UKRAINIAN STATE COMMITTEE OF COMMUNICATIONS AND INFORMATIZATION

**ODESSA NATIONAL ACADEMY OF TELECOMMUNICATIONS**

**after A. S. POPOV**

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# PHYSICS

**Module 2. Electrical oscillations and waves**

# ELECTROMAGNETIC OSCILLATIONS and WAVES

## PART 2: COMPLEX TASK

for bachelor training of educational area 0924 –“Telecommunications”

APPROVED

by the Faculty Council

Protocol № 7

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The following methodical guide is about section “Electromagnetic oscillations and waves” of physics course for telecommunications technician. Four problems allow students to learn basics of electrical engineering and calculations techniques applied to determine main characteristics of electrical oscillations systems and electromagnetic waves. It contains sufficient theoretical information combined with examples of problems’ solutions.

Recommended for students of TE-group, educational area 0924 – “Telecommunications”.

CONFIRMED

at the Department session

Protocol № 6

from 27.02.2009

## MODULE STRUCTURE

**Module № 2. „ Electrical oscillations and waves" –** 72 hours total

**Lectures – 16 hrs, pract. trainings – 0 hrs, labs – 16 hrs, self-studies – 33 hrs.**

### LIST OF PRACTICAL TRAINING

| Number of lessons | Denomination of lessons | Hours |
|---|---|---|
| **Module № 2** | | |
| 1 | Mechanical harmonic oscillations. Methods of representation of harmonic oscillations. | 2 self |
| 2 | Calculation of parameters of harmonic electrical oscillations. | 2 lab |
| 3 | Damped oscillations. Parameters of damped oscillations. Quality factor of a system. | 2 self |
| 4 | Calculation of parameters of damping electrical oscillations. | 2 lab |
| 5 | Electrical driven oscillations. Alternative current. Phasors. | 2 self |
| 6 | Calculation of current and voltage in RLC circuit. | 2 lab |
| 7 | Electromagnetic waves. Energy flux density. | 2 self |
| 8 | Calculation of parameters of electromagnetic waves. | 2 lab |

## INTRODUCTION

All problems should be solved in a individual way, i.e. each student have its own variant, which is specified by instructor.

Appropriate homework must forego to solving a problem. The homework contains self-studying of theory of sections ***"Oscillations"*** and ***"Waves"*** of the course of physics for telecommunications technician.

- Complex task is consists of four problems: 3.2; 3.3; 3.4; 3.6. Student must implement on one task from each problem. Concrete number of statement on each stage and initial data in thirty variants is specified in Tables of task variants. Number of the variant is determining by the index of surname of student at a group journal.
- ***Report*** is implementing on individual exercise book. Writings should be made on one side of double-page spread.
- On the cover there is need to mark title of the work, ***number of the variant***, surname and initials of student, code of group.
- Calculation part it is necessary to dispose in order of numeration of the problems.

- Calculation part of any problem must contain ten points (see examples):

**1.** ***Title*** of the problem;
**2.** ***Complete statement*** of a task;
**3.** ***Short writing*** of statement;
**4.** ***Transformation*** to the ***system of units*** **SI** of a numerical data of given quantities**.**
**5.** ***Scheme*** or explanatory plan;
**6.** ***List of laws*** and ***formulas*** which explain the physical phenomena of theme of the problem. All ***denotations*** at the formulas need an ***explanations;***
**7.** ***Literal solution***. From the listed at point **6** formulas it is necessary to make system of equations and to obtain expression of each desired quantity through given quantities at the literal (symbolic) representation.
**8.** ***Checkout of measurement unit of*** each desired quantity on correspondence to the expected measurement unit. For this purpose each symbol in the formula of a literal solution to substitute with its measurement unit and realize the necessary transformations.
**9.** Only after correspondence of measurement units to the expected it is possible to find the ***numerical solution***. Calculation should be transacted with three significant digits.
**10.** ***Result*** of execution of calculated part.

At the end of work it is necessary to enumerate the list of the used literature.

Besides this guide is recommended to use literature from bibliography given at the end of this guide.

## Problem 3.2.
## SIMPLE HARMONIC OSCILLATIONS

### MAIN CONCEPTS

Oscillations are the periodic changes of any physical quantities.

**Simple harmonic oscillations.** The equation which describes the ***eigenmode*** of oscillations has the simple ***harmonic*** form:

$$\xi(t) = A \cdot \cos(\omega_0 \cdot t + \varphi_0), \qquad (1)$$

where $\xi(t)$ – physical quantity, which makes the oscillations; $A$ – oscillation amplitude; $\varphi_0$ – initial phase (phase constant).

The time parameters of oscillations' eigenmode are called *eigen-parameters* (or *natural parameters*) and are written down with index «0»:

***Cyclic eigenfrequency*** of oscillations (changing of oscillations' phase per one second):

$$\omega_0 = 2\pi / T_0 = 2\,\pi \cdot \nu_0\,, \qquad (2)$$

$\nu_0$ – ***eigenfrequency*** of oscillations (number of oscillations per one second);

$T_0$ – ***eigenperiod*** of oscillations (minimal time interval of repeating of the value of oscillating quantity).

In equation of oscillations (1) it is described both mechanical, and electromagnetic oscillations, therefore it is possible to set up correspondence of mechanical and electrical oscillations' parameters:

| **Mechanic oscillations** | **Electromagnetic oscillations** |
|---|---|
| $\xi(t) = x(t)$<br>– displacement from the equilibrium position of material point of oscillating device; | $\xi(t) = q(t)$<br>– charge of oscillating circuit capacitor; |
| The velocity of material point of oscillating device:<br>$\upsilon(t) = \dfrac{dx}{dt}$ ; | Current flowing through the inductance coil of the oscillating circuit:<br>$i(t) = \dfrac{dq}{dt}$ ; |
| Restoring force, acting on material point of oscillating device:<br>$F(t) = ma(t) = m\dfrac{d\upsilon}{dt} = m\dfrac{d^2x}{dt^2}$,<br>$m$ – mass of oscillating device. | Back EMF (self-induction) in the inductance coil of the oscillating circuit:<br>$\varepsilon_{BACK}(t) = -L\dfrac{di}{dt} = -L\dfrac{d^2q}{dt^2}$,<br>$L$ – inductance of the inductance coil. |
| Cyclic eigenfrequency of spring pendulum oscillations:<br>$\omega_0 = \sqrt{\dfrac{k}{m}}$ , | Cyclic eigenfrequency of oscillations in the oscillating circuit:<br>$\omega_0 = \sqrt{\dfrac{1}{LC}}$ , |

| | |
|---|---|
| $k$ – spring constant (stiffness of spring). | $C$ – electrocapacity of the capacitor. |
| Potential energy of elastic deformation: $$W_P(t) = \frac{k \cdot x(t)^2}{2}.$$ | Energy of capacitor electric field: $$W_E(t) = \frac{q(t)^2}{2C}.$$ |
| Kinetic energy of oscillating device: $$W_K(t) = \frac{m \cdot \upsilon(t)^2}{2}.$$ | Energy of magnetic field of inductance coil: $$W_M(t) = \frac{L \cdot i(t)^2}{2}.$$ |
| Total energy of oscillations is independence of time: $$W = W_P(t) + W_K(t) = m\omega_0^2 \cdot \frac{A^2}{2} = \text{const},$$ $A$ – amplitude of material point strain of oscillating device. | Total energy of oscillations is independence of time: $$W = W_E(t) + W_M(t) = m\omega_0^2 \cdot \frac{Q^2}{2} = \text{const},$$ $Q$ – amplitude of capacitor charge of oscillating circuit. |

**Electromagnetic simple harmonic oscillations.**

There are three main parameters, which are changing during oscillations in oscillating LC-circuit: $q(t)$ – charge of capacitor, $u_C(t)$ – voltage on capacitor, $i(t)$ – current flawing through coil. They have identical eigenfrequency, but amplitudes and initial phases are different. Relation between these quantities represented in two definition:

$$\text{current:} \quad i = \frac{dq}{dt} \qquad \text{and} \qquad \text{electrocapacity} \quad C = \frac{q}{u}.$$

**Mathematic rules***:*

***Differentiation rule*** of harmonic function:

1. The multiplication constant (amplitude) is necessary to take out of the derivative sign.

2. Derivative of harmonic function has *phase lead* relative to own function on $\pi/2$:

$$[\cos(x)]'_x = -\sin(x) = \cos(x + \frac{\pi}{2}) \qquad \text{and} \qquad [\sin(x)]'_x = \cos(x) = \sin(x + \frac{\pi}{2}).$$

3. The result of derivation of harmonic function is necessary to multiply on derivative of a phase on time (on $\omega_0$) (see example).

