi_D$, then the right parts of equations (20) and (21) are equal, so the left are equal too:

$$I_4 R_x = I_1 R_1. \tag{22}$$

Same for arms *BC* and *DC*

$$I_3 R_3 = I_2 R_2. \tag{23}$$

At the moment of balance current doesn't flow through the ampermeter, so at the point *B* current I_4 doesn't branch out, but completely flows through resistor R_3 so $I_4 = I_3$.

At the point *D* current I_1 also moves to the part *DC*. Then we can write equations (22), (23) as

$$I_4 R_x = I_1 R_1; \qquad I_4 R_3 = I_1 R_2.$$

If we divide the first equation into the second, we will get

$$\frac{R_x}{R_3} = \frac{R_1}{R_2}. \tag{24}$$

This is the condition of the bridge's balance

$$R_x = R_3 \frac{R_1}{R_2}. \tag{25}$$

From the formula (25) the unknown resistor can be calculated. Formula (25) can be derived with help of Kirchhoff's rules for the circuits *ABDA* and *BCDB*, and nodes *B* and *D*. It is proposed for students to make it by oneself.

4 Data processing
For representation of the result of measurand x it is necessary:
1) Obtain the sequence of data x_1, x_2, x_3, ..., x_n (reduse to a Table of measurements).
2) Calculate the average value of measurand:

$$<x> = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i. \tag{26}$$

3) Find an abmodality each measurement (in a Table of measurements):

$$\Delta x_1 = <x> - x_1; \quad \Delta x_2 = <x> - x_2; \quad ...; \quad \Delta x_n = <x> - x_n.$$

4) Square each abmodality and summarize (in a Table of measurements):

$$\sum_{i=1}^{n} (\Delta x_i)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + ... + (\Delta x_n)^2.$$

5) Find a statistical absolute error of measurements from Sdudent's equation:

$$\Delta x_{ST} = t_{\alpha,n} \cdot \sqrt{\frac{\sum_{i=1}^{n} (\Delta x_i)^2}{n(n-1)}}. \tag{27}$$

where α – confidence probability; n – number of measurements; $t_{\alpha;n}$ – Sdudent's coefficient.
6) Find a device absolute error of measurements

$$\Delta x_{DEV} = \frac{\beta \cdot x_{max}}{100}, \tag{28}$$

β – accuracy class of electrical measuring instrument, x_{max} – grid limit.
7) Find a total absolute error of measurements

$$\Delta x = \sqrt{(\Delta x_{ST})^2 + (\Delta x_{DEV})^2}. \tag{29}$$

7) Calculate relative error of measurements:

$$\delta = \frac{\Delta x}{<x>}. \tag{30}$$

8) Final result is represented by a confidence interval and relative error:

$$x = (<x> \pm \Delta x)_\alpha; \qquad \delta\% = \frac{\Delta x}{<x>} \cdot 100\%.$$

5 Work execution order and experimental data analysis
1. Mount the scheme of the Fig. 7, and switch to the arm *AB* resistor R_x.
2. Write down into equipment table the values of parameters of resistors R_1, R_2 and R_3, specifically – accuracy class β_1, β_2 and β_3.

3. Set resistor R_2 in the arm *DC*, equal to R_1, that the relation become $R_1/R_2=1$.
4. Balance the bridge by changing resistor R_3.
5. Write the values R_2, R_3 into the table of measurements.
6. Make four more measurements for other values of relation R_1/R_2, for example, 1/3; 1/5; 1/7 and 1/9.
7. Calculate the average value of measurand *<R>* according to (26).
8. Complete a Table of measurements.
9. Specify a value of confidence probability as α=0,95, then obtain the value of Sdudent's coefficient as $t_{0,95\,;\,5}$= 2,77.
10. Calculate a statistical absolute error of measurements ΔR_{ST} according to (27).
11. Calculate a device absolute error of measurements ΔR_{ST} according to (28).
12. Calculate a total absolute error of measurements ΔR according to (29).
13. Calculate a relative error of measurements δ_R according to (30).
14. Write a final result as a confidence interval and relative error.
15. Conclude about magnitudes of statistical and device absolute errors.

6 Control questions
1. What are the SI units for resistance?
2. What does resistance of a conductor depends on?
3. How it is possible to define a total resistance in serial and parallel connections of resistors?
4. Draw the scheme of Wheatstone bridge.
5. What is the point of Wheatstone bridge method of measurements of resistances?
6. Derive the relation between resistances of bridge's arms at balance with Ohm's law.
7. Derive the relation between resistances of bridge's arms at balance with Kirchhoff's laws.
8. How it is possible to calculate statistical absolute error and device absolute error of measurements.

7 Content of the report

Laboratory work № 2-2
I. Home work (answer the control questions given by an instructor).
…
II. Laboratory work 2-2 implementation protocol.
1)Topic: *MEASUREMENT OF RESISTANCE WITH WHEATSTONE BRIDGE.*
2) Goal: 1 Study the method of measurements by means of a bridge circuit.
2 2 Study the method of data processing.
3 Finding the resistance of conductors.

3) Setup schematic diagram

here R_x – unknown resistor;
R_1 – etalon resistor.
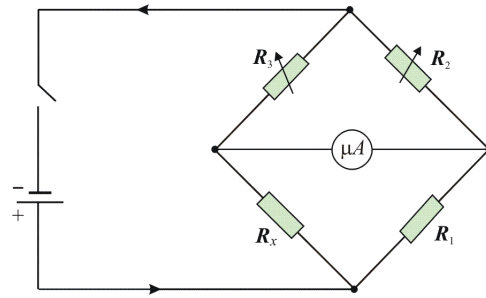
4) Table of measuring instruments:

№	Name	Type	Serial number	Grid limit	Grid unit	Accuracy class
1.	Etalon resistor R_1	МЛТ	-	10 kΩ	-	$\beta_1 = 1\ \%$
2.	One-decade resistance box R_2	Р33	-	90 kΩ	10 kΩ	$\beta_2 = 0{,}2\ \%$
3.	Four-decade resistance box R_3	Р33	-	9999 kΩ	1 Ω	$\beta_3 = 0{,}2\ \%$
4.	Microaperemeter (zero indicator)					

5) Equations for calculation:

1. At the ballance condition $I_{\mu A}=0$ of Wheatstone bridge the unknown resistance:

$$R_x = R_3 \frac{R_1}{R_2}.$$

2. Statistical absolute error of measurements:

$$\Delta R_x^{\mathrm{ST}} = t_{\alpha,n} \sqrt{\frac{\sum\limits_{i=1}^{n} (\Delta R_{xi})^2}{n(n-1)}},$$

where $\alpha = 0{,}95$ – confidence probability; $n = 5$ – number of measurements;
$t_{0,95\,;\,5} = 2{,}77$ – Sdudent's coefficient.

3. Device absolute error of measurements:

$$\Delta R_x^{\mathrm{DEV}} = \langle R_x \rangle \cdot \delta = \langle R_x \rangle \cdot \sqrt{\left(\frac{\beta_1}{100}\right)^2 + \left(\frac{\beta_2}{100}\right)^2 + \left(\frac{\beta_3}{100}\right)^2},$$

Where $\langle R_x \rangle$ – average value of measurand; δ – relative error of measurements; β_i – accuracy class of electrical measuring instrument.

4. Total absolute error of measurements:

$$\Delta R_x = \sqrt{(\Delta R_x^{\mathrm{ST}})^2 + (\Delta R_x^{\mathrm{DEV}})^2}.$$

6) Table of measurements: $R_1 = 10\ k\Omega = 10000\ \Omega$;

№	R_2, Ω	R_3, Ω	R_{xi}, Ω	$\Delta R_{xi}, \Omega$	$(\Delta R_{xi})^2, \Omega^2$
1.	10 000				
2.	30 000				
3.	50 000				
4.	70 000				
5.	90 000				
	Average value $\langle R_x \rangle =$	...		$\sum\limits_{i=1}^{5} (\Delta R_{xi})^2 =$	...

