UKRAINIAN STATE COMMITTEE OF COMMUNICATIONS AND INFORMATIZATION

ODESSA NATIONAL ACADEMY OF TELECOMMUNICATIONS

after A. S. POPOV

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PHYSICS

Module 1. Electrical current and magnetic field of a current

ELECTROMAGNETISM

PART 2: COMPLEX TASK

for bachelor training of educational area 0924 –"Telecommunications"

APPROVED

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Protocol № 7

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The following methodical guide is about section "Electromagnetism" of physics course for telecommunications technician. Four problems allow students to learn basics of electrical engineering and calculations techniques applied to determine main characteristics of electrical systems and magnetic field. It contains sufficient theoretical information combined with examples of problems' solutions.

Recommended for students of TE-group, educational area 0924 – "Telecommunications".

CONFIRMED

at the Department session

Protocol № 6

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MODULE STRUCTURE

Module № 1. „ Electrical current and magnetic field of a current” – 72 hours total

Lectures – 16 hrs, pract. trainings – 0 hrs, labs – 16 hrs, self-studies – 33 hrs.

LIST OF PRACTICAL TRAINING

Number of lessons	Denomination of lessons	Hours
	Module № 1	
1	Vector of electric intensity. Electrostatic force. Electric potential. Difference of the potential. Work done by the electric field moving a charge in electric field. Electro capacity of conductors’ systems. Energy of electric field.	2 self
2	The principle of superposition of electric fields. Calculation of resultant electric intensity.	2 lab
3	Laws of direct current.	2 self
4	Calculation of branched circuits.	2 lab
5	Magnetic induction and magnetic intensity. The Ampere force. The Lorentz force. Motion of charged particles in magnetic field.	2 self
6	The principle of magnetic fields’ superposition. Calculation of resultant magnetic intensity.	2 lab
	Electromagnetic induction. Self-induction law.	2 self
8	Electromagnetic induction law. Calculation of EMF of the induction.	2 lab

INTRODUCTION

All problems should be solved in a individual way, i.e. each student have its own variant, which is specified by instructor.

Appropriate homework must forego to solving a problem. The homework contains self-studying of theory of sections ***"Electrostatics"***, ***"Direct current"*** and ***"Electromagnetism"*** of the course of physics for telecommunications technician.

- Complex task is consists of four problems: 1.1; 1.5; 2.1; 2.4. Student must implement on one task from each problem. Concrete numbers of statement on each problem and initial data in thirty variants are specified in Tables of task variants. Number of the variant is determining by the index of surname of student at a group journal.
- ***Report*** is implementing on individual exercise book. Writings should be made on one side of double-page spread.
- On the cover there is need to mark title of the work, ***number of the variant***, surname and initials of student, code of group.
- Calculation part it is necessary to dispose in order of numeration of the problems.
- Calculation part of any problem must contain ten points:

1. ***Title*** of the problem;
2. Complete ***statement*** of a task;
3. ***Short writing*** of statement***;***
4. ***Transformation*** of a value of given quantities to the ***system of units*** **SI.**
5. Explained ***scheme*** or figure;
6. List of ***laws*** and ***formulas*** which reflect the physical phenomena of theme of the problem. All ***denotations*** at the formulas needs an ***explanations;***
7. From resulted formulae at point **6** it is necessary to make system of equations and to present solution of the task or its part at the letter kind, where sought quantity must be present through given quantity at letter (symbolic) designate.
8. ***Checking*** of ***units*** of measurement of target quantities on correspondence to the expected. For this necessary to substitute into the formula of letter solution in place of symbol of each quantity its unit of measurement and realize the necessary transformations.
9. Only after coincidence of the units of measurements to the expected it is necessary to substitute into the formula of letter solution the ***numerical values*** of measurements and execute the calculations (see examples of execution of calculated part). Calculation is transacted with three significant digits.
10. ***Result*** of execution of calculated part.

At the end of work it is necessary to enumerate the list of the used literature.

Besides this guide is recommended to use literature from bibliography given at the end of this guide.

Problem 1.1
THE PRINCIPLE OF SUPERPOSITION OF ELECTRIC FIELDS

MAIN CONCEPTS

Electric field is usually described by two basic quantities – vector of electric intensity and electric potential

$$\vec{E} = \vec{F} / q_0; \qquad \varphi = W_n / q_0,$$

where $\vec{F}$ – force acted with a test point charge q_0**,** which is located in the given point of a field; W_n – potential energy of a charge q_0, in the given point.

Intensity and potential of electric field of the point charge q at a distance of r of the charge

$$E = \frac{q}{4\pi\varepsilon_0\varepsilon \cdot r^2}; \qquad \varphi = \frac{q}{4\pi\varepsilon_0\varepsilon \cdot r},$$

where $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m – permittivity of vacuum; ε – relative permittivity of medium (for air ε=1).

For the uniform field the dependence between the intensity and potential of the electric field

$$E = \frac{\varphi_1 - \varphi_2}{r_{1,2}},$$

where $r_{1,2}$ – the distance between equipotential lines with the potentials φ_1 and φ_2, E – the electric intensity in the middle of the equipotential lines with the potentials φ_1 and φ_2.

The modulus of electric field intensity:

1) ***uniformly charged sphere*** with a radius of R at a distance of r from the centre of sphere

a) $E = 0$, for $r < R$;

б) $$E = \frac{Q}{4\pi\varepsilon_0\varepsilon \cdot r^2}, \quad \text{for } r \geq R,$$

where $Q=\sigma \cdot S$ – a charge of sphere, σ – surface charge density of sphere (charge of 1m^2 of a surface), $S=4\pi R^2$ – surface area of sphere.

2) ***endless uniformly charged cylinder*** with a radius of R at a distance of r from the axis of the cylinder.

a) $E = 0$, for $r < R$;

б) $$E = \frac{\tau}{2\pi\varepsilon_0\varepsilon \cdot r} \quad \text{for } r \geq R,$$

where $Q=\tau \cdot L$ – a charge of cylinder, τ – linear charge density (charge of 1m of a length), L– length of cylinder.

3) endless ***uniformly charged plate***

$$E = \frac{\sigma}{2\varepsilon\varepsilon_0},$$

σ – surface charge density of plate.

The superposition principle: the electric field created by one charge is independent from positions of other charges. Then the resultant vector is a vector sum of vectors of electric field individual charges:

$$\vec{E}_{\mathrm{RES}} = \vec{E}_1 + \vec{E}_2,$$

therefore:

a) the ***direction*** of the resultant vector is defined by parallelogram rule of the vectors' additions (see example 1);

b) the ***magnitude*** of the resultant vector is defined by cosine theorem:

$$E_{\mathrm{RES}} = \sqrt{E_1^{\,2} + E_2^{\,2} + 2E_1E_2\cos\alpha},$$

where α – angle between vectors.

The ***electrostatic force*** acting on a test charge q

$$F = q \cdot E_{\mathrm{RES}}.$$

EXAMPLES OF PROBLEM SOLUTION

Example 1. The sphere with radius R=5 *cm* and the endless uniformly plate are charged from the surfaces with the charge density σ_1=10 nC/m^2 and σ_1=–15 nC/m^2 correspondingly. The sphere's centre is situated on a distance of ℓ=10 *cm* from the plate. Find the electric intensity at the point A, which is situated on a distance of a=5 *cm* from the sphere's surface and b=10 *cm* from the plate; force, which will act on the point charge q_0=0,1 *nC*, if it is put to the point A.

Input data:

σ_1=10 nC/m^2=$10\cdot10^{-9}$ C/m^2

σ_2=–15 nC/m^2=$-15\cdot10^{-9}$ C/m^2

R=5 *cm* =$5\cdot10^{-2}$ *m*

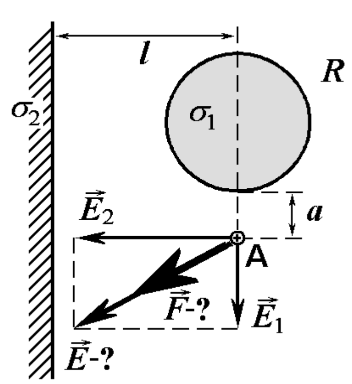
l=10 *см* =$10\cdot10^{-2}$ *м*

a=5 *cm*=$5\cdot10^{-2}$ *m*

b=10 *cm* =$10\cdot10^{-2}$ *m*

q_0=0,1 *nC*=$0{,}1\cdot10^{-9}$ *C*

Find: E, F – ?



Solution:

According to the principle of superposition of electric fields: the sphere creates the electric field irrespective of the plate position in the space, and vice versa: the plate electric field is independent of the sphere position. That's why the resultant intensity equals the ***vector sum*** of the individual intensities:

$$\vec{E} = \vec{E}_1 + \vec{E}_2.$$

a) We must ***show the direction*** of the resultant vector of electric intensity defined by parallelogram rule ***to the diagrammatic drawing*** (see picture).

b) We define the ***magnitude*** of the resultant vector by cosine theorem:

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\alpha}\,.$$

Let's find the individual intensities creating with the sphere and plate.

The ***sphere's field intensity*** is in the point at a distance of r from its centre

$$E_1 = \frac{|Q_1|}{4\pi\varepsilon_0 r^2}, \tag{1}$$

where ε_0=8.85·10^{-12} F/m – the vacuum permittivity; Q_1 – the charge of sphere.

Let's find the charge of the sphere through the surface charge density σ_1 and the area of sphere surface S=4πR^2:

$$Q_1 = 4\pi\sigma_1 R^2; \qquad r = a + R.$$

In this equation from the point A to the centre of the sphere distance r is defined as a sum of the distance a to the surface of sphere and radius of sphere R.

Inserted these expressions in the formula (1), we'll get

$$E_1 = \frac{4\pi R^2|\sigma_1|}{4\pi\varepsilon_0(a+R)^2} = \frac{R^2|\sigma_1|}{\varepsilon_0(a+R)^2}. \tag{2}$$

The ***plate electric intensity*** of the uniformly charged with the surface density σ_2

$$E_2 = \frac{|\sigma_2|}{2\varepsilon_0}. \tag{3}$$

The vector $\vec{E}_1$ is directed along to field line from the sphere, as the sphere is positively charged. The vector $\vec{E}_2$ is directed to the plate, as the plate is negatively charged.