***Integration rule*** of harmonic function:

1. The multiplication constant (amplitude) is necessary to take out of the integral sign.

2. Integral from harmonic function has *phase lag* from own function on $\pi/2$:

$$\int \cos(x)dx = \sin(x) = \cos(x - \frac{\pi}{2}) \quad \text{and} \quad \int \sin(x)dx = -\cos(x) = \sin(x - \frac{\pi}{2}).$$

3. The result of integration of harmonic function is necessary to divide on derivative of a phase on time (on $\omega_0$) (see example).

## EXAMPLE OF PROBLEM SOLUTION

**Example 1.** The oscillation circuit consists of coil by inductance of $L = 25$ $mH$ and capacitor. Current in circuit changes by the law $i(t) = I_m \cdot \cos\omega_0 t$, where $I_m = 20$ $mA$ and $\omega_0 = 10^4$ rad/$s$. 1) To get the equation of changing during the time charge of capacitor and voltage on the capacitor and on the coil. 2) To define total energy of oscillations in circuit.

*Input data:*

$I_m$=20 $mA$ = 0,02 $A$;

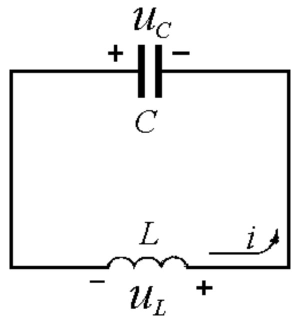
$\omega_0 = 10^4$ rad/$s$;

$L = 25$ $mH$ = 0, 025 $H$ ;

$I(t) = I_m \cdot \cos(\omega_0 t)$.

*Find:*

$q(t)$, $u_C(t)$, $\varepsilon_{BACK}(t)$, $W$ – ?




*Solution:*

1) From definition of the current

$$i = \frac{dq}{dt}$$

we find expression of charge from current (as the integration – is the mathematical function, inverse to differentiation):

$$q = \int i dt .$$

Let's substitute the input equation of current oscillations in this expression and integrate. We get the ***equation of oscillations of charge of capacitor*** :

$$q(t) = \int I_m \cos(\omega_0 t) dt = \frac{I_m}{\omega_0} \sin \omega_0 t = Q_m \cos(\omega_0 t - \frac{\pi}{2}),$$

$$\text{with amplitude } Q_m = \frac{I_m}{\omega_0}, \tag{1.1}$$

where $I_m$ – current oscillation amplitude; $\omega_0$ – cyclic eigenfrequency of oscillations.

In (1.1) we consider that integral from harmonic function has *phase lag* from own function on $\pi / 2$.

From definition of the electrocapacity

$$C = \frac{q}{u}$$

we find the expression of dependence of the voltage on capacitor from charge of its plate:

$$u_C = \frac{q}{C} .$$

Let’s substitute the equation (1.1) in this expression we get the ***equation of oscillations of voltage on capacitor***:

$$u_C(t) = \frac{Q_m}{C}\cos(\omega_0 t - \frac{\pi}{2}) = \frac{I_m}{\omega_0 C}\cos(\omega_0 t - \frac{\pi}{2}) = U_{mC}\cdot\cos(\omega_0 t - \frac{\pi}{2}),$$

$$\text{with amplitude } U_{mC} = \frac{I_m}{\omega_0 C}. \tag{1.2}$$

Electrocapacity can be found with the formula of oscillations’ cyclic eigenfrequency in the oscillating circuit:

$$\omega_0 = \sqrt{\frac{1}{L\cdot C}}, \text{ whence}$$

$$C = \frac{1}{\omega_0^2 L}, \tag{1.3}$$

where $L$ – circuit inductance. Substituted in the formula (1.2) the perceived equation for $C$ (1.3) finally we obtain:

$$u_C(t) = \omega_0 L I_m \cdot \cos(\omega_0 t - \frac{\pi}{2}). \tag{1.4}$$

From the 2nd Kirchoff’s rule voltage on the inductance coil equal to Back EMF:

$$u_L(t) = -\varepsilon_{BACK}(t) = L\frac{di}{dt}.$$

Let’s substitute the input equation of current oscillations in this expression and differentiate. We get the ***equation of oscillations of voltage on coil*** :

$$u_L(t) = L\frac{d}{dt}I_m\cos(\omega_0 t) = \omega_0 L\cdot I_m\cdot\cos(\omega_0 t + \frac{\pi}{2}) = U_{mL}\cdot\cos(\omega_0 t + \frac{\pi}{2}),$$

$$\text{with amplitude } U_{mL} = \omega_0 L\cdot I_m. \tag{1.5}$$

In (1.5) we consider that derivative of harmonic function has *phase lead* relative to own function on $\pi/2$.

2. Total energy of oscillations in circuit equals the sum of energy of electric field in capacitor $W_E$ and energy of magnetic field in coil $W_M$:

$$W = W_E + W_M;$$

$$W(t) = \frac{C\cdot u_C(t)^2}{2} + \frac{L\cdot i(t)^2}{2} = \frac{CU_{mC}{}^2}{2}\cos^2(\omega_0 t - \frac{\pi}{2}) + \frac{LI_m^2}{2}\cos^2(\omega_0 t).$$

Substituting the expression for $C$ from the formula (1.3), amplitude of oscillations of voltage on capacitor (1.2) and input equation of current oscillations, we get

$$W = \frac{LI_m^2}{2}\cos^2(\omega_0 t - \frac{\pi}{2}) + \frac{LI_m^2}{2}\cos^2(\omega_0 t) = \frac{LI_m^2}{2}(\sin^2\omega_0 t + \cos^2\omega_0 t) = \frac{LI_m^2}{2}. \tag{1.6}$$

Let's note, a total energy of oscillations has no time dependence, because absence of power loss.

Let’s check, whether the right part of equation of amplitude (1.1) gives the unit of charge [$C$], of equation of amplitude (1.4) the unit of voltage [$V$] and the formulas of amplitude (5) the unit of energy [$J$].

$$[q]=\frac{[I_m]}{[\omega_0]}=\frac{A}{s^{-1}}=A\cdot s=C\,;$$

$$[u_C]=[\omega_0]\cdot[L]\cdot[I_m]=c^{-1}\cdot H\cdot A=\frac{Wb\cdot A}{s}=\frac{T\cdot m^2}{s}=\frac{N\cdot m}{A\cdot m\cdot s}=\frac{J}{C}=V\,;$$

$$[W]=[L]\cdot[I_m^2]=H\cdot A^2=\frac{Wb\cdot A^2}{A}=T\cdot m^2\cdot A=\frac{N\cdot m^2\cdot A}{A\cdot m}=N\cdot m=J\,.$$

Substituted numerical values, let's write down the equation of changing $q$ and $u_C$ with numerical coefficients and calculate the full energy of oscillations in circuit

$$q(t)=\frac{0{,}02}{10^4}\sin 10^4 t=2\cdot10^{-6}\cdot\cos(10^4 t+\frac{\pi}{2})\,C\,;$$

$$u_C(t)=10^4\cdot2{,}5\cdot10^{-2}\cdot0{,}02\cos(10^4 t-\frac{\pi}{2})=5\cos(10^4 t-\frac{\pi}{2})\,V\,;$$

$$u_L(t)=10^4\cdot2{,}5\cdot10^{-2}\cdot0{,}02\cos(10^4 t+\frac{\pi}{2})=5\cos(10^4 t+\frac{\pi}{2})\,V\,;$$

$$W=\frac{2{,}5\cdot10^{-2}\cdot0{,}02^2}{2}=5\cdot10^{-6}\,J\,.$$

***Results***: $q(t)=2\cdot10^{-6}\sin(10^4 t)\,C$, $\quad u_C(t)=5\cos(10^4 t-\frac{\pi}{2})\,V$,

$u_L(t)=5\cos(10^4 t+\frac{\pi}{2})\,V$, $\quad W=5\cdot10^{-6}\,J$.