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7) Quantities calculation: …

8) Final results:

$R_x = (<R_x> \pm \Delta R_x)_\alpha \ \Omega = (\ ... \pm ... \ )_{0,95} \ \Omega;$ $\qquad\qquad \delta_{R_x} = \dfrac{\Delta R_x}{\langle R_x \rangle} \cdot 100\% = ... \ \%.$

9) Conclusions:

10) Work done by: **Work checked by:**

LABORATORY WORK № 2-3
DETERMINATION OF E.M.F. OF CURRENT SOURCE

1 Goal of the work:
Determine the EMF of a current source by:
a) Comparison method
b) Method of known resistance

2 Main concepts
2.1 Ohm's law (see main concepts of laboratory work № 2-2).

2.2 Distribution of potential along the closed circuit (see main concepts of laboratory work № 2-2).

2.3 Kirchhoff's rules (see main concepts of laboratory work № 2-2).

3 Description of experimental setup and methodology of measurements
Devices and outfits: Two sources of current, voltmeter, microamperemeter, miliamperemeter, potentiometer, two resistors, set of resistances.

The stand of laboratory table has two sources. EMF of one of them defines by the compensation method; EMF of another defines by the direct measurement meathod.

3.1 Measurement of EMF of a source with the compensation method.
If we connect the voltmeter (Fig. 8) directly to the terminals of a current source, then it will not show us the EMF of source. Voltmeter shows the potential difference, or the voltage between the points A and B, to which it is connected.

As it was shown earlier

$$\varphi_B - \varphi_B = \varepsilon - Ir.$$

Potential difference, which shows the voltmeter, is smaller than EMF on the magnitude of voltage drop on the internal resistance of the source. If Ir is smaller than voltmeter's error, then the value of EMF of the source can be taken equal to the index of voltmeter, and the measurement error has to be taken equal to the voltmeter's

Figure 8 – A voltmeter connected directly to EMF source terminals.

error. If the voltage drop inside the source is greater than voltmeter's error or even unknown, then we have to use compensation method to determine the EMF.

Compensation method is based on a fact that the potential difference on the poles of the source equals EMF, if the current in the source is absent. We can get this by another source, which is switched towards the first, as it is shown in Fig. 9, where ε_{aux} – EMF of auxiliary source, ε_x – EMF of the observable source, P – potentiometer; μA – microampermeter, which is used as zero–indicator. It is important, that the condition $\varepsilon_{aux} > \varepsilon_x$ was halt on. If we close the gate K_1, when the gate K_2 is opened, then current will flow through the circuit LCK_1L. On the part $CBAD$ current is absent. So, potentials of points B and C are the same. Potential of the point A is greater than potential of the point B on the quantity ε_x. If $\varphi_L - \varphi_C > \varepsilon_x$, then $\varphi_L > \varphi_{LA} > \varphi_C$, that's why we can always find such point D on potentiometer, which potential equals to potential of the point A ($\varphi_D = \varphi_A$). If we put cursor into the point D and close the gate K_2, current on the part AD won't flow, as $\varphi_A - \varphi_D = 0$. Current won't flow through the parts AB and BC, which are connected serially with the part AD. We say, that at this moment ε_x is being compensated. Index of the ampermeter equals zero. Let's write Ohm's law for the part $CBAD$:
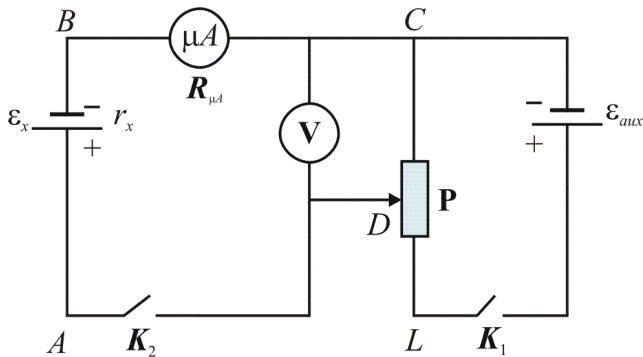
$$I(R_{\mu A} + r_x) = \varphi_C - \varphi_D + \varepsilon_x.$$

As $I=0$, then $\varphi_C - \varphi_D = U_V = \varepsilon_x$.

If we switch the voltmeter between points C and D, as shown in the Fig. 9, then at the moment of compensation its index will be equal to EMF ε_x, as voltmeter measures the potential difference $\varphi_D - \varphi_C = U_V$. We can equate the error of measurements with the voltmeter's error.

In order to measure EMF of the source with high accuracy, we don't use voltmeters, but special devices, which are called *potentiometers of direct current* PDCL and PDCV. Operation of these devises is based on the compensation method.

3.2 Measurement of EMF of the source with known resistance method
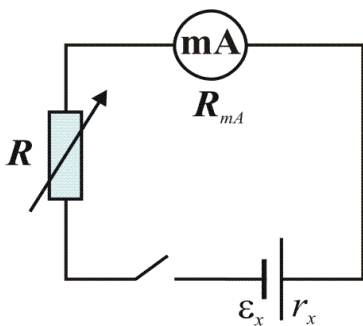
For measuring of EMF we need to mount the scheme, shown in the Fig. 10, which consists of observable source with unknown EMF ε_x and unknown resistance r_x, and also of miliampermeter and set of resistances R (Fig. 10).

Let with the resistance of a set equal R_1, microampermeter shows current I_1.

Let's write Ohm's law for whole circuit:

$$\varepsilon_x = I_1 R_1 + I_1(R_{mA} + r_x).$$

In this equation there are two unknown quantities ε_x and $(R_{mA} + r_x)$. That's why let's compose one more equation with these unknowns. Let's change the resistance of a set to R_2, miliampermeter will show I_2. Then

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$$\varepsilon_x = I_2R_2 + I_2(R_{mA} + r_x).$$

We've got a system of two equations with two unknowns. To get rid of (R_{mA} + r_x) let's multiply the first equation with I_2, and the second – with I_1, and subtract from the first, the second. We'll have

$$I_2\varepsilon_x - I_1\varepsilon_x = I_1\, I_2\, R_1 - I_1\, I_2\, R_2.$$

Hence

$$\varepsilon_x = \frac{I_1 I_2 (R_1 - R_2)}{I_2 - I_1}. \tag{31}$$

We can calculate EMF of the source with the formula (31). If the resistance of miliampermeter $R_{\mu A}$ is known, then we can define the internal resistance of the source r from two equations given above.

4 Data processing (see laboratory work № 2-2).

5 Work execution order and experimental data analysis
5.1 Compartison method
1. Mount the scheme Fig. 9. Turn on both observable and auxiliary sources. Resistance box R_2 simulate the sources internal resistance.
2. Set the value R_2=0. Set the potentiometer slider in the middle. Lock the gate K_1, and then K_2.
3. Moving the potentiometer slider, obtain absence of current through microampermeter.
4. Write voltmeter indications at the moment of compensation into the table of measurements.
5. Make five more measurements for other values of resistance box R_2, for example, 1, 3; 5; 7 and 9.
6. Calculate average value of EMF <ε_x> and complete a Table of measurements.
7. Calculate statistical absolute (27), device (28) and total (29) absolute errors $\Delta\varepsilon$ of measurements.
8. Calculate a relative error of measurements δ_ε.
9. Write a final result as a confidence interval and relative error.
10. Conclude about magnitudes of statistical and device absolute errors.

5.2 Direct method
1. Mount the scheme Fig. 8. Turn on only an observable source. Resistance box R_2 simulate the sources internal resistance.
2. Set the value R_2=0. Lock the gate K_2. Write voltmeter indications U_i into the Table of measurements.
3. Make five more measurements for other values of resistance box R_2, for example, 1, 3; 5; 7 and 9. It is equivalent to a changing of a sources.
4. Calculate a relative error $\beta_i^{\mathrm{DIR}} = \frac{\varepsilon_x - U_i}{\varepsilon_x} 100\%$ of each direct measurement (for each another sourse).
5. Represent the final result by a draph of dependence error of direct method of

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measurements β_i^{DIR} from relative magnitude of internal resistance of a source r/R_V.