As the vector $\vec{E}_1$ and vector $\vec{E}_2$ are mutually perpendicular and $cos90^{O}$=0, then cosine theorem is transformed to a Pythagorean theorem:

$$E = \sqrt{E_1{}^2 + E_2{}^2 + 2E_2E_2\cos 90} = \sqrt{E_1{}^2 + E_2{}^2}\,. \tag{4}$$

Putting (2) and (3) in (4) and removing a common factor $1/\varepsilon_0$ beyond the radicand, we'll get

$$E = \frac{1}{\varepsilon_0}\sqrt{\frac{R^2\sigma_1^{\ 2}}{(a+R)^2} + \frac{\sigma_2^{\ 2}}{4}} \ . \qquad (5)$$

2. The magnitude of force, which is exerted with the point charge q_0,which is situated in the electrostatic field, is defined with the formula

$$\mathrm{F} = q_0 E. \qquad (6)$$

We check if the formula (5) gives unit of intensity *V/m*, and the formula (6) the unit of force *N*.

$$[E] = \frac{1}{[\varepsilon_0]}\{[\sigma^2]\}^{1/2} = \frac{1}{1F/m}\{1\frac{C^2}{m^4}\}^{1/2} = \frac{1C\cdot m}{1\ F\cdot m^2} = \frac{1C\cdot V}{1C\cdot m} = 1V/m;$$

$$[F] = [Q][E] = 1C\cdot 1V/m = \frac{1C\cdot 1J/C}{m} = \frac{1N\cdot m}{m} = 1N.$$

Let's make substitution in the formulas (5) and (6) the value of quantities in the units of SI-system and make the evaluations

$$E = \frac{1}{8{,}85\cdot 10^{-12}}\sqrt{\frac{0{,}05^2\cdot 10^{-16}}{(0{,}05+0{,}05)^2} + \frac{(1{,}5\cdot 10^{-8})^2}{4}} = 1{,}02\cdot 10^3 V/m.$$

$$F = 10^{-10}\cdot 1{,}02\cdot 10^3 = 1{,}02\cdot 10^{-7}\ N.$$

A direction of force coincides with a direction of vector $\vec{E}$. (as $q_0 > 0$), that is shown in the picture.

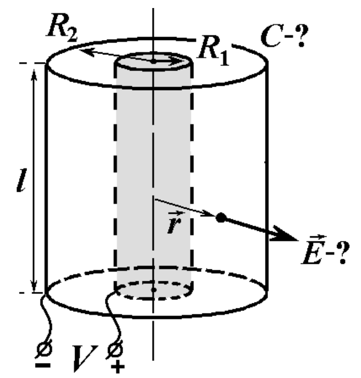
Results: $E = 1{,}02\cdot 10^3 V/m.$, $F = 1{,}02\cdot 10^{-7}\ H$.

Example 2. The air cylindrical capacitor consists of two coaxial cylinders with radiuses R_1 =1 *cm* and R_2 =3 *cm*. The length of the cylinders is *L*=50 *cm*. The capacitor was charged with the voltage *U*=100 *V*.

Find: 1) the electro capacity of capacitor; 2) field intensity in the capacitor at a distance of *r*=2 *cm* from the axle of the cylinders.

Input data:
R_1= 1*cm* =0,01*m;*
R_2=3*cm* = 0,03*m;*
L=50 *cm* =0.5*m;*
U=100 *V;*
r =2*cm* =0,02*m;*

Find: *C*; *E* – ?



Solution:

The electro capacity of the air (ε=1) cylindrical capacitor can be found with the formula

$$C = \frac{2\pi\varepsilon_0 L}{\ln(R_2 / R_1)}, \qquad (1)$$

where ε_0=8.85·10^{-12}*F/m* – the vacuum permittivity; *L* – the length of cylinders; R_1 and R_2 – radiuses of cylinders.

Let's find the vector of electric intensity at the distance of *r* from the axle of cylinders, at the point *A*, we'll use the principle of superposition of electric fields

$$\vec{E} = \vec{E}_1 + \vec{E}_2,$$

where $\vec{E}_1$ – electric intensity at the point *A*, created with the inner cylinder; $\vec{E}_2$ – electric intensity of the exterior cylinder at the same point. As it is necessary to find the intensity ***at the distance of r < R₂***, then $E_2 = 0$ and $E = E_1$. Supposing, that the cylinder is rather long ($r << L$), the necessary intensity is found with the formula of calculation of the field intensity of the endless cylinder

$$E = \frac{\tau}{2\pi\varepsilon_0 r}, \qquad (2)$$

where $\tau = Q / L$ – the linear density of the cylinder charge. When one applies the voltage *U* to the terminals of capacitor, then the charge *Q* will be induced on the cylinders:

$$Q=CU. \qquad (3)$$

Putting the expressions for τ and *Q* to the equation (2), we obtain

$$E = \frac{CU}{2\pi\varepsilon_0 rl}. \qquad (4)$$

We check if the formula (1) gives unit of capacitance *F*, and the formula (4) the unit of electric intensity *V/m*.

$$[C] = \frac{1\,F/m \cdot 1m}{1} = F; \qquad [E] = \frac{1\,F \cdot 1V}{F/m \cdot m \cdot m} = 1V/m.$$

Let's make substitution in the formulas (1) and (4) the value of quantities in the units of SI-system and make the calculations:

$$C = \frac{2 \cdot 3{,}14 \cdot 8{,}85 \cdot 10^{-12} \cdot 0{,}5}{\ln(0{,}03/0{,}01)} = 2{,}53 \cdot 10^{-11} F;$$

$$E = \frac{2{,}53 \cdot 10^{-11} \cdot 100}{2 * 3{,}14 \cdot 8{,}85 \cdot 10^{-12} \cdot 0{,}02 \cdot 0.5)} = 4{,}55 \cdot 10^{3} V/m.$$

Results*:* $E = 4{,}55 \cdot 10^{3} V/m$.

INDIVIDUAL TASKS FOR PROBLEM 1.1
THE PRINCIPLE OF SUPERPOSITION OF ELECTRIC FIELDS

In accordance with your variant to solve one of the following problems listed below. The number of problem statement and all necessary input data are reduced in the Table 1.1. If there is letter index in the number of the problem statement, then you should only answer the question, which corresponds to this index.

1 The electrostatic field is created with two endless parallel plates, which are charged uniformly with surface densities σ_1 and σ_2. Find force, which acts in this field on the point charge Q_1, if it's situated:

a) between the planes;

b) outside of planes.

2 The point charge Q_1 is situated in the centre of uniformly charged sphere with a radius R. Find the electrostatic intensity in two points, which lies from the centre at a distance of r_1 and r_2, if:

a) the charge of sphere equals Q_2;

b) surface density of the sphere charge equals σ_2.

3 The long thread, uniformly charged with the linear density τ_1, is situated on an axis of the long cylinder, which has the radius of R. The cylinder is uniformly charged with the linear density of τ_2. Define the electrostatic field intensity in the other points:

1) at a distance of r_1 from the thread;

2) at a distance of l from the surface of cylinder.

4 Two long parallel threads are uniformly charged with the linear densities of τ_1 and τ_2. The distance between the thread is l. Define the electrostatic field intensity at the point, which is situated at a distance of r_1 from the first thread and r_2 from the second thread.

5 The electrostatic field is created with the uniformly charged endless plate and sphere. The surface charge density of the plate is σ_1. The radius of sphere is R, the surface charge density of the sphere is σ_2. The sphere centre is situated at a distance of l from the plane. Find the electric intensity at the point, which is situated between the sphere and the plate at a distance of r_1 from the plate.

TABLE OF TASK VARIANTS

Table 1.1

Variant	Problem statement number	Q_1, nC	Q_2, nC	τ_1, nC/m	τ_2, nC/m	σ_1, nC/m^2	σ_2, nC/m^2	R, cm	l, cm	r_1, cm	r_2, cm
1	1a	+0.2	–	–	–	+2	+4	–	–	–	–
2	2a	–3	+6	–	–	–	–	6	–	3	10
3	4	–	–	–6	–9	–	–	–	6	6	6
4	5	–	–	–	–	+10	+40	4	10	2	–
5	3	–	–	–2	+3	–	–	5	5	4	–
6	1б	+0.2	–	–	–	–2	–3	–	–	–	–
7	2б	+0.2	–	–	–	–	+3	10	–	1	20
8	4	–	–	+3	+4	–	–	–	5	3	4
9	5	–	–	–	–	–4	–9	6	12	3	–
10	1a	+0.1	–	–	–	+2	–3	–	–	–	–
11	2б	+0.1	–	–	–	–	–10	5	–	3	10
12	3	–	–	+1	+2	–	–	4	5	1	–
13	1б	+0.4	–	–	–	–5	+3	–	–	–	–
14	2a	+1	+5	–	–	–	–	8	–	5	10
15	4	–	–	+10	–5	–	–	–	10	10	10
16	5	–	–	–	–	–6	+8	2	5	1	–
17	4	–	–	–40	–30	–	–	–	5	4	3
18	1a	+0.3	–	–	–	–4	–1	–	–	–	–
19	2a	–2	–4	–	–	–	–	5	–	2	6
20	4	–	–	+8	+4	–	–	–	8	8	8
21	5	–	–	–	–	+4	–6	5	10	4	–
22	3	–	–	–3	–4	–	–	6	3	3	–
23	1б	+0.1	–	–	–	+3	+1	–	–	–	–
24	2б	–0.4	–	–	–	–	–8	8	–	5	10
25	4	–	–	–8	+6	–	–	–	5	4	3
26	1a	–0.4	–	–	–	–2	+4	–	–	–	–
27	3	–	–	+2	–4	–	–	7	5	4	
28	2б	–0.5	–	–	–	–	+9	10	–	5	15
29	5	–	–	–	–	–3	–8	4	18	6	–
30	4	–	–	+4	+6	–	–	–	10	4	6

Problem 1.5
DIRECT CURRENT

MAIN CONCEPTS

Ohm's law (simple circuits):

a) for homogeneous subcircuit:

$$I = \frac{U}{R} ,$$

where I – a current through the subcircuit; U – voltage on the subcircuit connection terminal; R – the resistance of the subcircuit;

Voltage on homogeneous subcircuit:

$$U = IR.$$

b) for subcircuit containing EMF source:

$$I = \frac{\varepsilon - \Delta\varphi_{2,1}}{R + r},$$

where $\Delta\varphi_{2,1} = \varphi_2 - \varphi_1 = U$ – potential difference (or voltage) on subcircuit connection terminal, ε – value of Electro Motive Force (EMF) of source, which contains in the subcircuit; r – internal source resistance;

Voltage on branch of circuit containing EMF source:

$$U = \varepsilon - IR - Ir.$$

c) For the closed circuit (when it is possible to reduce to one equivalent source and one resistor):

$$I = \frac{\varepsilon}{R + r} ,$$

where R – external resistance of circuit, r – internal resistance of source.