## INDIVIDUAL TASKS FOR PROBLEM 3.2.
## ELECTROMAGNETIC SIMPLE HARMONIC OSCILLATIONS

*In accordance with your variant to solve one of the following problems listed below (The number of problem statement and all necessary input data are reduced in the table 3.2).*

**1** In the oscillating circuit, which consists of the capacitor with electocapacity of $C$ and coil with inductance of $L$, voltage on capacitor changes by dependence: $u_c(t) = U_{mc}\cdot\cos(\omega_0 t)$ . Write down the equation of changing $u_c(t)$ with numerical coefficients and get the equations of time changing of the charge on the plates of capacitor, current in circuit and energy of magnetic field.

**2** In the oscillating circuit with inductance of $L$ and electocapacity of $C$ current changes by law: $i(t)= I_m\cdot\cos(\omega_0 t)$. Write down the equation of changing current with numerical coefficients and get the equation of changing by time the voltage on capacitor, energy of electric field and energy of magnetic field.

**3** The oscillating circuit consists of coil with inductance of $L$ and capacitor with electocapacity of $C$. Charge on plates of capacitor changes by law $q(t) = Q_m \cdot \cos(\omega_0 t)$. Write down the equation of changing of charge with numerical coefficients and get the equations of changing by time voltage on capacitor, current in circuit and energy of magnetic field

## TABLE OF TASK VARIANTS

**Table 3.2**

| Variant | Statement | $C$, $\mu F$ | $L$, $mH$ | $\omega_0$, $rad/s$ | $Q_m$, $\mu C$ | $I_m$, $mA$ | $U_{mc}$, $mV$ |
|---|---|---|---|---|---|---|---|
| **1** | 1 | 1 | 10 | – | – | – | 200 |
| **2** | 3 | – | 10 | $10^3$ | 20 | – | – |
| **3** | 1 | 50 | – | $2 \cdot 10^3$ | – | – | 120 |
| **4** | 2 | 0,2 | 0,5 | – | – | 2 | – |
| **5** | 3 | – | 2 | $10^4$ | 1 | – | – |
| **6** | 1 | 0,1 | 1 | – | – | – | 300 |
| **7** | 3 | 2 | – | $10^5$ | 0,4 | – | – |
| **8** | 2 | – | 0,01 | $10^5$ | – | 50 | – |
| **9** | 3 | 4 | – | $5 \cdot 10^4$ | 0,6 | – | – |
| **10** | 1 | – | 0,2 | $10^5$ | – | – | 100 |
| **11** | 3 | 10 | 25 | – | 2 | – | – |
| **12** | 1 | 4 | – | $5 \cdot 10^3$ | – | – | 250 |
| **13** | 2 | – | 10 | $2 \cdot 10^3$ | – | 6 | – |
| **14** | 3 | 100 | 0,1 | – | 8 | – | – |
| **15** | 1 | 5 | 0,5 | – | – | – | 300 |
| **16** | 2 | – | 0,05 | $2 \cdot 10^5$ | – | 7 | – |
| **17** | 3 | 10 | – | $5 \cdot 10^3$ | 3 | – | – |
| **18** | 1 | – | 2 | $5 \cdot 10^4$ | – | – | 200 |
| **19** | 2 | 20 | 2 | – | – | 9 | – |
| **20** | 1 | 2 | – | $5 \cdot 10^3$ | – | – | 500 |
| **21** | 2 | 1 | 0,1 | – | – | 10 | – |
| **22** | 3 | – | 50 | $2 \cdot 10^3$ | 0,5 | – | – |
| **23** | 1 | – | 5 | $2 \cdot 10^4$ | – | – | 400 |
| **24** | 3 | 10 | 1 | – | 6 | – | – |
| **25** | 2 | 0,1 | – | $10^6$ | – | 8 | – |
| **26** | 1 | – | 1 | $5 \cdot 10^3$ | – | – | 150 |
| **27** | 2 | 1 | 2,5 | – | – | 4 | – |
| **28** | 3 | 2,5 | – | $4 \cdot 10^4$ | 1 | – | – |
| **29** | 1 | 4 | 0,4 | – | – | – | 80 |
| **30** | 2 | – | 8 | $5 \cdot 10^3$ | – | 5 | – |

## Problem 3.3.
## DAMPED HARMONIC OSCILLATIONS

### MAIN CONCEPTS

**Damped harmonic oscillation.**

The system looses energy by a drag force

$$F_D = -r \cdot \upsilon,$$

or a voltage drope on the active resistor

$$u_R = R \cdot i,$$

therefore the amplitude has exponential decay on time

$$A(t) = A_0 \cdot e^{-\beta \cdot t}.$$

The equation of damped oscillations has a mode

$$\xi(t) = A_0 \cdot e^{-\beta \cdot t} \cdot \cos(\omega \cdot t + \varphi_0), \quad (3)$$

where $\xi(t)$ – physical quantity, which oscillates; $A_0$ – initial amplitude of oscillations; $\beta$ – damping coefficient; $\varphi_0$ – initial phase (phase constant).

Cyclic frequency of damped oscillations (is written with no index) less then eigenfrequency:

$$\omega = \sqrt{\omega_0^2 - \beta^2}. \quad (4)$$

**Parameters of linear damping oscillations:**

1) Relaxation of vibrations – lessening of amplitude in $e$ = 2,71 times. The ***time of relaxation***:

$$\tau = 1 / \beta. \quad (5)$$

2) As far as amplitude of damped oscillations uninterruptedly decreases $A(t)=A_0 \cdot e^{-\beta \cdot t}$ , then the value of oscillating quantity $\xi(t)$ will never repeat. That's why, the quantity

$$T_{CONV} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0{}^2 - \beta^2}} \quad (6)$$

is called ***conventional period*** – minimal time, during which the value of oscillating quantity $\xi(t)$ will be equal peak magnitude (amplitude).

3) ***Decay decrement*** is a relation of two neibouring amplitude:

$$D = A_t / A_{t+T} = e^{-\beta T}. \quad (7)$$

4) ***Logarithmic decay decrement*** (damping constant):

$$\delta = \ln D = \beta T_{CONV}, \quad (8)$$

5) ***Quality factor*** of system

$$Q = \omega_0 / 2\beta. \quad (9)$$

In equation of oscillations (2) it is described both mechanical, and electromagnetic oscillations, therefore it is possible to set up correspondence of mechanical and electrical oscillations' parameters:

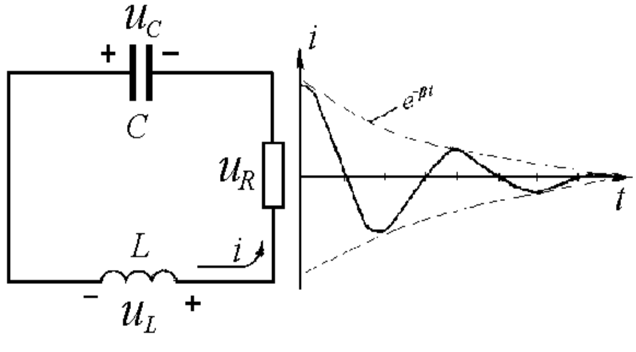
| **Mechanic oscillations** | **Electromagnetic oscillations** |
|---|---|
| $\xi(t) = x(t)$<br>– displacement from the equilibrium position of material point of oscillating device; | $\xi(t) = q(t)$<br>– charge of oscillating circuit capacitor; |
| ***Parameters of a system:*** | |
| $k$ – spring constant (stiffness of spring).<br>$m$ – mass of oscillating device.<br>$r$ – drag coefficient. | $C$ – electrocapacity of the capacitor.<br>$L$ – inductance of the inductance coil.<br>$R$ – resistance of circuit. |
| ***Damping coefficient:*** | |
| $\beta = \dfrac{r}{2m}$; | $\beta = \dfrac{R}{2L}$; |
| ***Cyclic frequency of damped oscillations:*** | |
| $\omega = \sqrt{\dfrac{k}{m} - \left(\dfrac{r}{2m}\right)^2}$; | $\omega = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2}$; |
| ***Quality factor of system:*** | |
| $Q = \dfrac{1}{r}\sqrt{mk}$. | $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$. |

## EXAMPLE OF PROBLEM SOLUTION

**Example 2.** The oscillating *RLC*-circuit consists of capacitor, and coil of inductance of 2 *mH* and resistor. At the initial moment of time charge on the capacitor plates is maximal and equals $q_0=Q_0=2\mu C$. Conventional period of oscillations 1 *ms*, logarithmic decay decrement is 0,8.
1) To write down the equation of oscillations of charge with numerical coefficients.
2) To define the capacity of capacitor and the resistance of resistor.