6. Conclude, when the magnitude of error of direct method of measurements is less.

6 Control questions

1) What is potential, voltage (potential difference), EMF of the source, voltage drop?
2) How does Ohm's law look for uniform part of circuit, for non-uniform part of circuit, for the whole circuit?
3) When do potentials difference on the poles of the source equal to its EMF?
4) What is compensation method of measurement EMF of sources? Draw the scheme.
5) What is the method of EMF measurement by the known resistance? Draw the scheme and derive the formula for calculation ε_x and r.
6) How potentials of different points of the circuit can be calculated, if all EMFs of all sources and resistances of all parts of the circuit are known?
7) How do potentials of different points of the circuit in this work experimentally define?
8) How do current and an internal resistance of sources define in this work?

7 Content of the report

Laboratory work № 2-3

I. Home work (answer the control questions given by an instructor).

…

II. Laboratory work № 2-3 implementation protocol.

1) **Topic:** *DETERMINATION OF E.M.F. OF CURRENT SOURCE*.

2) **Goal:** Studying EMF measurement methods: a) compensation method; b) method of direct measurements.

3) **Scheme of device:**
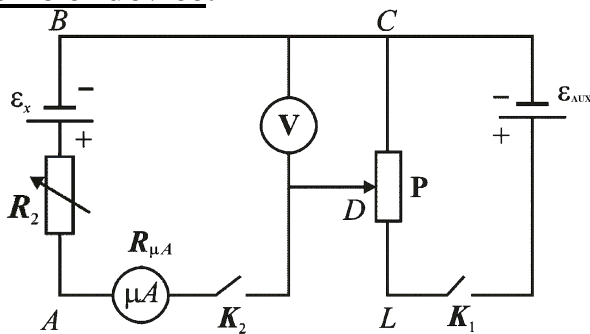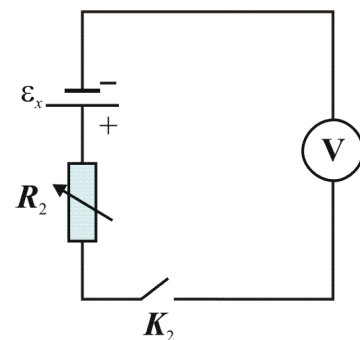
a) compensation method   b) direct measurements

4) **Table of measuring instruments:**

№	Name	Type	Serial number	Grid limit	Grid unit	Accuracy class
1.	One-decade resistance box R_2					
2.	Voltmeter					
4.	Microampermeter (zero-indicator)					

5) **Equations for calculation:**

5.1) Compensation method: $U_V = IR_V = \varepsilon_x - Ir$, where $r = r_x + R_2$.

When $I = 0$ (compensation condition), then $U_V = \varepsilon_x$.

Statistical absolute error of measurements we estimate by Sdudent's equation (27).

Device error of voltmeter from (28): $\Delta\varepsilon_x^{DEV} = \dfrac{\beta_V \cdot \varepsilon_{max}}{100}$, where β_V - is the accuracy

class of voltmeter, ε_{max} - its grid limit. Total absolute error we find by equation (29).

5.2) Direct measurement method: $U_V = IR_V = \varepsilon_x - Ir$, where $r = r_x + R_2$.

Relative error $\beta_i^{DIR} = \dfrac{<\varepsilon_x> - U_i}{<\varepsilon_x>} 100\%$, where $<\varepsilon_x>$ – average value of EMF from 5.1).

6) <u>Table of measurements:</u>

		Compensation method			Direct measurement method		
№	$r,\ k\Omega$	$\varepsilon_x,\ V$	$\Delta\varepsilon_{xi},\ V$	$(\Delta\varepsilon_I)^2,\ V^2$	$U,\ V$	$\beta_i^{DIR},\ \%$	r/R_V
1	$r_x + 0$						
2	$r_x + 10$						
3	$r_x + 30$						
4	$r_x + 50$						
5	$r_x + 70$						
6	$r_x + 90$						

average value $<\varepsilon_x>=$ **...** $\displaystyle\sum_{i=1}^{5}(\Delta\varepsilon_{xi})^2 =$ **...**

7) <u>Calculation of quantities:</u>   …

8) <u>Final results:</u>
8.1)

$\varepsilon_x = (<\varepsilon_x> \pm \Delta\varepsilon_x)_\alpha V = (\ ... \pm ... \ )_{0,95} V;$

$\delta_{\varepsilon_x} = \dfrac{\Delta\varepsilon_x}{\langle\varepsilon_x\rangle}\cdot 100\% = $ **...** $\%.$

8.1) Graph of β_2^{DIR} from r/R_V dependence:

9) <u>Conclusions:</u>

10) <u>Work done by:</u>   <u>Work checked by:</u>

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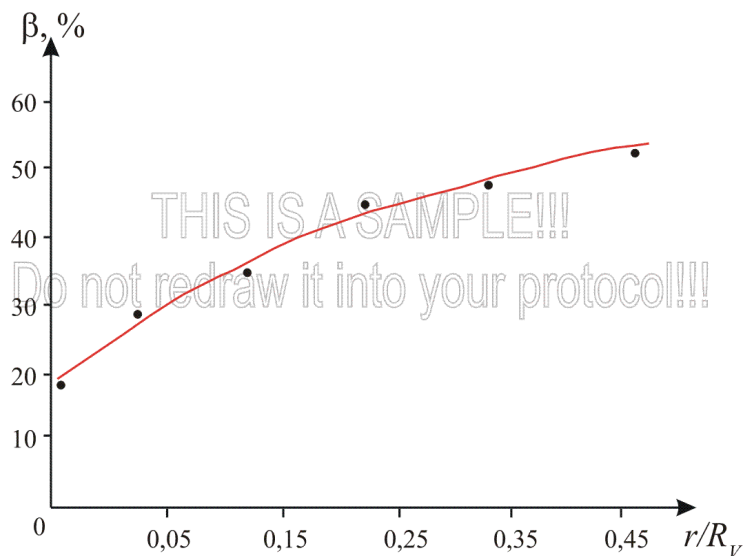
LABORATORY WORK № 2-4
POWER IN THE DIRECT CURRENT CIRCUIT

1 Goal of the work:

1 Study total power, useful power and electrical efficiency dependence on current and external load resistance.

2 Finding the condition of matching of source to a circuit loading.

2 Main concepts

2.1 Power in a DC circuit

Power of a current source *is quantity, which is numerically equal to work, done by the source per unit of time.*

For direct current flowing in the circuit, there must be a source, where the extraneous forces act. Work of extraneous forces has required to move charge q between poles of the source is equal

$$A_{EXTR}= \varepsilon q,$$

where ε – EMF of the source. As the result, the charges, concentrated by the extraneous forces on the poles, get potential energy, which is equal Eq. If we close the circuit on internal and external resistances then the current will flow in it.

Potential energy will change into kinetic energy of charge's motion. The last will change into a heat, if there are no other energy transformations (for example, into mechanical in electric motors etc.).

According to the energy conservation law, energy, that is generated along the whole circuit by the time t equals to work of extraneous forces Eq, done by this time.

Total power P_T, is a power which is *generated by source*:

$$P_T = \frac{A_{EXTR}}{t} = \frac{\varepsilon q}{t} = \varepsilon I .$$ (32)

Useful power P_U, is a power, which is *emit on the external part of the circuit*:

$$P_U=IU,$$ (33)

where U – potential difference on the poles of the source.

The ***power loss P_L*** due to heating of the internal resistance of source:

$$P_L=I^2r.$$ (34)

The commitment of energy consumptions of the current source is characterized by the efficiency η. ***Efficiency*** *of the source is the quantity, which is numerically equal to ratio of useful power P_U due to total power P_T*:

$$\eta =\frac{P_U}{P_T} .$$ (35)

If we take into account equations (39) and (40) we can write

$$\eta =\frac{U}{\varepsilon} .$$ (36)

2.2 Dependence of total power, useful power and efficiency of the source from the external load resistance. Maximum power theorem

According to Ohm's law, current in the circuit equals

$$I = \frac{\varepsilon}{R+r},$$

where R – external load resistance of the circuit, r – internal resistance of the source. If we put this expression into the formula for **total power** (32), we'll get

$$P_T = \frac{\varepsilon^2}{R+r}. \tag{37}$$

1) It is evident, that a total power gets the greatest value, when denominator is minimal, when $R=0$, beacose ε and r are not changing for given source. This is the case of a short circuit.