Kirchoff's rules (branched circuits):

For a branched circuits solution use the two Kirchoff's rules (see example 3):

1st – junction rule: $\sum I_i = 0$.

where $\sum I_i$ – algebraic sum of a current into the junction (which is positive when a current flows in the junction, and negative when a current flows out of the junction);

2nd – closed loop rule: $\sum I_i R_i + \sum I_i r_i = \sum \varepsilon_i$,

$\sum I_i R_i$ – algebraic sum of voltage drop on external resistors around any closed loop, and $\sum I_i r_i$ – algebraic sum of the voltage drop on internal resistance of sources (which are positive, when the direction of a current coincides with chosen one in advance direction of path-tracing);

$\sum \varepsilon_i$ – algebraic sum of the source electromotive force of the closed loop (which are positive, when the direction of extraneous force work (from – to + inside the source) coincides with chosen one in advance direction of path-tracing).

EXAMPLE OF PROBLEM SOLUTION

Example 3 The electric circuit consists of two sources of EMF $\varepsilon_1 = 20\ V$, $\varepsilon_2 = 5\ V$ and three resistors R_1, $R_2 = 19\Omega$ and $R_3 = 10\Omega$. A current $I_1 = 0{,}2\ A$ flows through the inner resistances of the sources $r_1 = 2\Omega$, $r_2 = 1\Omega$ and through the resistor R_1, at the direction, shown in the picture.

Find: 1) resistance R_1 and a current, which flows through the resistors R_2 and R_3;

2) the potential difference between the points A and B.

Input data: ε_1=20 V

ε_2=5 V;

r_1=2 Ω;

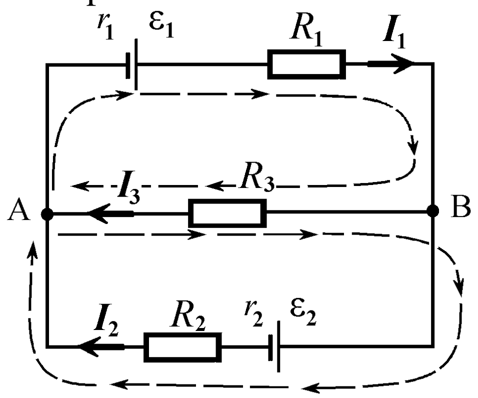
r_2=1 Ω;

R_2=19 Ω;

R_3=10 Ω;

I_1=0,2 A.

Find: I_2, I_3, R_1, $\Delta\varphi_{BA}$–?



Solution:

1. Let's use the Kirchhoff's rules for the solution of the branched chain.

In order to find one magnitude of resistance and two magnitudes of a current, it's necessary to make three equations. Before compiling the equations it's necessary arbitrarily to choose:

a) direction of currents (if they aren't set in the condition); b) direction of path-tracing.

The direction of a current I_1 is set, and let's choose directions of currents I_2 and I_3, as it's shown on the scheme. Let's agree to trace-paths clockwise (dashed line on the scheme). The given scheme has two junctions: A and B. In order to compile the equations with the first Kirchhoff's rule it's necessary to take into account, that a current, which flows in the junction, enters to the equation with plus-sign, and it is necessary to write a current, which flows out of the junction with minus-sign.

With the first Kirchhoff's rule for the junction A

$$I_1 - I_2 - I_3 = 0. \qquad (1)$$

It is no sense to compile the equation for the junction B, as it reduces to the equation (1).

We'll get two more necessary equations with the second Kirchhoff's rule. It is necessary to follow the sign rules: a) the voltage drop (product IR or Ir) enters to the equation with plus-sign, if the current's direction coincides with the direction of path-tracing, in other case – with minus-sign; b) EMF enters to the equation with plus-sign, if it enlarges the potential in the direction of path-tracing (pass through the source from minus to plus), in other case – with minus-sign.

With the second Kirchhoff's rule for the closed loop $B\varepsilon_1 R_1 A R_3 B$ and $BR_3 A R_2 \varepsilon_2 B$:

$$I_1 r_1 + I_1 R_1 + I_3 R_3 = \varepsilon_1; \qquad (2)$$

$$-I_3R_3+I_2R_2+I_2R_2=-\varepsilon_2. \qquad (3)$$

The set magnitudes inserted in the equation (1), (2), (3), we'll get the set of equations

$$\begin{cases}0.2-I_3-I_2=0;\\ 0.2\cdot 2+0.2R_1+10I_3=20;\\ -10\ I_3+19I_2+I_2=-5.\end{cases} \Rightarrow \begin{cases}0.2-I_3-I_2=0; & (4)\\ 0.2R_1+10\ I_3=19.6; & (5)\\ 20\ I_2-10I_3=-5. & (6)\end{cases}$$

Let's find I_3 from the equation (4) and insert to the equation (6)

$$I_3-0.2-I_2;\quad 20\ I_2-2+10\ I_2=-5.$$

Whence $I=-0{,}1$ A.

Minus-sign in the sense of a current I_2 means, that the current's direction I_2 has been chosen reverse to the acting. In reality current I_2 runs from junction *B* to the junction *A*.

From the equation (4) we search out I_3:

$I_3=0{,}2-(-0{,}1);\qquad I_3=0.3$ A.

From the equation (5) we search out R_1

$$R_1=\frac{19{,}6\cdot 10\cdot 0{,}3}{0{,}2}=83\ \Omega.$$

2. The difference of potential $U=\Delta\varphi_{A,B}=\varphi_B-\varphi_A$ can be found, if we use Ohm's law for Non-uniform site of a chain (subcircuit) in a proper way, for a example $B\varepsilon_1R_1A$.

$$I=\frac{\varepsilon-\Delta\varphi_{\mathrm{A,B}}}{R+r}. \qquad (7)$$

In Ohm's law it's taken into consideration, that positive direction of current's strength coincides with the direction of foreign forces work of the source, which fits the enlarging of the potential. Then the required potential difference is

$\Delta\varphi_{\mathrm{A,B}}=U=\varepsilon_1-I_1(R_1+r_1).$

We make the calculations

$\Delta\varphi_{\mathrm{A,B}}=20-0.2\,(83+2)=3V.$

Results*:* $I_2=-0{,}1\ A$; $I_3=0{,}3\ A$; $R_1=83\ \Omega$, $U=3V$.

INDIVIDUAL TASKS FOR PROBLEM 1.5
BRANCHED CIRCUITS

To compose the scheme from three adjacent branches, which are shown in the picture 4. The numbers of branches, EMF of sources ε_i, the inner resistance of sources r_i, the external resistance of branches R_i (or the current I_i, which flows along one of the branches from the point A to B) set with the variants in the table. 1.5. Find: 1) magnitudes of quantities, which are indicated in the last column of the Table 1.5;

2) the potential difference from the points *A* and *B*.

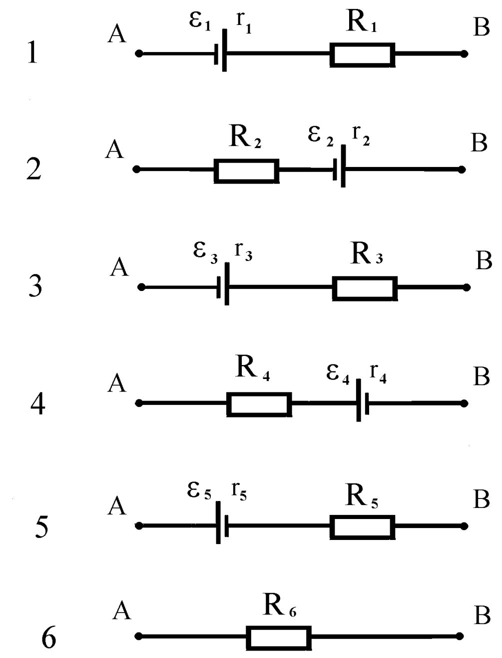
Example of the scheme, which corresponds 25th variant is shown in the picture 4,*a*.