*Input data:*
$L = 2\ mH = 0{,}002\ H$;
$Q_0 = 2\ \mu C = 2\cdot10^{-6}\ C$;
$T_{CONV} = 1 s = 10^{-3}\ s$;
$\delta = 0{,}8$.

*Find:* $q(t)$, $C$, $R$ – ?



*Solution:*

1) Oscillations in circuit will be damping. Let's write the equation of damped oscillations of charge in a general view:

$$q(t) = Q_0 e^{-\beta \cdot t} \cos(\omega t + \varphi_0), \tag{2.1}$$

where $Q_0$ – the initial amplitude of charge, $\beta$ – damping coefficient; $\omega$ – cyclic frequency of damped oscillations; $\varphi_0$ – initial phase.

From a definition of the conventional period $T_{CONV}=2\pi/\omega$ we express cyclic frequency of damped oscillations:

$$\omega = 2\pi / T_{CONV}. \tag{2.2}$$

From a definition of initial value of oscillating quantity $q_0=Q\cdot\cos(\varphi_0)$ and considering that the oscillations beginning from the position of maximal charge on the capacitor $Q$, we find the initial phase of oscillations:

$$\varphi_0 = arc\cos(q_0/Q) = arc\cos(1) = 0. \tag{2.3}$$

From a definition of the logarithmic decay decrement is $\delta=\beta T_{CONV}$, whence the coefficient of damping

$$\beta=\delta / T_{CONV}. \tag{2.4}$$

Let's check, whether the right part of the formula (2.2) gives the unit of cyclic frequency [*rad/s*], and the left part of the formula (2.4) – measurement unit of damping coefficient [1/*s*]:

$[\omega] = rad / s$;

$[\beta] = 1 / s$.

Let's substitute the numerical values in the formulas (2.2) and (2.4)

$$\omega = \frac{2\pi}{10^{-3}} = 2\pi \cdot 10^3 \ s^{-1} = 6{,}28 \cdot 10^3 \, rad / s;$$

$$\beta = \frac{0{,}8}{10^{-3}} = 800 \, s^{-1}.$$

Let' write down the equation of oscillation of charge with numerical coefficients

$$q = 2 \cdot 10^{-6} \cdot e^{-800 \cdot t} \cdot \cos(2\pi \cdot 10^3 t)\, C. \tag{2.5}$$

2) Let's substitute the equation of eigenfrequency $\omega_0 = 1/\sqrt{LC}$ into the definition of cyclic frequency of damped oscillations $\omega = \sqrt{\omega_0^2 - \beta^2}$:

$$\omega = \sqrt{\omega_0^2 - \beta^2} = \sqrt{\frac{1}{LC} - \beta^2} \quad \Rightarrow \quad \omega^2 = \frac{1}{LC} - \beta^2 \tag{2.6}$$

and find expression of capacity of capacitor:

$$C = \frac{1}{L(\omega^2 + \beta^2)}. \tag{2.7}$$

From a definition of the damping coefficient $\beta=R/2L$ we obtain resistance of resistor $R$:

$$R = 2\beta L. \tag{2.8}$$

Let's check, whether the right part of the formula (2.7) gives the unit of electrocapacity [$F$], and the left part of formula (2.8) – the unit of resistance [Ω]:

$$[C]=\frac{1}{H\cdot s^{-2}}=\left[\frac{L=\Phi/I}{H=Wb/A}\right]=\frac{s^2\cdot A}{Wb}=\left[\frac{\varepsilon=-d\Phi/dt}{V=Wb/s}\right]=\frac{s^2\cdot A}{V\cdot s}=\left[\frac{I=dQ/dt}{A=C/s}\right]=$$

$$=\frac{C}{V}=\left[\frac{C=Q/U}{F=C/V}\right]=F;$$

$$[R]=\frac{H}{s}=\left[\frac{L=\Phi/I}{H=Wb/A}\right]=\frac{Wb}{A\cdot s}=\left[\frac{\varepsilon=-d\Phi/dt}{V=Wb/s}\right]=\frac{V\cdot s}{A\cdot s}=\left[\frac{I=U/R}{A=V/\Omega}\right]=\Omega.$$

Let's make the calculations:

$$C=\frac{1}{2\cdot10^{-3}(4\pi^2\cdot10^6+800^2)}=1{,}25\cdot10^{-5}\ F;\qquad R=2\cdot300\cdot2\cdot10^{-3}=3{,}2\ \Omega.$$

***Results***: 1) $q(t)=2\cdot10^{-6}e^{-800\cdot t}\cos(2\pi\cdot10^3t)\,C$,
2) $C$=1,25·$10^{-5}$ $F$, $R$ = 3,2 Ω.

## INDIVIDUAL TASKS FOR PROBLEM 3.3. DAMPED HARMONIC OSCILLATIONS.

*In accordance with your variant to solve one of the following problems listed below (The number of problem statement and all necessary input data are reduced in the table 3.3).*

**1** Load of mass of $m$, suspended on the spring with stiffness of $k$, oscillate in viscous medium with drag coefficient of $r$. The equation of oscillations of load has view $x(t)=A_0\cdot e^{-\beta t}\cdot\cos\omega t$. Logarithmic decay decrement of oscillations is δ.

a) By the values of quantities, given in the table 3.3, find necessary parameters and write down the equation of vibrations with numerical coefficients.

b) Find the system quality factor.

c) Find the quantity, which is indicated in the last column of table.

**2** The oscillation circuit consists of capacitor with capacity of $C$, coil of inductance of $L$ and resistor of resistance of $R$. Current in circuit changes by law $i(t)=I_0\cdot e^{-\beta t}\cdot\sin\omega t$. Logarithmic decay decrement of oscillations is δ.

a) By the values of quantities, given in the table 3.3, find necessary parameters and write down the equation of current's oscillations with numerical coefficients.

b) Find the system quality factor.

c) Find the quantity, which is indicated in the last column of table.

## TABLE OF TASK VARIANTS

**Table 3.3**

| Variant | Statement | $k$, $N/m$ | $r$, $kg/s$ | $m$, $g$ | $A$, $cm$ | $C$, $\mu F$ | $L$, $mH$ | $R$, $\Omega$ | $I_0$, $mA$ | $\beta$, $s^{-1}$ | $\omega$, $rad/s$ | $\delta$ | Find |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 2 | – | – | – | – | 5 | 2 | 12 | 1 | – | – | – | $\boldsymbol{\delta}$ |
| **2** | 2 | – | – | – | – | 10 | – | – | 5 | 300 | – | 0,9 | ***L*** |
| **3** | 2 | – | – | – | – | – | 10 | 6 | 5 | – | – | 1,2 | ***C*** |
| **4** | 2 | – | – | – | – | – | – | 20 | 3 | – | $10^4$ | 2 | ***L*** |
| **5** | 1 | – | 0,5 | 100 | 4 | – | – | – | – | – | – | 2 | ***k*** |
| **6** | 2 | – | – | – | – | – | – | 40 | 6 | 600 | – | 1,7 | ***C*** |
| **7** | 1 | – | – | 250 | 3 | – | – | – | – | 2 | – | 1,1 | ***r*** |
| **8** | 2 | – | – | – | – | – | – | 50 | 1 | 800 | – | 1,8 | ***L*** |
| **9** | 2 | – | – | – | – | – | 8 | – | 8 | 400 | – | 1,5 | ***C*** |
| **10** | 1 | – | – | 150 | 2 | – | – | – | – | 5 | – | 1,6 | ***k*** |
| **11** | 2 | – | – | – | – | 0,5 | 5 | 80 | 2 | – | – | – | $\boldsymbol{\delta}$ |
| **12** | 2 | – | – | – | – | – | 15 | – | 5 | 700 | – | 2 | ***R*** |
| **13** | 1 | 70 | 1,6 | 200 | 1 | – | – | – | – | – | – | – | $\boldsymbol{\delta}$ |
| **14** | 2 | – | – | – | – | – | 5 | – | 7 | 4000 | – | 1,9 | ***C*** |
| **15** | 1 | 40 | – | – | 2 | – | – | – | – | 8 | – | 1,8 | ***r*** |
| **16** | 2 | – | – | – | – | – | 5 | 16 | 4 | – | $5 \cdot 10^3$ | – | $\boldsymbol{\delta}$ |
| **17** | 1 | – | 1,2 | 80 | 1 | – | – | – | – | – | – | 1,5 | ***k*** |
| **18** | 2 | – | – | – | – | 2 | – | – | 2 | 900 | – | 1,6 | ***R*** |
| **19** | 1 | – | – | 40 | 1 | – | – | – | – | 6 | – | 1,2 | ***k*** |
| **20** | 2 | – | – | – | – | – | – | 10 | 4 | – | $2 \cdot 10^4$ | 1,5 | ***L*** |
| **21** | 2 | – | – | – | – | – | 1 | 8 | 6 | – | – | 1,4 | ***C*** |
| **22** | 1 | 50 | – | – | 3 | – | – | – | – | 4 | – | 1,4 | ***r*** |
| **23** | 2 | – | – | – | – | 4 | – | – | 3 | 2000 | – | 1,3 | ***R*** |
| **24** | 1 | 50 | 0,8 | 50 | 2 | – | – | – | – | – | – | – | $\boldsymbol{\delta}$ |
| **25** | 2 | – | – | – | – | 0,2 | – | – | 4 | 5000 | – | 1,1 | ***L*** |
| **26** | 1 | – | – | 20 | 1 | – | – | – | – | – | 30 | 1,3 | ***k*** |
| **27** | 2 | – | – | – | – | – | – | 74 | 9 | 800 | – | 1,1 | ***C*** |
| **28** | 1 | – | 1,1 | – | 5 | – | – | – | – | – | 25 | 1,4 | ***k*** |
| **29** | 2 | – | – | – | – | 4 | 4 | 16 | 2 | – | – | – | $\boldsymbol{\delta}$ |
| **30** | 1 | – | – | 120 | 2 | – | – | – | – | 3 | – | 1,2 | ***r*** |