2) When R grows, total power decreases (Fig. 11). When $R = \infty$, $P_T = 0$.

Formula of **useful power** (33) can be changed, if we use $U\ IR$, and $I=\varepsilon / (R+r)$.

$$P_U = IU = I^2 \cdot R; \qquad P_U = \frac{\varepsilon^2 R}{(R+r)^2}. \tag{38}$$

Graph of dependence of useful power from external resistance is shown in the Fig. 11. As it follows from the formula (38):

1) $P_U = 0$, when $R=0$ (short circuit) and when $R = \infty$ (open circuit). It is evident, that with some intermediate value of R useful power has to be maximal.

2) In order to find this value R, we need to differentiate equation (38) with R and make the first derivative vanish:

$$\frac{dP}{dR} = \frac{\varepsilon^2}{(R+r)^2} - \frac{2\varepsilon^2 R}{(R+r)^3} = \frac{\varepsilon^2(R+r-2R)}{(R+r)^3} = \frac{\varepsilon^2(r-R)}{(R+r)^3} = 0.$$

Obtained expression changes into zero when $R - r = 0$. So, useful power has maximum, when the external resistance equals to the internal ($R = r$).

Now let's derive the law of the **source efficiency** dependence from the external resistance, using Ohm's law:

$$\eta = \frac{U}{\varepsilon} = \frac{IR}{I(R+r)} = \frac{R}{R+r} = \frac{1}{1+r/R} \tag{39}$$

From that formula it follows:

1) when $R = 0$, $\eta = 0$, as $r/R = \infty$ (short circuit); in this case, total power P_T is maximal, but all of it disinterestedly dissipates in the source;

2) when $R = r$, $\eta = 0,5 = 50\%$; in this case $P_U = P_T / 2$;

3) when $R = \infty$, $\eta = 1 = 100\%$ - maximal efficiency; in this case useful power equals total power, but each of them equals zero.
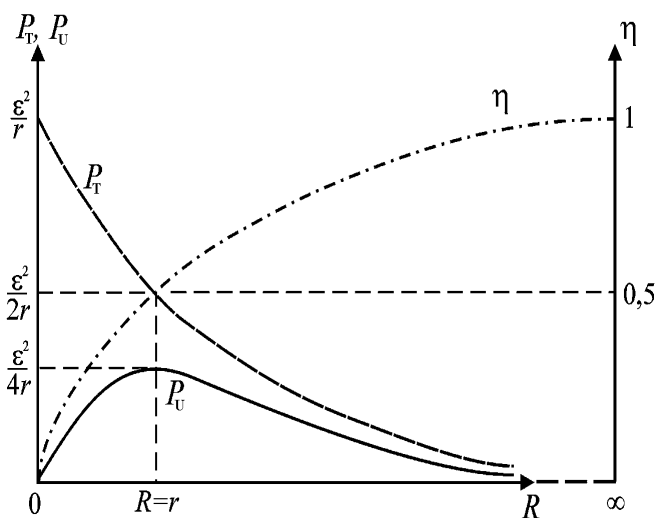

Figure 11 – Maximum power theorem

Graph of efficiency dependence from R is shown in the Fig. 11.

Generalizing all considered above we can conclude that, *in order to obtain maximum useful power from a source with a finite internal resistance, the external load resistance must be made with the same way as that of the source.* This statement is known as the **Maximum power theorem** it get the condition of matching of source to a circuit loading. Note that the theorem applies to maximum power but not maximum efficiency.

2.3 Dependence of total power, useful power and efficiency of the source from a current

Let's derive formulas of dependencies P_T, P_U, η from a current, for a source, in which EMF and internal resistance are constant.

Total power of the source is directly proportional to a current:
$$P_T = I\varepsilon.$$
Total power vanish $P_T = 0$, when $I = 0$ (open circuit). Total power has the largest value $P_T = \varepsilon^2 / r$, when current is maximal $I_{SC} = \varepsilon / r$ (short cirquit).

Useful power in a complicated manner depend on current:
$$P_U = IU, \text{ but } U = \varepsilon - Ir, \text{ so } P_U = I\varepsilon - I^2 r.$$
Graph of dependence $P_U = f(I)$ is the second order curve (parabola). Evaluation gives
$$P_U = I(\varepsilon - Ir).$$
Useful power vanish $P_U = 0$, when $I = 0$ (open circuit). Useful power also vanish $P_U = 0$ when $\varepsilon - Ir = 0$, or $I_{SC} = \varepsilon / r$ (short circuit).

It's evident that with any intermediate value of a current useful power must be maximal. In order to find this value of current, we need to differentiate expression (40) with current and make first derivative vanish:
$$\frac{dP}{dI} = \varepsilon - 2Ir = 0 \Rightarrow I = \varepsilon / 2r.$$
Such current, that equals to the half of short circuit current, occurs when $R + r = 2r$, or $R = r$. We've got the same result, as in the case of dependence of useful power from R.

Efficiency dependence from a current can be derived, if in equation (36) substitute the expression $U = \varepsilon - Ir$. Then
$$\eta = \frac{U}{\varepsilon} = \frac{\varepsilon - I_r}{\varepsilon} = 1 - \frac{r}{\varepsilon} \cdot I . \tag{40}$$

Graph of efficiency dependence from current represents a straight line with negative angular coefficient.

$\eta = 1$ when $I = 0$ (open circuit, $P_U = P_T = 0$).

$\eta = 0$ when $I_{SC} = \varepsilon / r$ (short circuit, P_T maximal, but $P_U = 0$).

3 Description of experimental setup and methodology of measurements

Devices and outfits: Source of a current, voltmeter, miliampermeter, potentiometer, resistance box.

In order to research the dependence of power and efficiency of the source from a current, we mount the scheme, which is shown in the Figure 12.

With using only two terminals: middle one and one of the extreme we can set potentiometer, which is on the panel of laboratory table, as changeable resistance. When we move a car of potentiometer, we change a current in the circuit and voltage on the external resistance. After obtaining some indexes of miliampermeter and correspondent to the indexes of voltmeter, we need to calculate the value of total power (32), useful power (33) and efficiency of the source (36).

With the received data we have to build graphs of total and useful powers and efficiency of the source versus current.

In order to research the dependence of total and useful powers and efficiency of the source from the external resistance of the circuit, we mount the scheme, that is shown in the Fig. 13. We use resistance box as an external resistance. When we change resistance of the box, we change current in the circuit. For some values of current and resistance we need to calculate the value of total power (37), useful power ($P=I^2R$) and efficiency of the source (39). With the received data we have to build graphs of total and useful powers and efficiency of the source versus external resistance of the circuit.
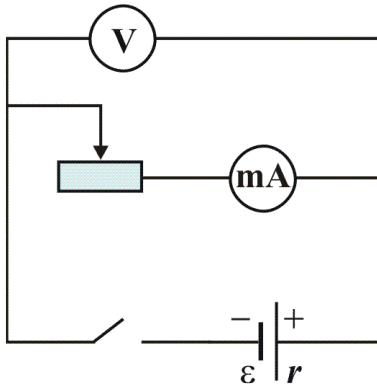
Figure 12 – Scheme for researching of power and efficiency dependence from a current
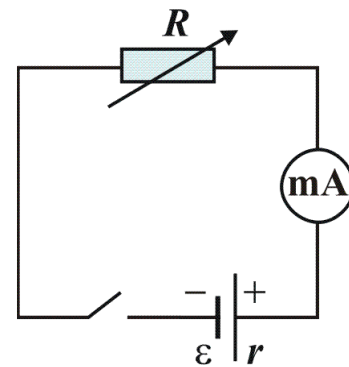
Figure 13 - Scheme for researching of power and efficiency dependence from external resistance

4 Work execution order and experimental data analysis

Researching useful power, total power and efficiency of the source dependence from external resistance of a circuit

1. Mount scheme Fig 13. Set resistance box as changeable resistance.
2. Determine minimal and maximal values of a current in a circuit. Divide that interval on 9 – 10 approximately equal parts for gridding of a current axis.
3. Make 10 – 11 measurements of a current at the points of obtained grid with matching values of resistances. Results of measurements write into the table of measurements.
4. Be sure to make measurement at $U = \varepsilon / 2$ (exceptional point).
5. Calculate total, useful power and efficiency. Results of calculations write into the table of measurements.
7. Plot the dependence of P_T, P_U and η versus load resistance on one graph.
8. Evaluate from graph a value of internal resistance of a sourse.
9. Conclude, when the circuit works with maximal efficiency and with maximal useful power.
10. Conclude about the condition of matching of source to a circuit loading.