TABLE OF TASK VARIANTS

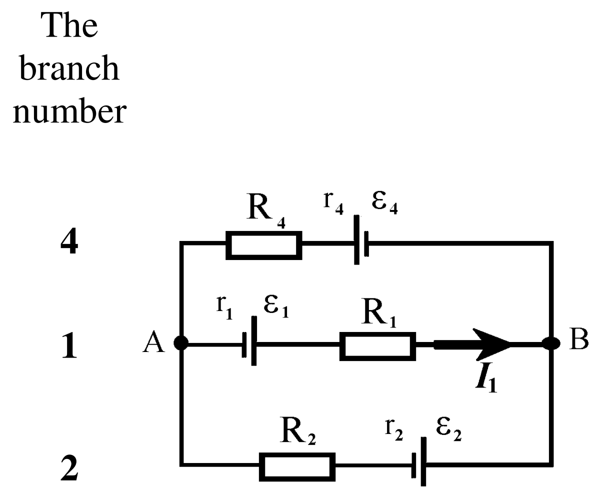
Table 1.5

Variant	The branch number	ε_i, V	r_i, Ω	R_i, Ω	I_i, A	Find
1	1, 2, 3	$\varepsilon_1=11$, $\varepsilon_2=4$, $\varepsilon_3=6$	$r_1=r_2=r_3=0$	$R_1=25$, $R_2=50$, $R_3=10$	–	$\mathbf{I_1, I_2, I_3}$
2	4, 5, 6	$\varepsilon_4=9$, $\varepsilon_5=10$	$r_4=1$, $r_5=2$	$R_4=19$, $R_5=38$	$I_6=0{,}1$	$\mathbf{I_4, I_5, R_6}$
3	1, 2, 4	$\varepsilon_1=16$, $\varepsilon_2=5$, $\varepsilon_4=7$	$r_1=r_2=r_4=0$	$R_2=30$, $R_4=50$	$I_1=0{,}4$	$\mathbf{I_2, I_4, R_1}$
4	5, 4, 1	$\varepsilon_1=9$, $\varepsilon_4=6$, $\varepsilon_5=2$	$r_1=r_4=r_5=0$	$R_4=50$, $R_5=10$	$I_1=0{,}2$	$\mathbf{I_4, I_5, R_1}$
5	1, 2, 6	$\varepsilon_1=10$, $\varepsilon_2=8$	$r_1=2$, $r_2=1$	$R_1=8$, $R_2=19$, $R_6=60$	–	$\mathbf{I_1, I_2, I_6}$
6	3, 2, 1	$\varepsilon_2=4$, $\varepsilon_3=5$	$r_1=r_2=r_5=0$	$R_1=30$, $R_2=40$, $R_3=20$	$I_1=0{,}1$	$\mathbf{I_2, I_3, \varepsilon_1}$
7	1, 4, 6	$\varepsilon_1=8$, $\varepsilon_4=2$	$r_1=2$, $r_4=1$	$R_1=18$, $R_4=39$, $R_6=80$	–	$\mathbf{I_1, I_4, I_6}$
8	1, 4, 2	$\varepsilon_2=11$, $\varepsilon_4=7$	$r_1=r_2=r_4=0$	$R_1=50$, $R_2=20$, $R_4=30$	$I_1=0{,}1$	$\mathbf{I_2, I_4, \varepsilon_1}$
9	2, 1, 3	$\varepsilon_1=9$, $\varepsilon_2=8$, $\varepsilon_3=1$	$r_1=r_2=r_5=0$	$R_1=50$, $R_2=20$, $R_3=10$	–	$\mathbf{I_1, I_2, I_3}$
10	4, 1, 5	$\varepsilon_4=4$, $\varepsilon_5=2$	$r_1=r_4=r_5=0$	$R_1=25$, $R_4=50$, $R_5=10$	$I_1=0{,}4$	$\mathbf{I_4, I_5, \varepsilon_1}$
11	1, 3, 2	$\varepsilon_2=16$, $\varepsilon_3=3$	$r_1=r_2=r_5=0$	$R_1=70$, $R_2=20$, $R_3=10$	$I_1=0{,}1$	$\mathbf{I_2, I_3, \varepsilon_1}$
12	6, 4, 1	$\varepsilon_1=3$, $\varepsilon_4=7$	$r_1=2$, $r_4=1$	$R_1=78$, $R_4=39$	$I_6=0{,}1$	$\mathbf{I_1, I_4, R_6}$
13	5, 4, 1	$\varepsilon_4=4$, $\varepsilon_5=14$	$r_1=r_4=r_5=0$	$R_1=90$, $R_4=20$, $R_5=40$	$I_1=0{,}1$	$\mathbf{I_4, I_5, \varepsilon_1}$
14	4, 6, 5	$\varepsilon_4=10$, $\varepsilon_5=5$	$r_4=2$, $r_5=1$	$R_4=33$, $R_5=19$	$I_6=0{,}3$	$\mathbf{I_4, I_5, R_6}$
15	1, 6, 4	$\varepsilon_1=4$, $\varepsilon_4=3$	$r_1=2$, $r_4=1$	$R_1=18$, $R_4=9$, $R_6=60$	–	$\mathbf{I_1, I_4, I_6}$
16	4, 1, 6	$\varepsilon_1=2$, $\varepsilon_4=12$	$r_1=3$, $r_4=2$	$R_1=97$, $R_4=18$	$I_6=0{,}1$	$\mathbf{I_2, I_4, R_6}$
17	4, 1, 5	$\varepsilon_1=22$, $\varepsilon_4=8$, $\varepsilon_5=4$	$r_1=r_4=r_5=0$	$R_1=25$, $R_4=50$, $R_5=10$	–	$\mathbf{I_1, I_4, I_5}$
18	2, 1, 6	$\varepsilon_1=20$, $\varepsilon_2=6$	$r_2=1$	$R_1=82$, $R_2=29$, $R_6=10$	$I_1=0{,}2$	$\mathbf{I_2, I_6, r_1}$
19	2, 3, 1	$\varepsilon_1=19$, $\varepsilon_2=4$, $\varepsilon_3=5$	$r_1=r_2=r_3=0$	$R_2=20$, $R_3=10$	$I_1=0{,}2$	$\mathbf{I_2, I_3, R_1}$
20	4, 1, 6	$\varepsilon_1=13$, $\varepsilon_4=1$	$r_4=1$	$R_1=27$, $R_4=24$, $R_6=40$	$I_1=0{,}3$	$\mathbf{I_4, I_6, r_1}$
21	2, 1, 4	$\varepsilon_1=12$, $\varepsilon_2=9$, $\varepsilon_4=5$	$r_1=r_2=r_4=0$	$R_1=30$, $R_2=60$, $R_4=20$	–	$\mathbf{I_1, I_2, I_4}$
22	2, 1, 6	$\varepsilon_1=8$, $\varepsilon_2=6$	$r_1=3$	$R_1=27$, $R_2=9$, $R_6=25$	$I_2=0{,}1$	$\mathbf{I_1, I_6, r_2}$
23	5, 1, 4	$\varepsilon_1=19$, $\varepsilon_4=6$, $\varepsilon_5=2$	$r_1=r_4=r_5=0$	$R_4=50$, $R_5=10$	$I_1=0{,}2$	$\mathbf{I_4, I_5, R_1}$
24	1, 6, 2	$\varepsilon_1=18$, $\varepsilon_2=15$	$r_1=2$, $r_2=1$	$R_1=58$, $R_2=9$, $R_6=30$	–	$\mathbf{I_1, I_2, I_6}$
25	4, 1, 2	$\varepsilon_2=4$, $\varepsilon_4=2$	$r_1=r_2=r_4=0$	$R_1=50$, $R_2=20$, $R_4=80$	$I_1=0{,}2$	$\mathbf{I_2, I_4, \varepsilon_1}$
26	1, 6, 5	$\varepsilon_1=8$, $\varepsilon_5=6$	$r_1=2$, $r_5=3$	$R_1=8$, $R_5=12$, $R_6=10$	-	$\mathbf{I_1, I_5, I_6}$
27	2, 4, 5	$\varepsilon_2=8$	$r_2=2$, $r_4=1$, $r_5=5$	$R_2=18$, $R_4=14$, $R_5=25$	$I_4=0{,}2$, $I_5=0{,}3$	$\mathbf{I_2, \varepsilon_4, \varepsilon_5}$
28	3, 6, 4	$\varepsilon_3=36$, $\varepsilon_4=9$	$r_3=2$, $r_4=1$	$R_3=16$, $R_4=8$	$I_6=0{,}5$	$\mathbf{I_4, I_3, R_6}$
29	3, 1, 5	$\varepsilon_3=40$, $\varepsilon_5=30$	$r_1=r_5=2$, $r_3=5$	$R_3=35$, $R_1=28$, $R_5=28$	$I_1=0{,}7$	$\mathbf{I_5, I_3, \varepsilon_1}$
30	2, 3, 4	$\varepsilon_2=20$, $\varepsilon_4=40$, $\varepsilon_3=10$	$r_2=10$, $r_4=15$, $r_3=5$	$R_2=110$, $R_4=105$	$I_3=0{,}2$	$\mathbf{I_4, I_2, R_3}$

THE PICTURES FOR PROBLEM 1.5



Picture 4. – Structure of branches of composite circuit



Picture 4, a. – Example of building of a scheme for 25th variant

Problem 2.1
THE PRINCIPLE OF MAGNETIC FIELDS' SUPERPOSITION
MAIN CONCEPTS

Biot-Savart-Laplace law:

$$d\vec{H} = \frac{1}{4\pi} \cdot \frac{I \cdot [d\vec{l} \times \vec{r}]}{r^3} ,$$

where $d\vec{H}$ – a vector of magnetic intensity of a field, which is created of the element of a current $\boldsymbol{I}\,d\vec{l}$; $\vec{r}$ – a radius-vector from the element $\boldsymbol{I}\,d\vec{l}$ to the target point.

The modulus of magnetic field intensity:

a) the ***endless straight wire*** with a current ***I*** at a distance of ***r*** from the wire

$$H = \frac{I}{2\pi r} ;$$

b) the ***segment of straight wire*** with a current ***I*** at a distance of $r_\perp$ from the axis of a segment

$$H = \frac{I}{4\pi \cdot r_\perp}(\cos\alpha_1 + \cos\alpha_2) ,$$

where α_1 and α_2 – angles, which are created radius-vectors, traced from the ends of a conductor to the target point at which field is being calculated;

c) at the centre of the ***current-carrying coil*** of the radius R with a current *I*

$$H_C = \frac{I}{2R};$$

d) at the axis of the ***current-carrying coil***

$$H_A = \frac{IR^2}{2(R^2 + a^2)^{3/2}},$$

where ***a*** – is the distance from the centre of coil to the target point at the axis;

e) at the axis of the ***infinitely long solenoid***

$$H = I \cdot n_0 = \frac{IN}{L},$$

where N – the number of wire coils in the solenoid; L – the length of the solenoid; n_0 – the quantity of coils on the unit of solenoid length.

The superposition principle: the magnetic field created with one current is independent from positions of other currents. Then the resultant vector of magnetic intensity is a vector sum of magnetic intensity vectors of individual currents:

$$\vec{H}_{RES} = \vec{H}_1 + \vec{H}_2,$$

therefore: 1) the ***direction*** of the resultant vector is defined by parallelogram rule of the vectors' additions; 2) the ***magnitude*** of the resultant vector is defined with cosine theorem:

$$H_{RES} = \sqrt{H_1^2 + H_2^2 + 2H_1H_2\cos\alpha},$$

where α – angle between the vectors.