## Problem 3.4.
## DRIVEN HARMONIC OSCILLATIONS

### MAIN CONCEPTS

**Series oscillatory circuit.**

***Ohm's law for the alternate current*** is represented the formula (11) together with expressions (10) and (12) – (17):

When the external electromotive force of AC-generator (generator voltage) has a simple harmonic view:

$$\varepsilon_{ext}(t) = \varepsilon_m \cdot \cos(\Omega \cdot t + \varphi_{0\varepsilon}) , \qquad (10)$$

where $\varepsilon_m = \varepsilon_{RMS}\sqrt{2}$ – generator ***peak*** voltage ($\varepsilon_{RMS}$ – its ***root mean square*** value voltmeter of which is indicated); $\Omega = 2\pi f$ – cyclic frequency of generator voltage ($f$ – frequency of generator); $\varphi_{0\varepsilon}$ – initial phase of generator voltage,

then steady state oscillations of public ***current*** in a circuit will be described by an equation:

$$i(t) = I_m \cdot \cos(\Omega t + \varphi_{0I});$$
$$I_m = \varepsilon_m / Z \text{ – peak current,} \qquad (11)$$
$$\varphi_{0I} = \varphi_{0\varepsilon} + \Delta\Phi \text{ – initial phase of current.}$$

***Impedance of series*** oscillatory circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2} , \qquad (12)$$

where $R$ – resistance of cirquit;

$$X_C = 1/\Omega C \text{ – capacitive reactance;}$$
$$X_L = \Omega L \text{ – inductive reactance;} \qquad (13)$$

***Phase difference*** ($\Delta\Phi = \varphi_{0I} - \varphi_{0\varepsilon}$) between the current and generator voltage

$$\Delta\Phi = arc\text{tg}[(X_C - X_L) / R] , \qquad (14)$$

CONSIDERATION: if to take into account that the initial phase of current $\varphi_{0I} = \varphi_{0\varepsilon} - \Delta\Phi$, then $\Delta\Phi = arc\text{tg}[(X_L - X_C) / R]$.

The public current in a circuit $i(t)$ determines individual voltages on circuit devices:

***Voltage on resistor***

$$u_R(t) = i(t) \cdot R = U_{mR} \cdot \cos(\Omega t + \varphi_{0R});$$
$$U_{mR} = I_m R \text{ – peak voltage,} \qquad (15)$$
$$\varphi_{0R} = \varphi_{0I} = \varphi_{0\varepsilon} + \Delta\Phi \text{ – initial phase.}$$

***Voltage on inductor***

$$u_L(t) = -\varepsilon_{BACK} = L \cdot \text{d}i(t) / \text{d}t = U_{mL} \cos(\Omega t + \varphi_{0L});$$
$$U_{mL} = I_m X_L \text{ – peak voltage,} \qquad (16)$$
$$\varphi_{0L} = \varphi_{0I} + \pi/2 = \varphi_{0\varepsilon} + \Delta\Phi + \pi/2 \text{ – initial phase.}$$

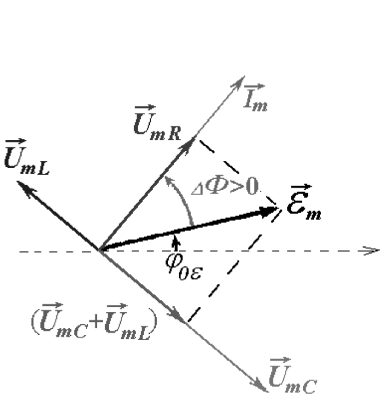
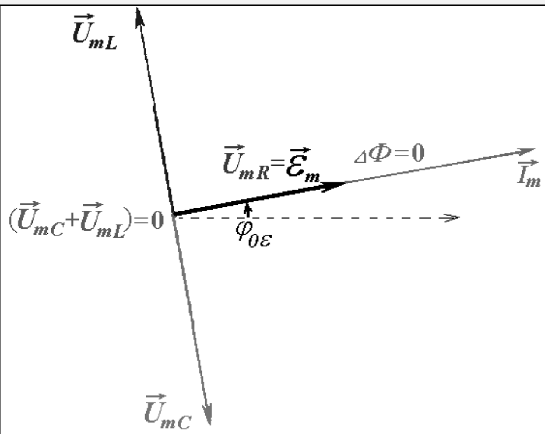
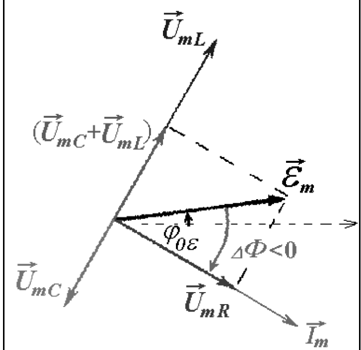
***Voltage on capacitor***

$$u_C(t)=\frac{q}{C} = \frac{1}{C}\int i(t)dt = U_{mC}\cdot\cos(\Omega t+\varphi_{0C}),$$

$$U_{mC}= I_m X_C \text{ – peak voltage,} \qquad (17)$$

$$\varphi_{0C} = \varphi_{0I} - \pi/2 = \varphi_{0\varepsilon}+\Delta\Phi-\pi/2 \text{ – initial phase.}$$

The peak current as well as peak voltages on circuit devices are strongly depends on generator frequency. Magnitude of a current and distribution between the voltages in series RLC-circuit at three various regions of frequency spectrum are shown below.

| **Low frequency** | **Resonance $\Omega_R$** | **High frequency** |
|---|---|---|
| ***Reactance***: | | |
| $X_C > X_L$. Considerably predominate capacitive reactance. | $X_C = X_L$; Lowest resistance: $Z=R$ | $X_C < X_L$. Considerably predominate inductive reactance. |
| ***Reactive voltage***: | | |
| $U_{mL} < U_{mC} = X_C\cdot\varepsilon_m / Z$. Predominate capacitive voltage. | $U_{mC} = U_{mL} = Q\cdot\varepsilon_m$; | $U_{mC} < U_{mL} = X_L\cdot\varepsilon_m / Z$. Predominate inductive voltage. |
| ***Current***: | | |
| $I_m \approx \varepsilon_m / \sqrt{R^2 + X_C^2}$. Low current. | $I_m = \varepsilon_m / R$; Heavy current. | $I_m \approx \varepsilon_m / \sqrt{R^2 + X_L^2}$. Low current. |
| ***Phase difference*** ($\Delta\Phi=\varphi_{0I} - \varphi_{0\varepsilon}$) between the current and generator voltage: | | |
| $\Delta\Phi > 0$; | $\Delta\Phi = 0$; | $\Delta\Phi < 0$. |
| | | |
| Current and voltage phasors at $t$=0 are represented on vector voltage diagram: | | |
|  |  |  |

## EXAMPLE OF PROBLEM SOLUTION

**Example 3.** Simple harmonic external EMF with the frequency $10^3$ *Hz* applied to the series oscillatory circuit of *RLC*-filter. The elements of filter have a nominal value: resistance 100 Ω, inductance 40 *mH* and capacity 1 μ*F*. The voltage on capacitor varies with time by the law: $u_C(t) = U_{mC}\cdot \cos(\Omega t)$, with the reading of an voltmeter $U_{\mathrm{RMS}\,C} = 20\ V$.