5 Control questions

1) What is total power, useful power, and efficiency of a source?
2) Write formulas of total power, useful power and efficiency dependencies on a current. What values of a current do they equal zero with? Are there maximal values?
3) Derive what value of a current useful power is maximal with?
4) Write formulas of total power, useful power and efficiency dependencies on external resistance. What values of external resistance do they equal zero with? Are there maximal values?
5) Derive what value of external resistance useful power is maximal with?
6) Derive a formula of efficiency dependence on external resistance. Analyze an obtained formula. Build a graph.
7) Derive a formula of total power dependence on external resistance. Analyze an obtained formula. Build a graph.
8) Derive a formula of useful power dependence on external resistance. Analyze an obtained formula. Build a graph.
9) What is efficiency of a source?
10) What is useful power? Where does it generate?
11) What is total power of a source?

6 Content of the report

Laboratory work № 2-4

I. Home work (answer the control questions given by an instructor)**.**

…

II. Laboratory work 2-4 implementation protocol.

1) Topic: *POWER IN A DIRECT CURRENT CIRCUIT.*

2) Goal:
1 Study total power, useful power and electrical efficiency dependence on current and external load resistance.
2 Finding the condition of matching of source to a circuit loading.

3) Setup scheme
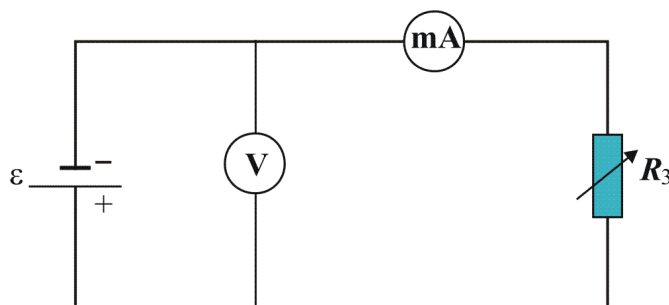
4) Table of measuring instruments:

№	Name	Type	Serial number	Grid limit	Grid unit	Accuracy class
1.	Four-decade resistance box R_3					
2.	Voltmeter					
4.	Miliampermeter					

5) Equations for calculations:

1. $P_T\,[mW] = I\,[mA]\cdot\varepsilon;$ 2. $P_U\,[mW] = I^2\cdot R\cdot 10^{-3};$ 3. $\eta = (P_U\,/\,P_T)\cdot 100\,\%$.

6) Table of measurements: $R = R_3.$ $\varepsilon = U_V$, when $I = I_{min}$ (or $R_3 = 9\,999\,\Omega$).

№	$I,\,mA$	$R,\,\Omega$	$P_T,\,mW$	$P_U,\,mW$	$\eta,\,\%$
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
$\varepsilon\,/\,2$					

7) Calculation of quantities:

8) Final results:
Dependences of
total power P_T,
useful power P_U and
efficiency η
versus load resistance:
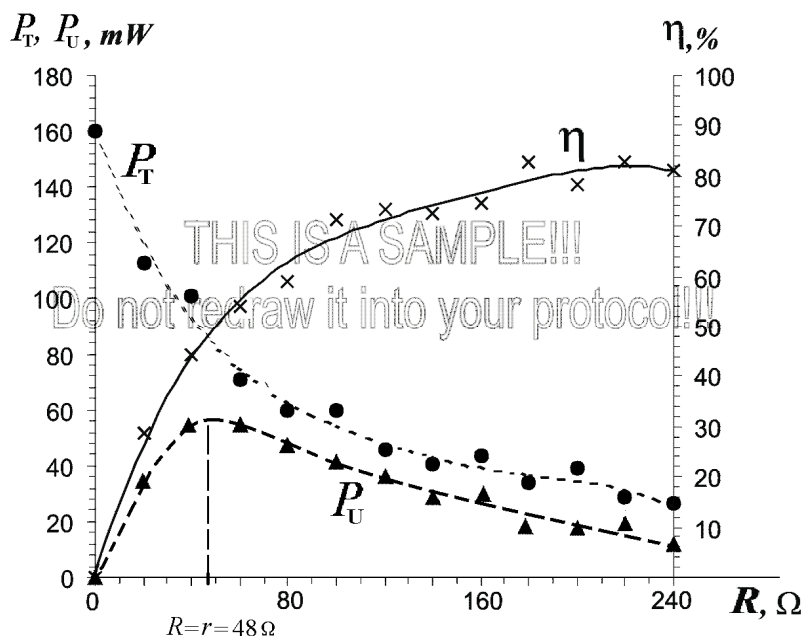
9) Conclusions:
9a) …
9b) …

10) Work done by: Work checked by:

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LABORATORY WORK № 3-1
STIDYING MAGNETIC FIELD VIA TANGENT-COMPASS

1 Goal of the work:

1 Study of method of a vector diagrams for representation of force fields.

2 Finding the horizontal component of a magnetic field intensity vector in the laboratory via tangent-compass.

2 Main concepts

2.1 Magnetic field of a current. Induction and intensity of magnetic field

Any moving electric charge creates a magnetic field, main property of which is that the other charges, which are moving in this field, experiences a force. It is well known that wires with parallel currents appeal and with anti-parallel – repeal. Explanation of it is that the magnetic field of moving charges of first wire acts on moving charges of the second. Permanent (stable in time) magnetic field doesn't act on static charges. Magnetic field, surrounding permanent magnets, creates with orientation in one specified direction of atomic micro-currents, which are directed motion of electrons in atoms.

Now, let's consider magnetic field created with currents running in different shaped conductors.

Product of a current *I* and infinitesimal segment of wire *dl* is called **element of the current *Idl***. Element of the current is a vector, whose direction coincides with that of a current. Element of the current in Magnetism is similar to test charge in Electrostatics – it is a probe with the help of which we can explore magnetic field. As it has been established by Ampere, any element of the current, placed in a magnetic field, will experience a force:

$$dF = I \cdot [\vec{dl} \times \vec{B}], \text{ or, in scalars }, dF = B \cdot Idl \sin(\vec{dl} \stackrel{\wedge}{\ } \vec{B}). \qquad (41)$$

Quantity $\vec{B}$ is a vector of magnetic induction. It has same direction as a force, with which magnetic field acts on a north pole of magnetic arrow. Equation (1) gives us a physical sense of $\vec{B}$:

$$B = dF / Idl \sin(\vec{dl} \stackrel{\wedge}{\ } \vec{B}). \qquad (42)$$

Vector of magnetic induction *numerically equal to force, with which magnetic field acts on a unit element of current (I·dl=1A·m), placed perpendicularly to vector of magnetic induction (sin($\vec{dl} \stackrel{\wedge}{\ } \vec{B}$)=1).*

Note that the direction of Ampere's force doesn't coincide with that of vector of magnetic induction. Its direction defines by a "***left hand rule***": *if you place left hand so that $\vec{B}$ falls into open palm, and four fingers point the direction of current, then extended thumb points in the direction of Ampere's force* (Fig. 14).
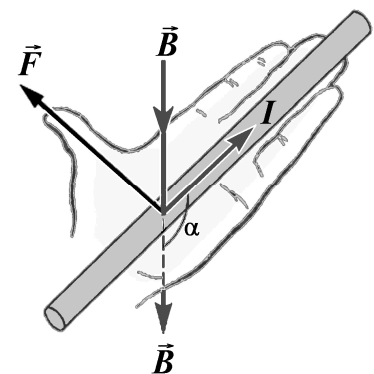