The relation between magnetic induction $\vec{B}$ and magnetic intensity $\vec{H}$:

$$\vec{B} = \mu_0\mu\vec{H},$$

where $\mu_0 = 4\pi\cdot 10^{-7}$ *H/m* – permeability of vacuum; μ – magnetic permeability (for air $\mu=1$).

EXAMPLES OF PROBLEM SOLUTION

Example 4. The endless direct wire is located perpendicularly to the plane of the current coil and is situated at a distance of $a=5cm$ from its centre. A current in the wire $I_1=10A$, a current in the coil $I_2=3A$. Current's direction is shown in the picture. Radius of coil $R=3cm$. Find the magnetic field induction in the centre of coil.

Input:

$I_1=10A$;

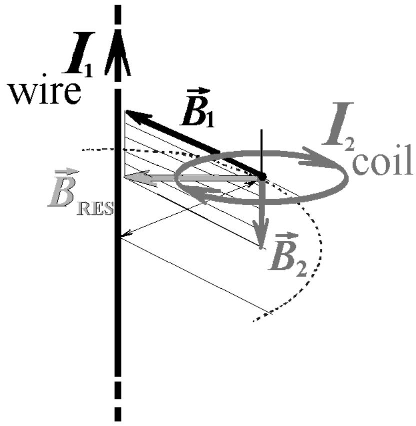
$I_2=3A$;

$a=5cm=5\cdot 10^{-2}\,m$;

$R=3cm=3\cdot 10^{-2}\,m$.

Find:

B – ?



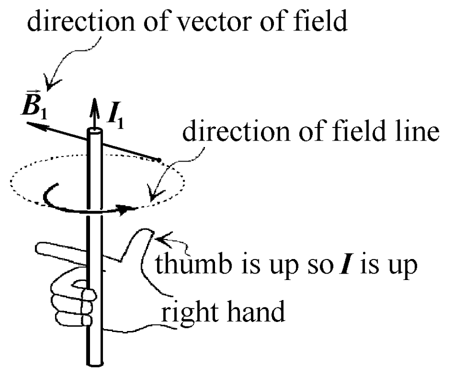
Picture a) Parallelogram rule

Solution: In accordance with the superposition principle of magnetic fields the magnetic induction $\vec{B}$ equals the *vector sum* of the magnetic field inductions $\vec{B}_1$ and $\vec{B}_2$ at the centre of coil, which are created in this point with the individual currents I_1 and I_2:

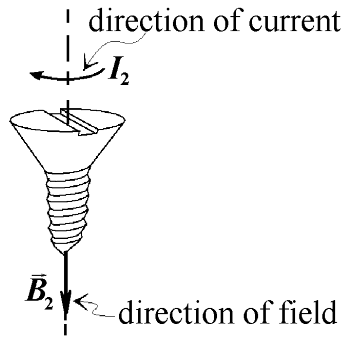
$$\vec{B} = \vec{B}_1 + \vec{B}_2 .$$

1) We must show the direction of the resultant vector of magnetic induction defined by parallelogram rule to the diagrammatic drawing (see **Picture a)**).

Let's find direction of a vector $\vec{B}_1$ with the *right-hand rule* (see **Picture b)**). The thumb direction should coincide with a direction of a current I_1. The curled fingers points in the direction of the field line. Vector of magnetic field lies along a tangent to the field line. The straight forefinger of right hand points in the direction of vector of magnetic field $\vec{B}_1$.



Picture b) Right-hand rule



Picture c) Screw rule

Let's find direction of a vector $\vec{B}_2$ with the *screw rule* (see **Picture c)**). Rotational motion of screw should coincide with a direction of a current I_2. Translational movement of screw points in the direction of vector of magnetic field $\vec{B}_2$.

The resultant vector $\vec{B}$ is defined with parallelogram rule of the vectors' additions (see **Picture b)**).

2) We define the magnitude of the resultant vector with cosine theorem:

$$B = \sqrt{B_1{}^2 + B_2{}^2 + 2B_1B_2 \cos\alpha} .$$

As the vector $\vec{B}_1$ is perpendicular to a current I_1, and the vector $\vec{B}_2$ lies in the plane of a current I_1, then angle is between the vectors $\alpha=90^{o}$ ($\cos 90^{o}=0$). Therefore cosine theorem will be conversed to a Pythagorean theorem:

$$B = \sqrt{B_1{}^2 + B_2{}^2} . \qquad (1)$$

We find the magnetic field induction of the infinitely long straight wire with a current in the air with the formula

$$B_1 = \mu\mu_0 \frac{I_1}{2\pi r}, \qquad (2)$$

where $\mu_0=4\pi\cdot10^{-7}$ *H/m* – permeability of vacuum; $r=a$ – distance from the wire to the target point.

The magnetic field induction in the centre of the coil with radius of R

$$B_2 = \mu\mu_0 \frac{I_2}{2R}. \qquad (3)$$

Substituting (2) and (3) in the formula (1) in the given case μ=1, we'll get:

$$B = \sqrt{\frac{\mu_0{}^2 I_1{}^2}{4\pi^2\alpha^2} + \frac{\mu_0{}^2 I_2{}^2}{4R^2}}, \qquad \text{or} \qquad B = \frac{\mu_0}{2}\sqrt{\frac{I_1{}^2}{\pi^2\alpha^2} + \frac{I_2{}^2}{R^2}}. \qquad (4)$$

Check if the right part of the formula gives us (4) the unit of magnetic induction [T]

$$[\hat{A}] = [\mu_0] \cdot \left[\frac{[I^2]}{[R^2]}\right]^{1/2} = \frac{H}{m} \cdot \left[\frac{[A^2]}{[m^2]}\right]^{1/2} = \frac{Wb \cdot A}{A \cdot m \cdot m} = \frac{T \cdot m^2}{m^2} = \grave{O}.$$

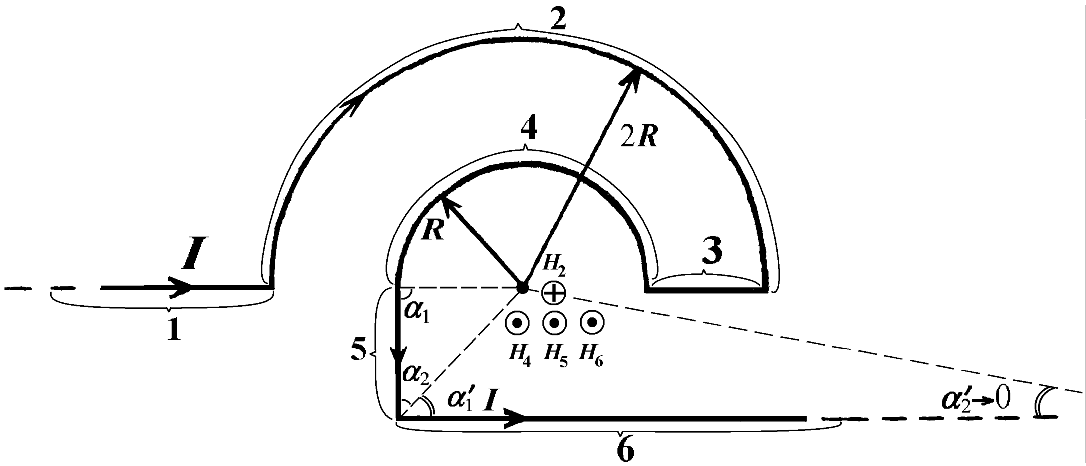
Let' make the calculations:

$$\hat{A} = \frac{4.3.14 \cdot 10^{-7}}{2} \cdot \frac{10^2}{3{,}14^2 \cdot 0{,}03^2} = 7{,}45 \cdot 10^{-5}\, \grave{O}.$$

Result: $B = 7{,}45 \cdot 10^{-5}\, T$.

Example 5. Over the infinitely long wire, so bent, as it's shown in the picture, current flow 5A. Radius of the arc 5cm. Find the intensity of the magnetic field in the point O.

Input data:
I=5 A;
R=5·10^{-2} m.
Find:
H – ?



Solution:

We'll find $\vec{H}$ the magnetic field intensity in the point O using the superposition principle of fields. In the given case wire can be divided into six segments: four straight segments: №1, №3, №5 and №6 and two arcs of the semicircles: №2 and №4 (see picture). The magnetic field intensity in the target point is equal to the vector sum of the intensities, which are created with six segments:

$$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 + \vec{H}_4 + \vec{H}_5 + \vec{H}_6. \qquad (1)$$

The magnitude of magnetic intensity vector, which is created with the element of a current $I d\vec{l}$, is found from Biot-Savart-Laplace law:

$$dH = \frac{1}{4\pi} \cdot \frac{I \cdot dl \cdot r \cdot \sin\alpha}{r^3},$$

where α – angle between the radius-vector to the target point and the element of a current $Id\vec{l}$.

As the point O lies on the axis of the segment №1 and №3, the radius-vectors lies on the direction of corresponding currents. Then α_1=α_2=0 and H_1=H_3 = 0

$$\vec{H} = \vec{H}_2 + \vec{H}_4 + \vec{H}_5 + \vec{H}_6. \qquad (2)$$

The vectors' directions of intensity of straight segments №5 and №6 are found with the *right-hand rule*, and the vectors' directions of intensity of semicircles: №2 and №4 are found with the *screw rule*.