1) To rebuild the equation of changing of current in circuit, of voltage on resistor, voltage on capacitor, of voltage on inductor and EMF, applied to circuit with numerical coefficients.

2) To build the vector voltage diagram at $t=0$.

3) To find the values of external EMF – ε, voltages – $u_R$, $u_C$, $u_L$ at the moment of time of $t_1 = T/8$ ($T$ – period of oscillations). To build the voltage diagram at $t_1 = T/8$.

*Input data:*

$R = 100\ \Omega$;

$L = 40\ mH = 0{,}04\ H$;

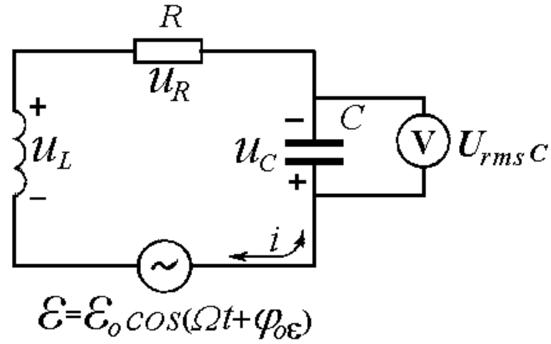
$C = 1\ \mu F = 10^{-6}\ F$;

$u_C(t) = U_{mc}\cos(\Omega t)$;

$U_{\mathrm{RMS}\,C} = 20\ V$;

$f = 10^3\ Hz$;

$t_1 = T/8$.




*Find:*

1) $i(t)$, $u_R(t)$, $u_L(t)$, $u_C(t)$, $\varepsilon(t)$ – ?

2) Phasors at $t=0$ – ?

3) phasors at $t_1$, $u_R(t_1)$, $u_L(t_1)$, $u_C(t_1)$, $\varepsilon(t_1)$ – ?

*Solution:*

1) Let's evaluate the peak voltage on capacitor $U_{mC}$ and cyclic frequency Ω of oscillations in circuit:

$U_{mC} = U_{\mathrm{rms}\,C}\cdot\sqrt{2}$; $[U_{mC}] = V$; $U_{mC} = 20\cdot\sqrt{2} = 28{,}3\ V$;

$\Omega = 2\pi f$; $[\Omega] = rad\cdot Hz = \mathrm{rad}/s$; $\Omega = 6{,}28\cdot 10^3\ \mathrm{rad}/s$.

Here $U_{\mathrm{rms}\,C}$ – root mean square value of voltage on capacitor voltmeter of which is indicated; $f$ –frequency of external EMF of generator.

From the view of the given equation $u_C(t) = U_{mc}\cos(\Omega t)$ we can conclude then $\varphi_{0C} = 0$.

Finally we obtain the equation of oscillations of voltage on capacitor with numerical coefficients:

$\underline{u_C(t) = 28{,}3\cos(6{,}28\cdot 10^3 t)\ V}$.

From **_Ohm's law for the AC_** voltage on capacitor with the peak voltage and initial phase there is a view:

$$u_C(t) = U_{mC}\cdot\cos(\Omega t + \varphi_{0C});$$
$$U_{mC} = I_m X_C; \qquad (3.1)$$
$$\varphi_{0C} = \varphi_{0I} - \pi/2.$$

where capacitive reactance is defined by the equation:

$$X_C = 1 / \Omega C\,; \qquad (3.2)$$

From this we obtain: $I_m = U_{mC} / X_C$; $\varphi_{0I} = \varphi_{0C} + \pi/2$.

Let's check dimensionality and make the calculations:

$[X_C] = 1/(F \cdot \text{rad}/s) = \Omega$; $X_C = 1/(6{,}28 \cdot 10^3 \cdot 10^{-6}) = 159\Omega$;

$[I_m] = V/\Omega = A$; $I_m = 28{,}3 / 159 = 0{,}178\ A$;

$[\varphi_{0I}] = \text{rad}$; $\varphi_{0I} = 0 + \pi/2 = \pi/2$.

From ***Ohm's law for the AC*** public current in circuit with the peak value and initial phase there is a view:

$$i(t) = I_m \cdot \cos(\Omega t + \varphi_{0I});$$
$$I_m = \mathcal{E}_m / Z\,; \qquad (3.3)$$
$$\varphi_{0I} = \varphi_{0\varepsilon} + \Delta\Phi,$$

where impedance of *series* oscillatory circuit and phase difference between the current and generator voltage are defined by the equations:

$$Z = \sqrt{R^2 + (X_C - X_L)^2}\,; \qquad (3.4)$$

$$\Delta\Phi = \text{arctg}[(X_C - X_L)/R]. \qquad (3.5)$$

Let's substitute numerical values in (3.3) and obtain the equation of current oscillations with numerical coefficients:

$\underline{i(t) = 0{,}178 \cdot \cos(6{,}28 \cdot 10^3 t + \pi/2)\ A.}$

Thus current in circuit has *phase lead* relative to the voltage on capacitor on $\pi/2$.

From ***Ohm's law for the AC*** voltage on resistor with the peak voltage and initial phase there is a view:

$$u_R(t) = U_{mR} \cdot \cos(\Omega t + \varphi_{0R});$$
$$U_{mR} = I_m R\,; \qquad (3.6)$$
$$\varphi_{0R} = \varphi_{0I} = \varphi_{0\varepsilon} + \Delta\Phi.$$

Let's check dimensionality and make the calculations:

$[U_{mR}] = A \cdot \Omega = V$; $U_{mR} = 0{,}178 \cdot 100 = 17{,}8\ V$;

$[\varphi_{0R}] = \text{rad}$; $\varphi_{0R} = \pi/2$.

Let's substitute numerical values in (3.6) and obtain the equation of oscillations of voltage on resistor with numerical coefficients:

$\underline{u_R(t) = 17{,}8 \cdot \cos(6{,}28 \cdot 10^3 t + \pi/2)\ V.}$

Thus voltage on resistor has *phase lead* relative to the voltage on capacitor on $\pi/2$.

From ***Ohm's law for the AC*** voltage on inductor with the peak voltage and initial phase there is a view:

$$u_L(t) = U_{mL} \cos(\Omega t + \varphi_{0L});$$
$$U_{mL} = I_m X_L\,; \qquad (3.7)$$
$$\varphi_{0L} = \varphi_{0I} + \pi/2.$$

where inductive reactance is defined by the equation:

$$X_L = \Omega L. \qquad (3.8)$$

Let's check dimensionality and make the calculations:

$[X_L] = \text{rad}/s \cdot H = \Omega$; $X_L = 6{,}28 \cdot 10^3 \cdot 0{,}04 = 251\ \Omega$;

$[U_{mL}] = A \cdot \Omega = V$; $U_{mL} = 0{,}178 \cdot 251 = 44{,}7$ $V$;
$[\varphi_{0I}]$ = rad; $\varphi_{0I} = \pi/2 + \pi/2 = \pi$.

Let's substitute numerical values in (3.7) and obtain the equation of oscillations of voltage on inductor with numerical coefficients:

$\underline{u_L(t) = 44{,}7\cos(6{,}28 \cdot 10^3 t + \pi)\ V.}$

Thus voltage on resistor has *phase lead* relative to the voltage on capacitor on $\pi$.

From ***Ohm's law for the AC*** external EMF of a generator there is a view:

$$\varepsilon_{\text{ext}}(t) = \varepsilon_m \cdot \cos(\Omega \cdot t + \varphi_{0\varepsilon}), \tag{3.9}$$

where the peak external voltage and the initial phase we find from equations (3.3):

$$\varepsilon_m = I_m \cdot Z; \tag{3.10}$$

$$\varphi_{0\varepsilon} = \varphi_{0I} - \Delta\Phi; \tag{3.11}$$

Let's check dimensionality and make the calculations:

$[Z] = \sqrt{\Omega^2 + (\Omega)^2} = \Omega$; $Z = \sqrt{100^2 + (159 - 251)^2} = 136\,\Omega$;

$[\varepsilon_m] = A \cdot \Omega = V$; $\varepsilon_m = 0{,}178 \cdot 136 = 24{,}1$ $V$;

$[\Delta\Phi]$ = rad; $\Delta\Phi = arc\text{tg}[(159 - 251)/100] = arc\text{tg}[-0{,}93] = -0{,}75\text{rad} = -0{,}24\pi \approx -\pi/4$;

$[\varphi_{0\varepsilon}]$ = rad; $\varphi_{0\varepsilon} = \pi/2 + \pi/4 = 3\pi/4$.