Figure 14 – Left hand rule

SI unit for $\vec{B}$ is Tesla. Tesla is the induction of such magnetic field which exerts a force of $1N$ on element of a current of $1\ A\cdot m$, placed perpendicularly to vector $\vec{B}$

$$1T = 1N\,/\,1A\cdot m.$$

Graphically magnetic field can be depicted with the help of magnetic field lines (lines of magnetic force). ***Magnetic field line** is a type of line for which the tangent line to the path at each point is parallel to the vector $\vec{B}$ at that point.*

Each element of a current generates a magnetic field around it. Induction of that field defines by Biot-Savart-Laplace law:

$$d\vec{B} = \frac{\mu\mu_0 I[d\vec{l}\times\vec{r}]}{4\pi r^3}, \quad \text{or, in scalars,} \quad dB = \frac{\mu\mu_0 \cdot Idl\cdot \sin(d\vec{l}\ ^\wedge\ \vec{r})}{4\pi r^2}, \qquad (43)$$
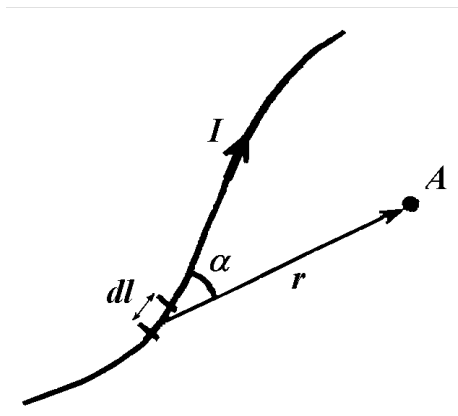
Figure 15 – llustration to the Biot – Savart –Laplace law

here, r – is the distance between the element of a current Idl and the point A at which the field induction dB is being computed (Fig. 15); $\mu_0 = 4\pi \cdot 10^{-7}\ H\,/\,m$ – magnetic constant (permeability of vacuum); μ – relative permeability of medium, it shows in what many times induction in a given medium is different from that of vacuum.

Quantity dB depends on medium, because certain substance magnetizes in external magnetic field and generates certain own magnetic field.

With dividing (43) on $\mu\mu_0$ we'll obtain quantity, which is independent from properties of medium – ***intensity of magnetic field***:

$$dH = \frac{Idl\sin(d\vec{l}\ ^\wedge\ \vec{r})}{4\pi r^2}. \qquad (44)$$

Equation (44) is a ***Biot-Savart-Laplace law*** for magnetic field intensity. It defines the value of magnetic field intensity dH for the field, generated by element of the current Idl. The relation between intensity and induction is given by equation

$$B = \mu\mu_0 H. \qquad (45)$$

Directions of $\vec{B}$ and $\vec{H}$ coincide only in isotropic homogeneous medium. As it follows from (45) the SI unit for intensity is *ampere per meter* (*A*/ *m*).

Field lines of intensity vector are constructed in such way that vector of intensity has directed along the tangent to the field line. Intensity of magnetic field is a magnetic quantity, which characterizes magnetic field of macro currents. Induction characterizes total field of micro and macro currents.

Wires usually have a finite length. However they can be represented as a huge number of elements of the current Idl. Magnetic intensity and induction at any point are vector sum of intensities $d\vec{H}$ and inductions $d\vec{B}$ of fields created by each element of the current separately:

$$\vec{H} = \sum d\vec{H} ; \qquad \vec{B} = \sum d\vec{B} .$$

This superposition principle allows to determine intensity and induction of fields, created by wires with different form. However, in general case, this operation stays quite complicated for wires with random shape.

Now, let's consider some special cases.

1.***Finite segment of current*** create a circular magnetic field (Fig.16). Magnetic intensity at a distance of r from axis of a current defined with the formula:

$$H = \frac{I}{4\pi r}(\cos\alpha_1 + \cos\alpha_2), \qquad (46)$$

here r – α_1 and α_2 - angles, which are created radius-vectors, traced from the ends of a conductor to the target point A at which field is being calculated.

Vectors of magnetic intensity $\vec{H}$ and induction $\vec{B}$ are perpendicular to a plane passing through a segment BC and point A.

Magnetic field lines are concentric circles around a straight current which are lying in perpendicular plane to a current. Direction of field lines can be determined by the rule of vector product, or by the ***right-hand rule*** (Fig. 16a): *if you wrap your right hand around the wire and your thumb points in the direction of the current I_1, then curled fingers will point in the direction of the magnetic field lines H_1. The straight forefinger of right hand point in the direction of vector of magnetic field $\vec{H}_1$.*

Figure 16 – Field lines of segment of a current
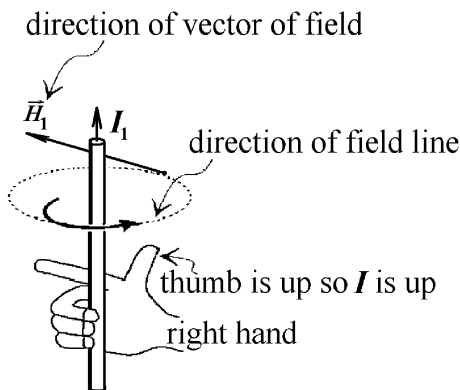
Figure 16a – Right-hand rule

2. ***Infinite straight current*** create a magnetic field same as a segment (Fig.17).

If length of a segment (Fig.16) increases then angles α_1 and α_2 will decrease until they become zero as in the case of infinite wire. As $\cos 0° = 1$, then

$$H_A = I / 2\pi r. \qquad (47)$$

Direction of $\vec{H}$ and field lines are the same as for finite segment (Fig.17).

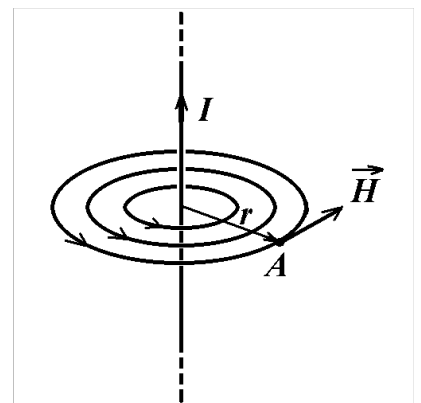**Figure 17** – Field lines of an infinite current

3. ***Circular current*** (*single coil*) (Fig.18). Magnetic intensity at the ***center*** of a circular current can be defined with the formula
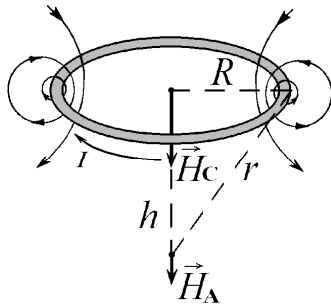
$$H_C = I / 2R, \qquad (48)$$
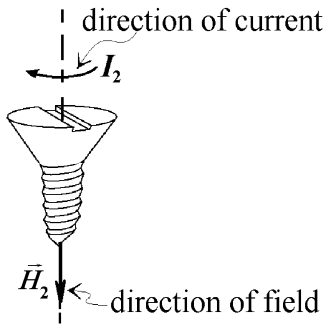
Figure 18 – Field lines of a circular current

here R – is the radius of a circle. Vector $\vec{H}_C$ is perpendicular to the plane of a coil in each point underlying in a plane of that coil. It can be derived from superposition principle applied to set of magnetic fields produced by each segment of wire dl separately.

In order to determine direction of vector H the **screw rule** (Fig. 18,a) is used: *if you turn the screw in a direction of a current I_2 then translational motion of screw will point in a direction of magnetic field $\vec{H}_2$.*

Intensity magnitude **on the axis** of the circle can be defined with the formula

$$H_A = IR^2 / 2(R+h)^{3/2}, \qquad (49)$$

where h – distance from the point in which the field is being calculated to the center of coil.