We find the vectors' directions of intensity with the right-hand screw rule. Vector $\vec{H}_2$ is perpendicularly directed to the plane of the picture from an observer, vectors $\vec{H}_4, \vec{H}_5, \vec{H}_6$ – to the observer. Taking into account the direction to an observer, we replace the vector equality (1) with scalar.

$$H=H_4+ H_5+ H_6 – H_2.$$

We find magnitudes of arcs' vectors of the semicircles №2 and №4 using the formula of calculation for the filed intensity in the centre of coil

$$H = \frac{I}{2R}.$$

Radiuses of semicircles R_2=2R and R_4=R (see picture). In the given case the magnetic field is created only with the half of such loop current, that's why

$$H_2 = \frac{I}{8R} \quad \text{and} \quad H_4 = \frac{I}{4R}. \qquad (3)$$

We find magnitudes of vectors of straight segments №5 and №6 using the formula of calculation for the filed intensity of segment of straight wire

$$H = \frac{I}{4\pi r_\perp}(\cos\alpha_1 + \cos\alpha_2).$$

where $r_\perp$ – the smallest distance from the wire to the point, where the intensity is situated; α_1 and α_2 – angels, which are created radius-vectors, traced from the ends of a conductor to the target point.

For the segment №5: $r_\perp$ =R; α_1=90^0; cosα_1=0; α_2=45°; cos α_2= √2/2, then

$$H_5 = \frac{I\sqrt{2}}{8\pi R}. \qquad (4)$$

For the segment №6: $r_\perp$ =R; α_1=45°; cosα_1=√2/2; $\alpha_2 \to 0$; cos $\alpha_2 \to 1$, then

$$H_6 = \frac{I}{4\pi R}\left(\frac{\sqrt{2}}{2} + 1\right). \qquad (5)$$

Inserted (3), (4) and (5) in the formula (2), we'll get

$$H = \frac{I}{4R} + \frac{I\sqrt{2}}{8\pi R} + \frac{I}{4\pi R}\left(\frac{\sqrt{2}}{2} + 1\right) - \frac{I}{8R} = \frac{I}{8R} + \frac{I\sqrt{2}}{4\pi R} + \frac{I}{4\pi R},$$

or

$$H = \frac{I}{8\pi R}\left(\pi + 2\sqrt{2} + 2\right). \qquad (5)$$

Check if the right part of the formula gives (5) the unit of the magnetic field intensity [*A/m*]

$$[H] = \frac{[I]}{[R]} = \frac{A}{m}.$$

Let's make the calculations:

$$H = \frac{5}{8 \cdot 3{,}14 \cdot 0{,}05}(3{,}14 + 2\sqrt{2} + 2) = 31{,}7 A/m.$$

Result: H = 31,7 *A/m*.

INDIVIDUAL TASKS FOR PROBLEM 2.1
THE PRINCIPLE OF MAGNETIC FIELDS' SUPERPOSITION

In accordance with your variant to solve one of the following problems listed below. The number of problem statement and all necessary input data are reduced in the table 2.1. Necessary drawing is allocated from Figures from problem 2.1.

1 Find magnetic induction, which is created with two infinitely long electric conductors. The picture number, in which the arrangements of conductors are shown, values of current I_1 and I_2, point, at which magnetic induction should be found, and necessary distances are given at the table. 2.1.

2 Find the magnetic field intensity, which is created with the infinitely long straight conductor and wire loop of radius R. The picture number, in which the situations of a conductor and loop are shown, the values of currents in the conductor I_1 and loop I_2, point, at which magnetic intensity should be found, and necessary distances are given at the table. 2.1.

3 A current I_1 flows along the infinitely long bent wire. The picture number, in which the wire's form is shown, the values of currents and necessary distances are given at the table 2.1. Find the magnetic induction at the point O.

4 The infinitely long wire, bent at an angle α, and circular contour of radius of R lie in the same plane. The picture number, in which the situations of contour and wire are shown, values of currents in the wire I_1 and contour I_2, angle α and necessary distances are given at the table 2.1. Find the magnetic field intensity in the centre of the circular contour.

TABLE OF TASK VARIANTS

Table 2.1

Variant	Statement number	Picture number*	Point	I_1, A	I_2, A	*R, sm*	*a, sm*	*b, sm*	*c, sm*
1	1	1	**A**	3	4	–	1	–	–
2	3	6	**O**	10	–	4	–	–	–
3	1	2	**A**	4	4	–	10	–	–
4	3	9	**O**	9	–	3	–	–	–
5	2	4	**O**	10	2	2	5	–	–
6	1	1	**B**	12	8	–	1	2	–
7	4	10	**O**	8	5	2	–	–	–
8	1	3	**B**	8	6	–	5	3	4
9	2	5	**B**	2	10	5	6	–	–
10	3	7	**O**	10	–	3	–	–	–
11	1	3	**A**	5	3	–	8	–	–
12	4	1	**O**	3	2	4	–	–	–
13	2	5	**A**	6	5	6	8	–	–
14	3	8	**O**	8	–	2	–	–	–
15	4	12	**O**	10	5	5	3	–	–
16	1	1	**C**	16	12	–	2	4	–
17	3	6	**O**	5	–	3	–	–	–
18	2	5	**O**	15	4	4	5	–	–
19	4	10	**O**	4	3	1	–	–	–
20	1	2	**B**	6	8	–	5	3	4
21	3	7	**O**	12	–	2	–	–	–
22	2	4	**A**	8	10	3	4	–	–
23	4	11	**O**	6	5	3	–	–	–
24	3	8	**O**	6	–	1	–	–	–
25	4	12	**O**	8	2	10	5	–	–
26	1	1	**B**	2	6	-	5	3	-
27	2	5	**O**	3	7	5	9	-	-
28	3	9	**O**	5	-	10	-	-	-
29	1	2	**B**	3	5	-	5	3	4
30	2	4	**A**	2	6	7	9	-	-

* See Figures for problem 2.1.

FIGURES FOR PROBLEM 2.1 (beginning)

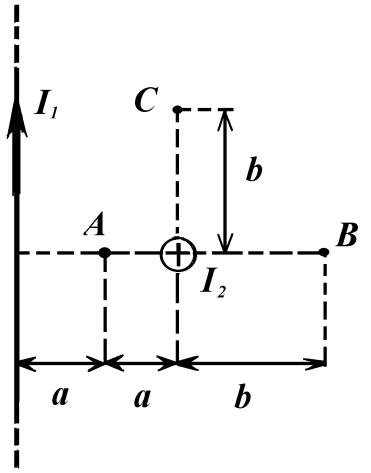


Figure 1

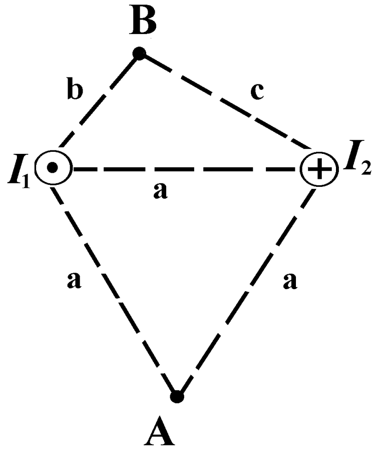


Figure 2

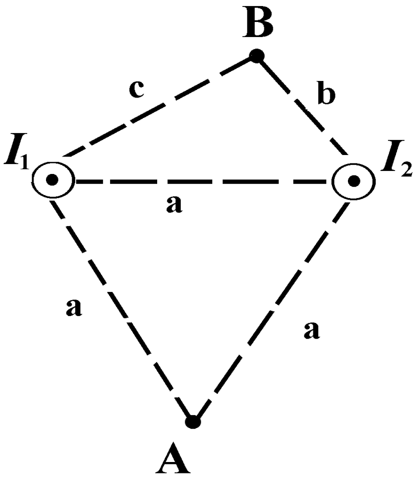


Figure 3

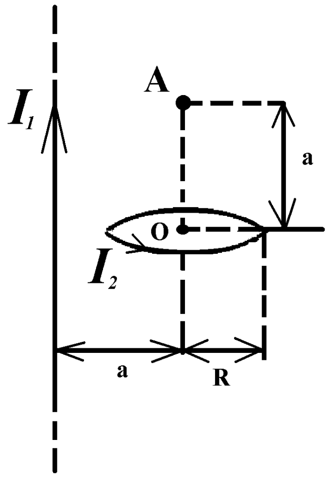


Figure 4

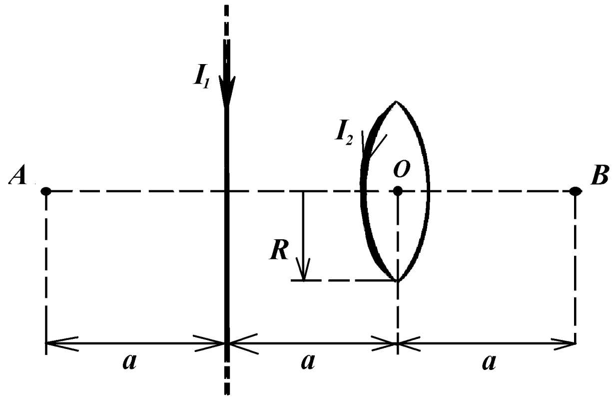


Figure 5

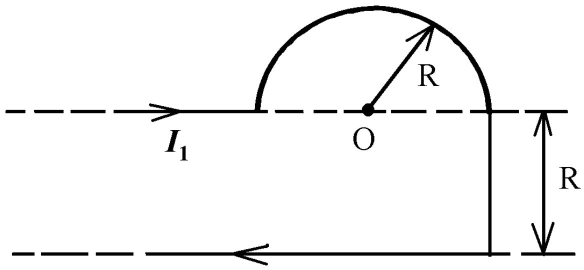


Figure 6

FIGURES FOR PROBLEM 2.1 (end)