Let's substitute numerical values in (9) and obtain the equation of oscillations of external voltage with numerical coefficients:

$\underline{\varepsilon_{\text{ext}}(t) = 24{,}1\cos(6{,}28 \cdot 10^3 t + 3\pi/4)\ V.}$

Thus external voltage has *phase lead* relative to the voltage on capacitor on $3\pi/4$.

2) For building the vector voltage diagram at $t$=0 let's write the equation of voltages in this moment of time without calculating the value of cosine:

$u_C(t = 0) = 28{,}3\cos(0)$ $V$;
$u_R(t = 0) = 17{,}8 \cdot \cos(\pi/2)$ $V$;
$u_L(t = 0) = 44{,}7\cos(\pi)$ $V$;
$\varepsilon_{\text{ext}}(t = 0) = 24{,}1\cos(3\pi/4)$ $V$;
$i(t = 0) = 0{,}178 \cdot \cos(\pi/2)$ $A$.
$\Delta\Phi = -\pi/4$;
$\varphi_{0\varepsilon} = 3\pi/4$.

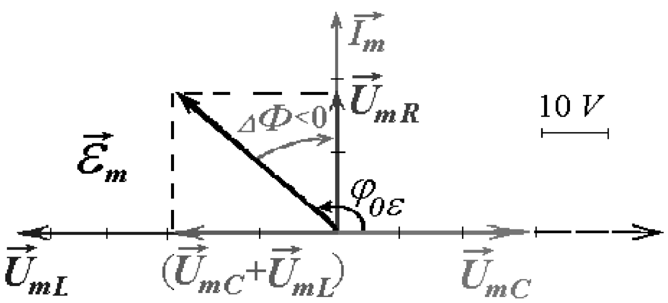



**Figure 3.4,a** – The vector voltage diagram at $t$=0.

In this case the argument of cosine is a phase of corresponding voltage at $t$ = 0.

For each phasor, the angle which is numerically equal to an initial phase, we lay off counterclockwise (for positive phase) relative to the axis $X$ (dashed axis at the **Fig. 3.4,a**). Initial phase of external voltage $\varphi_{0\varepsilon} = 3\pi/4$ and $\Delta\Phi \approx -\pi/4$ is shown at **Fig. 3.4,a**.

The magnitude of each phasor is equal to peak value of corresponding voltage. If one chooses the scale of voltage equal 10 $V/cm$, then the phasor length, representing the oscillation of voltage on capacitor, will equal 2,8 $cm$ (amplitude will be $U_{mc}$ = 28,3 $V$), this vector will be directed along the bearing axis, for the reason that the argument of cosine is equal to zero at $t = 0$.

In much the same way we build phasors, representing the oscillation of voltage on resistor, on inductor and external voltage of generator.

At the result we obtain, that according to the second Kirhchoff's rule, the phasor external EMF must be equal to the vector sum of the all voltage phasors (see **Fig. 3.4,a**):

$$\vec{\varepsilon}_m = \vec{U}_R + \vec{U}_{mC} + \vec{U}_{mL} \quad .$$

3) For finding values of external EMF and voltages at the moment of time of $t_1 = T/8$ we calculate the period of oscillations:

$T = 2\pi / \Omega$; $[T] = rad / \mathrm{rad}/s = s$; $T = 2 \cdot 3{,}14 / 6{,}28 \cdot 10^3 = 10^{-3}\ s$.

Let's substitute numerical value $t_1 = T / 8 = 10^{-3} / 8\ s$ in the equation of oscillations. For building the vector voltage diagram at $t_1 = T / 8$ we underline the equation of voltages in this moment of time without calculating the value of cosine and build phasors in the same way as **Fig. 3.4,a**:

$u_C(t_1) = 28{,}3\cos(6{,}28/8) = \underline{28{,}3\cos(\pi/4)}$ ;
$u_R(t_1) = 17{,}8 \cdot \cos(\pi/4+\pi/2) = \underline{17{,}8 \cdot \cos(3\pi/4)}$;
$u_L(t_1) = 44{,}7\cos(\pi/4+\pi) = \underline{44{,}7\cos(5\pi/4)}$;
$\varepsilon_{ext}(t_1) = 24{,}1\cos(\pi/4+3\pi/4) = \underline{24{,}1\cos(\pi)}$;
$i(t_1) = 0{,}178 \cdot \cos(\pi/4+\pi/2) = \underline{0{,}178 \cdot \cos(3\pi/4)}$;
$\Delta\Phi = -\pi/4$;
$\varphi_{0\varepsilon} = 3\pi/4$.

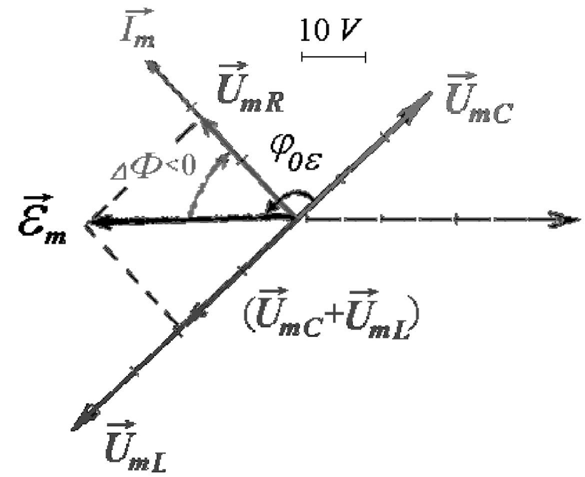



**Figure 3.4,b** – The vector voltage diagram at $t_1 = T/8$.

In this case the argument of cosine is a phase of corresponding voltage at $t_1 = T/8$. We note, that the phase of all voltages was incremented on $\pi/4$ and phasors have turned counterclockwise through an angle of $\pi/4$ (see **Fig. 3.4,b**).

Finally we calculate numerical values of the voltages at $t_1 = T/8$:

$u_C(t_1) = 28{,}3 \cdot 0{,}707 = 20\ V$;
$i(t_1) = 0{,}178 \cdot 0{,}707 = 0{,}126\ A$;
$u_R(t_1) = 17{,}8 \cdot 0{,}707 = 12{,}6\ V$;
$u_L(t_1) = 44{,}7 \cdot (-0{,}707) = -31{,}6\ V$;
$\varepsilon_{ext}(t_1) = 24{,}1 \cdot (-1) = -24{,}1\ V$.

***Results*:**

1) $u_C(t) = 28{,}3\cos(6{,}28 \cdot 10^3 t)\ V$; $i(t) = 0{,}178 \cdot \cos(6{,}28 \cdot 10^3 t+\pi/2)\ A$;
$u_R(t) = 17{,}8 \cdot \cos(6{,}28 \cdot 10^3 t+\pi/2)\ V$; $u_L(t) = 44{,}7\cos(6{,}28 \cdot 10^3 t+\pi)\ V$;
$\varepsilon_{ext}(t) = 24{,}1\cos(6{,}28 \cdot 10^3 t+3\pi/4)\ V$.
2) Phasors at $t=0$ see on **Fig. 3.4,a**.
3) Phasors at $t = T/8$ see on **Fig. 3.4,b**.
$u_C(t_1) = 20\ V$; $i(t_1) = 0{,}126\ A$; $u_R(t_1) = 12{,}6\ V$; $u_L(t_1) = -31{,}6\ V$; $\varepsilon_{ext}(t_1) = -24{,}1\ V$.

## INDIVIDUAL TASKS FOR PROBLEM 3.4.
## DRIVEN ELECTROMAGNETIC OSCILLATIONS

*In accordance with your variant to solve one of the following problems listed below (The number of problem statement and all necessary input data are reduced in the table 3.4).*

**1** Simple harmonic external EMF applied to the series oscillatory circuit of *RLC*-filter. Three nominal values of elements of filter and one equation of current or voltage oscillations in circuit are listed in the table 3.4. We used the following notations of quantities: ε – external EMF, $i$ – public current in circuit, $u_R$ – voltage on resistor, $u_C$ – voltage on capacitor, $u_L$ – voltage on inductor.