Figure 18,a – Screw rule

Solenoid.(Fig. 19) is a wire, wound around a cylindrical core. Length of a solenoid is in many times greater than its diameter. Inside the solenoid magnetic field there is uniform with intensity

$$H = IN / l = I \cdot n, \qquad (50)$$

where N – number of loops of wire in a solenoid, l – solenoids length, n – number of loops per unit length(winding density). Uniformity of a field is broken at ends of solenoid. Outside there is a weak field with respect to an inside field. Direction of $\vec{H}$ field lines can be defined in the same way as in the case of single coil.
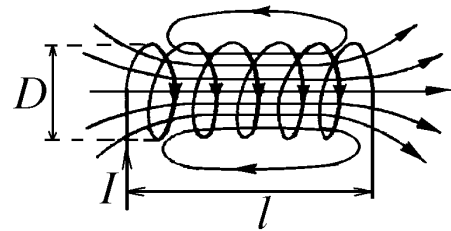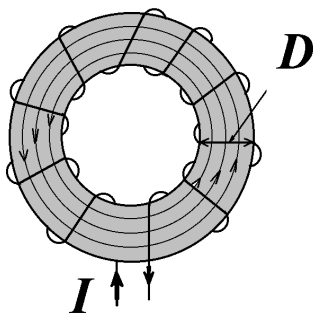

Figure 19 – Field lines of solenoid

Torus (Fig 20) is a wire wound around a toroidal core. Torus has no boundary effect. Magnetic field is completely concentrated inside the torus, $\vec{H}$ field lines are concentric circles. If the diameter of loops D is in many times smaller than the length of torus circle, the intensity can be defined with the formula (50):

$$H = IN / l = I \cdot n,$$

here l – is the length of midline of a torus.

Figure 20 - Torus

2.2 Earth's magnetic field

The Earth is a natural magnet. In different points on the Earth there is a magnetic needle, able to swing around horizontal and vertical axis, sets under the different angles with Earth's surface. It means that field lines and vector of intensity have different angles with its surface. *The magnetic poles* are the two positions on Earth's surface where the magnetic field is entirely vertical. There magnetic needle sets vertically. Magnetic field *S* pole is near the Earth's geographic North Pole and the other magnetic field *N* pole is near the Earth's geographic South Pole. The exact positions of geographical and magnetic poles don't coincide. The locations of the magnetic poles are not static; they wander as much as 15 km every year.

As it is seen from Fig. 21, the field lines come out of the Earth on the south hemisphere, and come in on the north one. Vertical plane, which is drawn through magnetic needle, is called *magnetic meridian plane.* Line of intersection of this plane with Earth's surface is known as *magnetic meridian.* In other words, *magnetic meridian* is the line, connecting the magnetic south and north poles. The angle between the magnetic and the geographic meridian is the *Magnetic declination* φ. The angle between vector of intensity and horizontal plane is the *Magnetic inclination i.*
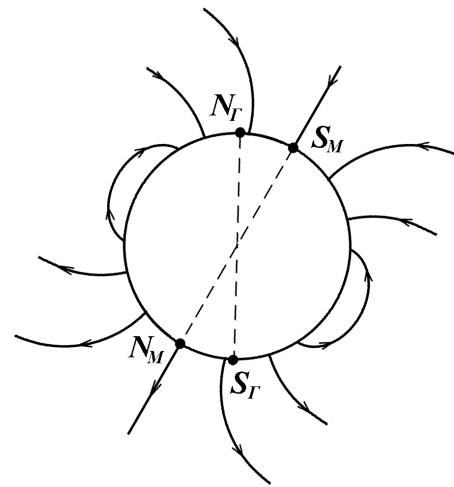
Figure 21 – Earth's magnetic field lines

Vector of intensity can be decomposed on two components – horizontal $\vec{H}_h$ and vertical $-\vec{H}_V$ (Fig. 22). The magnetic needle, able to swing randomly in horizontal plane, will point its north pole in the direction of $\vec{H}_h$. Magnetic declination, magnetic inclination, and magnitude of horizontal component of magnetic field intensity completely characterize Earth's magnetic field.

Magnitude of $\vec{H}_h$ can be defined by a tangent-compass. However, in the laboratory there are many electromagnetic devices which have built in permanent magnets. Superposition of those fields and Earth's magnetic field produces some resultant average field, which is proposed for students to investigate. In this laboratory work the magnitude and direction of horizontal component $\vec{H}_h$ of Earth's magnetic field determine with the help of tangent-compass in different points of laboratory desktop.
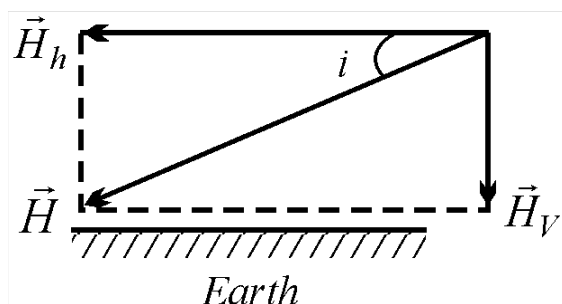
Figure 22 – Two components of Earth's magnetic field intensity

### 3 Description of experimental setup and methodology of measurements

Devices and outfits: tangent-compass, direct current source, set of resistances R_3, milliamperemeter.

Tangent-compass is a thin coil with N winds placed in vertical plane so, that the intensity in the center of this coil can be calculated with the formula:

$$H_C = IN \, / \, 2R = IN \, / \, D, \qquad (51)$$

here R – radius of a coil, D – its diameter, I – current, N – number of winds.

There is a magnetic needle with a scale (a compass) in the center of a coil. *Direction* of a horizontal component of Earth's magnetic field is determine simply by compass.
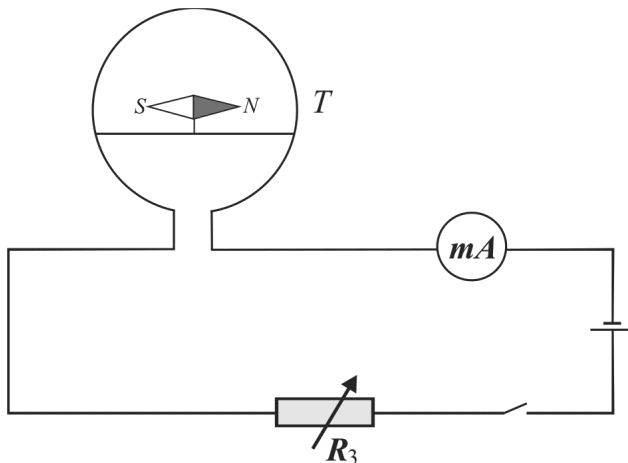
Figure 23 – Work 3-1 experimental setup scheme

For determination of a *magnitude* of horizontal component of Earth's magnetic field you'll have to mount the scheme Fig. 23, here T – tangent-compass, mA – milliamperemeter, R_3 – variable resistance.