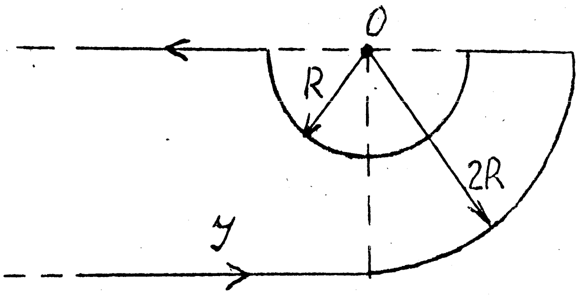


Figure 7

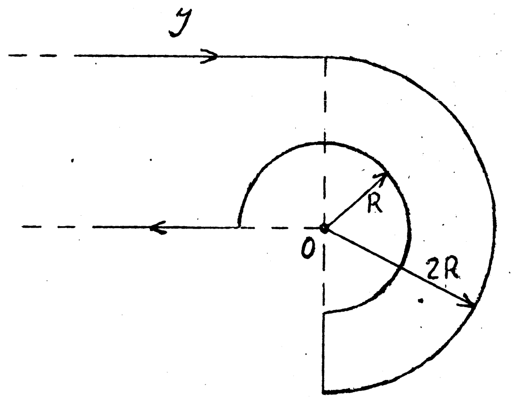


Figure 8

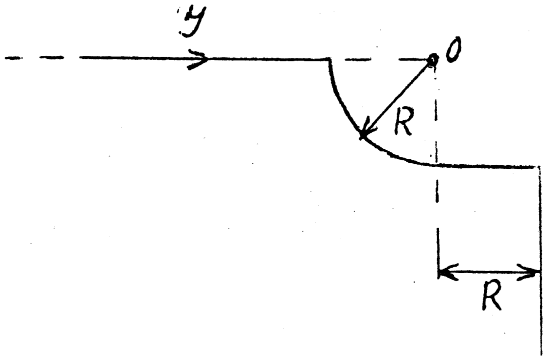


Figure 9

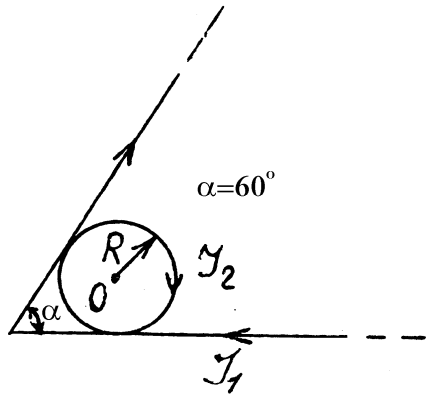


Figure 10

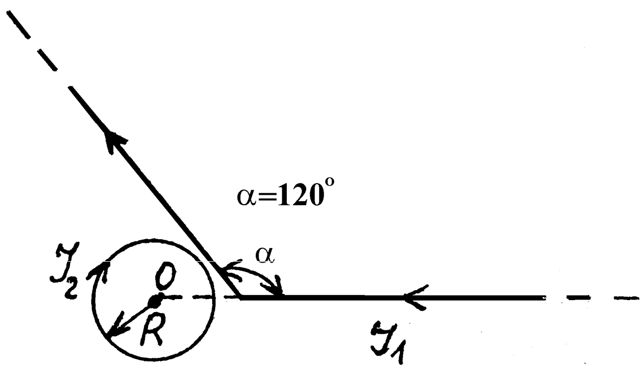


Figure 11

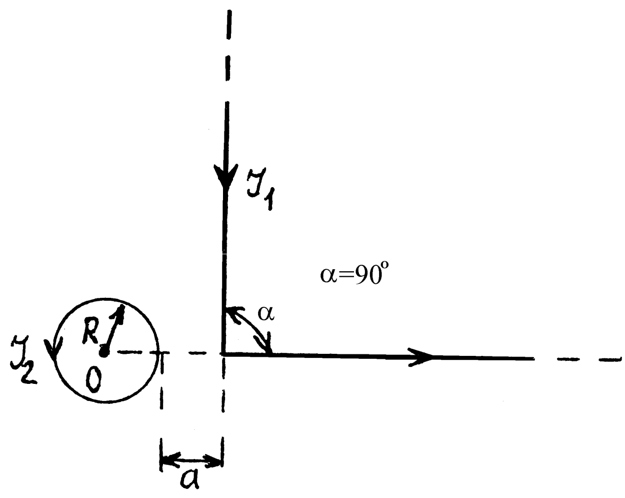


Figure 12

Problem 2.4
THE LAW OF ELECTROMAGNETIC INDUCTION

MAIN CONCEPTS

Flux of the vector of magnetic induction $\vec{B}$ through the plane contour in the homogeneous (uniform) magnetic field

$$\Phi = B \cdot S \cdot \cos\alpha,$$

where S – area of the contour; α – angel between vector $\vec{B}$ and normal $\vec{n}$ to the plane of the contour.

Full magnetic flux or magnetic flux interlinkage:

$$\phi = N\Phi,$$

where N – the quantity of loops of windings, which are transpierced by the flux Φ.

EMF of induction in the coil

$$\varepsilon_i = -\frac{d\phi}{dt}.$$

Charge, which flows over the closed contour by the changing of magnetic flux

$$Q = -\frac{\Delta\phi}{R},$$

where R – resistance of contour.

Magnetic field energy, stored by the coil with the inductance L, through which the current I flows

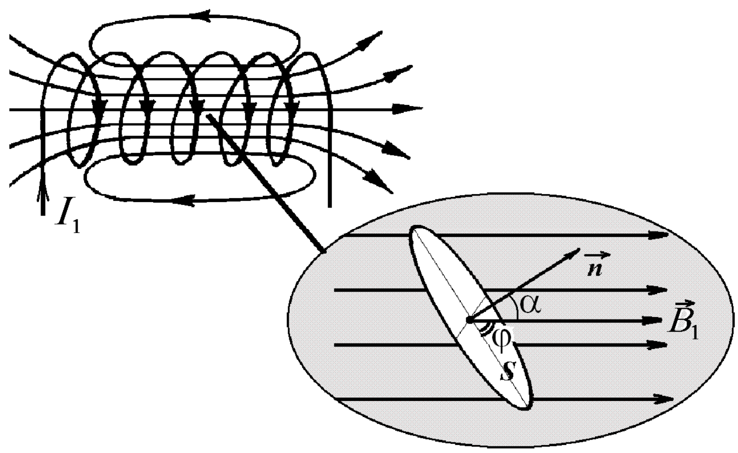
$$W = \frac{LI^2}{2}.$$

EXAMPLES OF PROBLEM SOLUTION

Example 6. In the middle of the long straight solenoid, which has 10 turns per centimeter, there is the circular contour with area 10 cm^2. Plane of contour is situated at an angle of 30^0 to an axis of the solenoid (look at the picture). A current 5*A* flows over the winding of solenoid. What average EMF is appears in the circular contour, if during 10 *ms* the direction of a current in solenoid is changed on the opposite one?

Input data:
$n_0 = 10\ cm^{-1} = 10^3\ m^{-1}$
$S = 10\ cm^2 = 10^{-3}\ m^2$;
$\varphi = 30^0$;
$I = 5A$;
$\Delta t = 10\ ms = 10^{-2}\ s$.

Find: $\langle\varepsilon\rangle$ – ?



Solution:

When a current in the coil of solenoid is changed, the magnetic flux of Φ, that runs through the circular contour, will change too. EMF of induction, which can be found in Faraday's law of electromagnetic induction $\varepsilon = -d\phi/dt$ appears owing to this in contour. They find instantaneous value of EMF of induction with this formula, but for finding the average value from the average speed of magnetic flux changing in course of time we should write

$$\langle\varepsilon\rangle = -\frac{\Delta\phi}{\Delta t} = N\frac{\Phi_1 - \Phi_2}{\Delta t}, \tag{1}$$

where $\Delta\phi = N\cdot(\Phi_2 - \Phi_1)$ - the interlinkage changing in time interval of Δt. Circular contour has a one loop N=1. In the middle part of long solenoid field will be homogeneous. In this case the magnetic flux is found with the formula: $\Phi = BS\cdot\cos\alpha$, where B – the solenoid magnetic field induction; S – area of contour; $\alpha = 90^0 - \varphi$ – angle between vector $\vec{B}$ and normal $\vec{n}$ to the plane of contour.

The magnetic field induction is in the middle of long solenoid

$$B = \mu\mu_0 I\cdot n_0, \tag{2}$$

where μ_0– a magnetic constant; μ– magnetic permeability of medium (in air μ=1) I – current strength in the winding of solenoid; n_0– quantity of turns per the unit of length of a solenoid.

Let's choose the direction of a current in the winding of solenoid, as it's shown in the picture I_1, and we'll find corresponding to the direction of vector $\vec{B}_1$ with the screw rule. Magnetic flux equals in this case

$$\Phi_1 = \mu_0 I n_0 S\cos\alpha. \tag{3}$$

After changing the current direction on opposite one and vector of magnetic induction will change its direction $\vec{B}_2 \uparrow\downarrow \vec{B}_1$.

Magnetic Flux

$$\Phi_2 = \mu_0 I n_0 S\cos\alpha_2 = -\mu_0 I n_0 S\cos\alpha. \tag{4}$$

Here it is necessary to take into account, that $\alpha_2 = 180^0 - \alpha$ and $\cos\alpha_2 = -\cos\alpha$. Inserting the expressions Φ_1 (3) and Φ_2 (4) in the formula (1) and taking into account, that $\alpha = 90^0 - \varphi$ and $\cos\alpha = -\cos\varphi$ (look at the picture), we get

$$\langle\varepsilon\rangle = \frac{2\mu_0 I n_0 S\cos\alpha}{\Delta t} = \frac{2\mu_0 I n_0 \sin\varphi}{\Delta t}.$$

Let's check if the right part of the calculating formula gives the unity of EMF difference [V]:

$$[\varepsilon] = \frac{H/m\cdot \grave{A}\cdot m^{-1}\cdot 1}{s} = \frac{Wb}{s} = V.$$

Let's make the calculations

$$\langle\varepsilon\rangle = \frac{2\cdot 4\cdot 4{,}14\cdot 10^{-7}\cdot 5\cdot 10^{3}\cdot 10^{-3}\sin 30^0}{10^{-2}} = 6{,}28\cdot 10^{-4}V.$$

Result: $\langle\varepsilon\rangle = 6{,}28\cdot 10^{-4}\ V.$

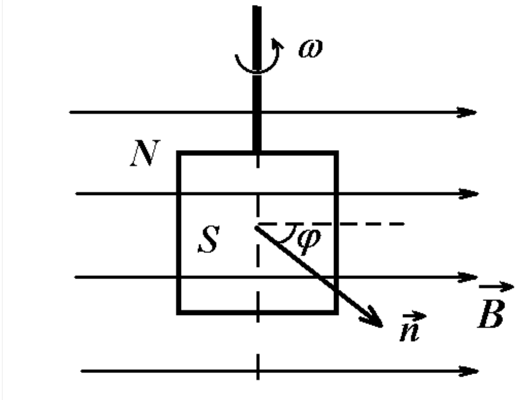
Example 7 In the homogeneous magnetic field with induction of 0,5 *T* short coil is uniformly rotated. Coil consists of 100 loops with the area of 100 cm^2. The axis of rotation is in the plane of loops of the coil and perpendicularly to the lines of magnetic field induction (see the picture).

Find:

1) maximal EMF, which appears into coil with its rotation with period of 0,1 *s*;
2) charge, which flows over the coil within the change of angle between coil's axis and magnetic induction's vector from $\varphi_1=0^o$ to $\varphi_2=90^o$, if the coil resistance is 2Ω.

Input data:
B=0,5 *T*;
N=100;
S=100 cm^2;
T=0,1 *s*;
$\varphi_1=0$; $\varphi_2=90;^o$
R=2 Ω.

Find:
ε_{max} ; Q – ?



Solution:

The magnetic flux which runs through the coil changes with the rotation of the coil, and in accordance with Faraday's law of electromagnetic induction in coil EMF of induction appears

$$\varepsilon = -\frac{d\phi}{dt}, \tag{1}$$

where the magnetic flux interlinkage in the homogeneous magnetic field with induction of B is found with the formula:

$$\phi = BSN \cdot \cos\varphi \tag{2}$$

where S – area of loops; φ – angle between normal $\vec{n}$ to the plane of coil's loops and vector $\vec{B}$; N – quantity of loops, which are transpierced with the flux $\Phi=BS\cdot\cos\varphi$. Coil during time of t will be turned an angle of $\varphi=\omega t$, when the rotation is uniform, ω – angular velocity of rotation, which related with the period of rotation as $\omega = 2\pi / T$.

Substituted expressions ω and φ in the formula (2) we'll get

$$\phi = BSN \cdot \cos\omega t = BSN \cdot \cos(\frac{2\pi}{T} \cdot t). \tag{3}$$

In order to find the instantaneous value of EMF of induction we'll substitute the expression for magnetic flux interlinkage (3) in the formula (1) and differentiate with time

$$\varepsilon = -\frac{d}{dt}[B \cdot S \cdot N \cdot \cos(\frac{2\pi}{T})] = BSN\frac{2\pi}{T}\sin(\frac{2\pi}{T} \cdot t).$$

The value of EMF will be maximum at that moment of time, if $\sin(\frac{2\pi}{T}t) = 1$.

Then $$\varepsilon_{\max} = BSN\frac{2\pi}{T}. \quad (4)$$

In order to find the charge of Q, which runs over the coil, let's apply Ohm's law for a closed circuit instantaneous value of EMF

$$\varepsilon_I = Ri, \quad (5)$$

where R – coil's resistance; i – instantaneous value of current in the coil.

Equating the right parts of equations (1) and (5), we'll get

$$-\frac{d\phi}{dt} = Ri.$$

As there is instantaneous value of current strength $i = dQ / dt$ then this equation can be rewritten in the type of

$$-\frac{d\phi}{dt} = R\frac{dQ}{dt},$$

whence

$$dQ = -\frac{1}{R}\cdot d\phi. \quad (6)$$

If one takes integral from expression (6), then we'll find charge, which flows through the coil with the change of magnetic flux interlinkage from ϕ_1 till ϕ_2:

$$\int_0^Q dQ = -\frac{1}{R}\int_{\phi_1}^{\phi_2} d\phi \quad \text{or} \quad Q = \frac{\phi_1 - \phi_2}{R}. \quad (7)$$

Changing the angle between coil's axis and vector $\vec{B}$ from φ_1 to φ_2, then the magnetic flux interlinkage will change from $\phi_1 = BSN\cdot\cos\varphi_1$ till $\phi_2 = BSN\cdot\cos\varphi_2$. Substituted expressions ϕ_1 and ϕ_2 in the formula (7), we'll find the sought-for charge:

$$Q = \frac{BSN}{R}(\cos\varphi_1 - \cos\varphi_2). \quad (8)$$

Let's check if the right part of the formula gives (4) the unit of EMF (B), and the right part of the formula (8) the unit of charge (C):

$$[\varepsilon_{\max}] = \frac{[B][S][N]}{[T]} = \frac{T\cdot m^2}{s} = \frac{N\cdot m^2}{A\cdot m\cdot s} = \frac{J}{C} = V;$$

$$[Q] = \frac{[B][S][N]}{[R]} = \frac{T\cdot m^2}{\Omega} = \frac{N\cdot m^2}{A\cdot m\cdot\Omega} = \frac{J}{V} = C.$$

Let's make the calculations:

$$\varepsilon_{\max} = \frac{2\cdot 3{,}14\cdot 100\cdot 0{,}5\cdot 10^{-2}}{0{,}1} = 31{,}4\ V;$$

$$Q = \frac{100\cdot 0{,}5\cdot 10^{-2}}{2}(\cos 0^o - \cos 90^o) = 0{,}25\ C.$$

***Results*:** $\varepsilon_{\max} = 31{,}4\ V$, $Q = 0{,}25\ C$.

INDIVIDUAL TASKS FOR PROBLEM 2.4
THE LAW OF ELECTROMAGNETIC INDUCTION

In accordance with your variant to solve one of the following problems listed below (The number of problem statement and all necessary input data are reduced in the table 2.4):

1 Coil of area of S has N loops of wire with resistance of R and is situated in the homogeneous magnetic field with induction of B. The loop's plane makes the angle of β with magnetic induction lines. Within the changing of magnetic field direction on the opposite one, the charge of Q runs over the coil. With the values of magnitudes, set in the table 2.4, find the magnitude, which is set in the last column of the table.

2 On the iron core solenoid with inductance of L and area of cross-section of S is put on the wire ring. The winding of solenoid has loops of N. When the circuit is closed, through the solenoid the current I is been settled during the time interval Δt. The average EMF of ε_{AVE} is induced in the wire ring. The magnetic permeability of iron is μ within these conditions. With the values of magnitudes, set in the table 2.4, find the magnitude, which is set in the last column of the table.

3 In the homogeneous magnetic field with induction of B rotating the short coil, which has N loops of diameter of D. The axis of rotation lies in the plane of loops of the coil and is perpendicular to the lines of magnetic field induction. Angular velocity of rotation is ω (frequency of rotation of ν, period of T). The maximal value of EMF of induction, which appears in the coil is ε_{MAX}. With the values of magnitudes, set in the table 2.4, find the magnitude, which is set in the last column of the table.

TABLES OF TASK VARIANTS

Table 2.4

Variant	Statement	N	S, cm^2	D, cm	L, cm	R, Ω	B, T	β	Q, mC	I, A	μ	ε_{AVE}, V	Δt, ms	ε_{MAX}, V	ω, rad/s	ν, $1/s$	T, ms	Find
1	3	–	–	12	–	–	0,4	–	–	–	–	–	–	30	60	–	–	***N***
2	1	100	80	–	–	20	0,2	30°	-	–	–	–	–	–	–	–	–	***Q***
3	3	100	–	10	–	–	–	–	–	–	–	–	–	15	90	–	–	***B***
4	2	500	4	–	30	–	–	–	–	2	–	0,4	2	–	–	–	–	**μ**
5	1	300	40	–	–	–	0,2	60°	4	–	–	–	–	–	–	–	–	***R***
6	3	500	–	20	–	–	0,3	–	–	–	–	–	–	90	–	–	–	**ω**
7	1	400	50	–	–	40	0,1	–	5	–	–	–	–	–	–	–	–	**β**
8	2	600	4	–	25	–	–	–	–	0,8	400	–	3	–	–	–	–	ε_{ave}
9	3	300	–	5	–	–	–	–	–	–	–	–	–	50	–	25	–	***B***
10	1	100	40	–	–	15	–	40°	6	–	–	–	–	–	–	–	–	***B***
11	3	–	–	18	–	–	0,2	–	–	–	–	–	–	60	–	15	–	***N***
12	1	200	50	–	–	10	0,3	60°	–	–	–	–	–	–	–	–	–	***Q***
13	3	100	–	8	–	–	–	–	–	–	–	–	–	80	–	–	20	***B***
14	2	800	3	–	20	–	–	–	–	0,5	–	0,3	1	–	–	–	–	**μ**
15	1	–	20	–	–	30	0,3	30°	2	–	–	–	–	–	–	–	–	***N***
16	3	100	–	8	–	–	0,4	–	–	–	–	–	–	50	–	–	–	***T***
17	1	200	20	–	–	5	0,3	–	48	–	–	–	–	–	–	–	–	**β**
18	3	200	–	20	–	–	0,1	–	–	–	–	–	–	–	–	20	–	ε_{maz}
19	2	700	8	–	40	–	–	–	–	0,6	600	–	2	–	–	–	–	ε_{ave}
20	3	–	–	15	–	–	0,1	–	–	–	–	–	–	40	–	–	30	***N***
21	1	500	10	–	–	25	–	20°	4	–	–	–	–	–	–	–	–	***B***
22	3	200	–	10	–	–	0,2	–	–	–	–	–	–	70	–	–	–	***v***
23	2	900	6	–	35	–	–	–	–	–	800	0,6	4	–	–	–	–	***I***
24	3	400	–	15	–	–	0,3	–	–	–	–	–	–	–	50	–	–	ε_{maz}
25	2	–	5	–	25	–	–	–	–	1	200	0,5	1	–	–	–	–	***N***
26	2	650	-	–	10	–	–	–	–	0,2	300	0,1	5	–	–	–	–	***S***
27	3	450	–	–	–	–	0,4	–	–	–	–	–	–	80	–	–	25	***D***
28	2	150	7	–	15	–	–	–	–	5	200	0,7	-	–	–	–	–	Δt
29	1	400	-	–	–	5	0,15	70°	6	–	–	–	–	–	–	–	–	***S***
30	2	900	3	–	-	–	–	–	–	0,4	500	0,2	6	–	–	–	–	***L***

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CONTENT

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Part 2: complex task