The method of measurements is in the following. When there is no current through coil the north pole of magnetic needle points in the direction of horizontal component $\vec{H}_h$ of Earth's magnetic field. The needle should lie in a plane of a coil.
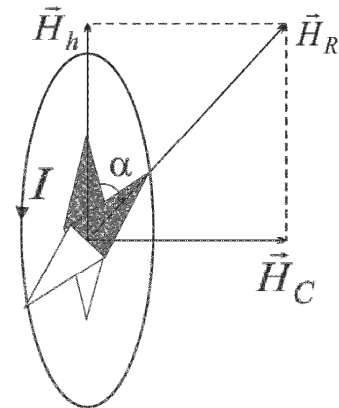
Appearance of a current will generate magnetic field with intensity H_C, perpendicular to the plane of a coil. Thus in a horizontal plane we'll have a superposition of investigated field H_h and field H_C, produced by the coil (Fig.24). The magnetic needle will point in the direction of the resultant field. It is conveniently that $\vec{H}_h$ and $\vec{H}_C$ was perpendicular. This can be achieved by setting the coil in such way that the magnetic needle has appeared in a plane of coils (position 1 on a Fig. 24). Obviously that $\vec{H}_h$ will lie in the same plane. Vector $\vec{H}_C$ is perpendicular to the plane of coils. Hence, $\vec{H}_h$ and $\vec{H}_C$ are perpendicular to each other. A parallelogram built on these vectors is rectangle. The resultant field vector $\vec{H}_R$ lies on a diagonal of such rectangle. As it follows from Fig. 24

Figure 24 – Determination of magnitude of intensity of Earth's magnetic field

$$H_C \, / \, H_h = \text{tg } \alpha, \qquad \text{then} \qquad H_h = H_C \, / \, \text{tg } \alpha \,.$$

Taking into account the formula (51), we'll have

$$H_h = IN \, / \, D \, \text{tg } \alpha \,. \qquad (52)$$

Tangent-compass has a scale for measuring angle α. If the coil is correctly

oriented the magnetic needle will set in a plane of coil on a zero mark in the absence of current. Presence of current makes needle turn on angle α in a horizontal plane, and points the direction of resultant magnetic field.

4 Work execution order and experimental data analysis

1. Mount the scheme (see Fig. 23).
2. Measure the diameter of a coil with help of a ruler (accurately for millimeters) for one time.
3. Draw the exploring part of desktop in a smaller scale into your report. In the middle of a sheet draw a line along its larger side (this line corresponds to midline of desktop, parallel to the laboratory stand). Draw three equidistant lines perpendicular to that line. You'll have to determine H_h in the points of intersection of these lines.
4. Set the tangent compass so, that the center of magnetic arrow appears in one of chosen points.
5. Inspect the tangent compass: the origin of of compass scale should coincide with a plane of coils.
6. Place a coil of tangent-compass so, that the plane of its coils coincides with vertical plane, passing through desktop midline (origin of of compass scale correspond with right side of desktop midline). **Determine the direction** of H_h. In order to do this, in the **absence of a current** angle β measures between direction of a needle and the origin of of compass scale (right side of plane of coils). Write its value into the table of measurements.
7. **Determine magnitude** of H_h. Place tangent-compass so, that magnetic needle appeares in the plane of coils in the absence of current. The needle should coincide with origin of of compass scale.
8. Turn the current on and using a resistance box R_3 achieve 30^0 or 45^0 deviation of a needle from origin position.
9. Write the scale readings of milliamperemeter and compass scale (value of angle α of deviation of a needle from its zero position) into the table of measurements.
10. Calculate the magnitude of H_h using the formula (52). Number of winds N is written on the label of a coil.
11. Do steps 4 – 9 for rest two points.
12. As a final result report must contain diagram of vectors $\vec{H}_h$ distribution (directions and magnitudes in three given points) on a desktop in the scale.
13. Conclude about the proper cause of difference of directions or magnitudes of these vectors in various points.

5 Control questions

1) What is magnetic field induction?
2) What is magnetic field intensity?
3) What are field lines of vector of intensity?
4) Draw field lines for a circular current. How to determine direction of intensity vector in the center of a circular current?
5) What is the point of Biot-Savart-Laplace law?

6) Write the formula for calculation of intensity in the center and on the axis of circular current.

7) What are the main features of magnetic field with respect to electrostatic field?

8) What is the circulation of vector of intensity?

9) What is the point of total current law?

10) Describe Earth's magnetic field. What are magnetic inclination and magnetic declination?

11) What is superposition principle? How the resultant intensity can be determined if the intensities of each of superposed fields are given.

12) How to determine the direction and magnitude of horizontal component of Earth's magnetic field intensity by tangent-compass? Draw a scheme. Derive a calculation formula.

6 Content of the report
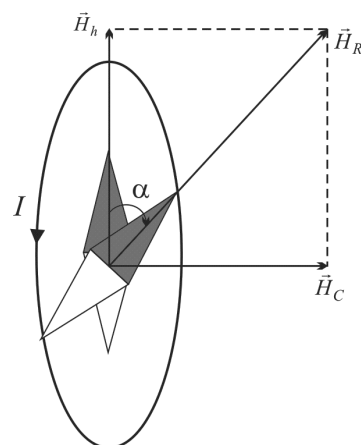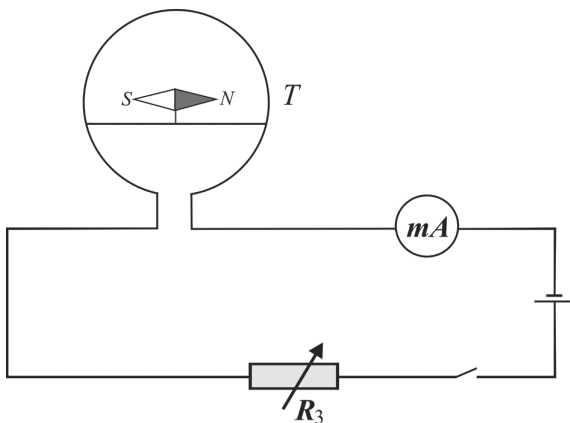
Laboratory work № 3-1

I. Home work.

…

II. Laboratory work № 3-1 implementation protocol.

1) Topic: *STUDYING OF A MAGNETIC FIELD VIA TANGENT-COMPASS.*

2) Goal:
1 Study of method of a vector diagrams for representation of force fields.
2 Finding the horizontal component of a magnetic field intensity vector in the laboratory via tangent-compass.

3) Setup schematic drawing:

4) **Table of measuring instruments:**

№	Name	Type	Serial number	Grid limit	Grid unit	Accuracy class (absolute error)
1	Miliampermeter					
	Four-decade resistance box R_3					
2	Tangent-compass			360°	5°	5°
3	Ruler					

5) **Equations for calculations:**

Horizontal component of Earth's magnetic field

$$H_h = \frac{I \cdot N}{D \cdot tg\alpha},$$

where I – a current through the coil of tangent-compass; N – number of winds of a coil; D – diameter of a coil; α – angle of deviation of a needle when the current turn on from its zero-current position.
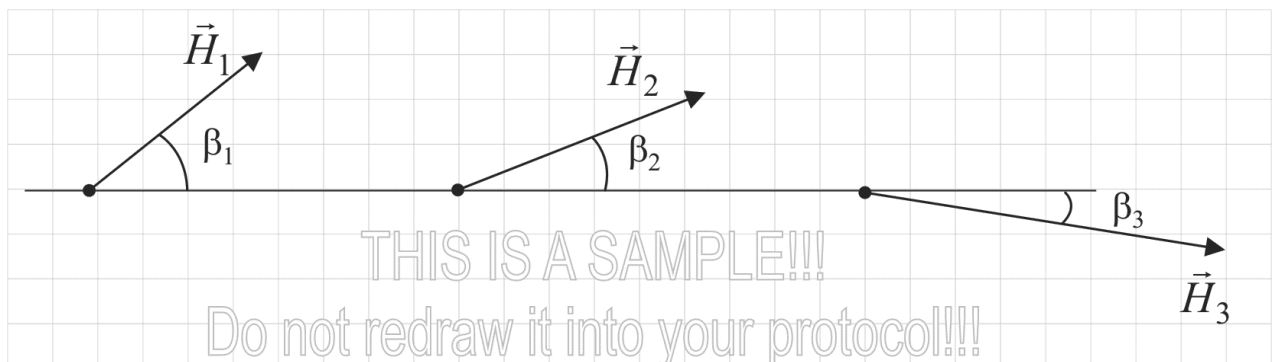
6) **Table of measurements**

$N = 100$; $D = \ldots$ *mm*.

№	$\beta,^{\circ}$	I, *mA*	$\alpha,^{\circ}$	H_h, *A / m*
1.				
2.				
3.				

$$tg(45^0) = 1; tg(30^0) = \frac{1}{\sqrt{3}} = 0{,}577$$

7) **Final results as a diagram:**

THIS IS A SAMPLE!!!
Do not redraw it into your protocol!!!

8) **Conclusions:**

9) **Work done by:**   **Work checked by:**

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