1) To rebuild five equations of all five electric oscillatory quantities in circuit with numerical coefficients.

2) To build the vector voltage diagram at $t$=0.

3) To find the values of external EMF – ε, voltages – $u_R$, $u_C$ , $u_L$ at the moment of time of $t_1 = T/4$ ($T$ – period of oscillations). To build the vector voltage diagram at $t_1 = T/4$.

**2** Simple harmonic external EMF on the resonance frequency applied to the series oscillatory circuit of *RLC*-filter. Three nominal values of elements of filter and equation of current or voltage oscillations in circuit are listed in the table 3.4. We used the following notations of quantities: ε – external EMF, $i$ – current in circuit, $u_R$ – voltage on resistor, $u_C$ – voltage on capacitor, $u_L$ – voltage on inductor.

1) To rebuild the equation of oscillations of all five quantities in circuit with numerical coefficients.

2) To build the vector voltage diagram at $t$=0.

3) To find the values of external EMF – ε, voltages – $u_R$, $u_C$ , $u_L$ at the moment of time of $t_1 = T/8$ ($T$ – period of oscillations). To build the vector voltage diagram at $t_1 = T/8$.

## TABLE OF TASK VARIANTS

**Table 3.4**

| Variant | Statement | $C$, $mcF$ | $L$, $mH$ | $R$, $\Omega$ | The equation of oscillations |
|---|---|---|---|---|---|
| **1** | 1 | 1 | 5 | 50 | $i(t)=0{,}6\cdot\cos(2\cdot 104\cdot t)$, $A$ |
| **2** | 1 | 100 | 10 | 15 | $u_R(t)=6\cdot\cos(500\cdot t)$, $V$ |
| **3** | 2 | 20 | 12,5 | 5 | $\varepsilon(t)=20\cdot\cos(\Omega_R\cdot t)$, $V$ |
| **4** | 1 | 0,4 | 0,5 | 25 | $u_L(t)=5\cdot\cos(5\cdot 10^4\cdot t)$, $V$ |
| **5** | 1 | 5 | 1,5 | 20 | $\varepsilon(t)=39{,}5\cdot\cos(2\cdot 10^4\cdot t)$, $V$ |
| **6** | 2 | 0,1 | 1 | 20 | $i(t)=1{,}5\cdot\cos(\Omega_R\cdot t)$, $A$ |
| **7** | 1 | 2 | 0,05 | 7,5 | $u_C(t)=5\cdot\cos(2\cdot 10^5\cdot t)$, $V$ |
| **8** | 2 | 0,5 | 0,2 | 5 | $u_R(t)=15\cdot\cos(\Omega_R\cdot t)$, $V$ |
| **9** | 1 | 4 | 0,2 | 5 | $i(t)=1{,}6\cdot\cos(2{,}5\cdot 10^4\cdot t)$, $A$ |
| **10** | 1 | 0,5 | 0,15 | 20 | $u_L(t)=12\cdot\cos(2\cdot 10^5\cdot t)$, $V$ |
| **11** | 1 | 10 | 25 | 75 | $u_R(t)=30\cdot\cos(10^3\cdot t)$, $V$ |
| **12** | 1 | 4 | 7,5 | 50 | $u_C(t)=20\cdot\cos(10^4\cdot t)$, $V$ |
| **13** | 1 | 25 | 20 | 20 | $\varepsilon(t)=19{,}3\cdot\cos(10^3\cdot t)$, $V$ |
| **14** | 2 | 4 | 2,5 | 4 | $u_L(t)=50\cdot\cos(\Omega_R\cdot t)$, $V$ |
| **15** | 1 | 0,5 | 0,2 | 30 | $i(t)=0{,}6\cdot\cos(2\cdot 10^5\cdot t)$, $A$ |
| **16** | 2 | 0,5 | 0,05 | 2 | $u_C(t)=30\cdot\cos(\Omega_R\cdot t)$, $V$ |
| **17** | 1 | 10 | 4 | 30 | $u_R(t)=6\cdot\cos(2{,}5\cdot 10^3\cdot t)$, $V$ |
| **18** | 1 | 0,2 | 1,5 | 100 | $u_C(t)=25\cdot\cos(10^5\cdot t)$, $V$ |
| **19** | 2 | 20 | 2 | 3 | $\varepsilon(t)=4{,}5\cdot\cos(\Omega_R\cdot t)$, $V$ |
| **20** | 1 | 2 | 10 | 50 | $u_L(t)=20\cdot\cos(10^4\cdot t)$, $V$ |
| **21** | 1 | 1 | 0,1 | 15 | $\varepsilon(t)=21{,}2\cdot\cos(5\cdot 10^4\cdot t)$, $V$ |
| **22** | 2 | 5 | 50 | 10 | $i(t)=2\cdot\cos(\Omega_R\cdot t)$, $A$ |
| **23** | 1 | 0,25 | 1,5 | 40 | $u_R(t)=24\cdot\cos(4\cdot 10^4\cdot t)$, $V$ |
| **24** | 1 | 10 | 1 | 15 | $u_L(t)=3\cdot\cos(5\cdot 10^3\cdot t)$, $V$ |
| **25** | 2 | 0,1 | 0,01 | 4 | $u_R(t)=6\cdot\cos(\Omega_R\cdot t)$, $V$ |
| **26** | 1 | 5 | 20 | 30 | $u_C(t)=20\cdot\cos(4\cdot 10^3\cdot t)$, $V$ |
| **27** | 1 | 2,5 | 2,5 | 30 | $\varepsilon(t)=29{,}7\cdot\cos(8\cdot 10^3\cdot t)$, $V$ |
| **28** | 2 | 0,5 | 0,8 | 5 | $u_L(t)=40\cdot\cos(\Omega_R\cdot t)$, $V$ |
| **29** | 1 | 2 | 2 | 15 | $i(t)=0{,}8\cdot\cos(2\cdot 10^4\cdot t)$, $A$ |
| **30** | 2 | 0,02 | 0,05 | 6 | $u_C(t)=25\cdot\cos(\Omega_R\cdot t)$, $V$ |

## Problem 3.6.
## ELECTROMAGNETIC WAVES (EMW).

### MAIN CONCEPTS

When a plane EMW propagates from the source (which is located at point $x_0$=0) along the positive direction of $x$-axis, the vector of electric field intensity will be changing along the $y$-axis, and the vector of magnetic field intensity will be changing along the $z$-axis, according to the ***equations of EMF***:

$$E_y(x,t)=E_m\cdot\cos(\omega t-kx+\varphi_0); \quad H_z(x,t)=H_m\cdot\cos(\omega t-kx+\varphi_0), \qquad (18)$$

where $E_m$ and $H_m$ – amplitudes of electric field intensity and magnetic field intensity in a wave correspondingly; $\varphi_0$ – initial phase of the wave source.

***Cyclic frequency*** $\omega$ $[rad\,/\,s]$ – is a changing of phase of a wave per second:

$$\omega = 2\pi / T = 2\pi f, \qquad (19)$$

here $T\,[s]$ – period is a time of one oscillation of waves' quantities;

$f$ $[Hz]$ – frequency is a number of oscillations of waves' quantities per second.

***Wave number*** $k$ $[rad\,/\,m]$ – is a changing of phase a wave per meter:

$$k = 2\pi / \lambda, \qquad (20)$$

here $\lambda$ $[m]$ – ***wavelength*** is a length of one oscillation (distance which is transited by a wave for a period).

***Phase velocity*** of propagation of EMW in medium

$$\upsilon = \frac{1}{\sqrt{\varepsilon\varepsilon_0\mu\mu_0}} = \frac{c}{n}, \qquad (21)$$

where ***speed of light*** (velocity of propagation of EMW in vacuum):

$$c = 1\Big/\sqrt{\varepsilon_0 \cdot \mu_0}; \qquad (22)$$

and ***refractive index***

$$n = \sqrt{\varepsilon\mu}; \qquad (23)$$

$\varepsilon_0$ and $\mu_0$ – electric and magnetic constants correspondingly;

$\varepsilon$ and $\mu$ – relational electric permittivity and magnetic permeability of medium (as a rule the transparent medium is non-magnetic $\mu$=1).

In one EMW the volume density of energy of electric field $w_C$ is equal to volume density of energy of magnetic field $w_L$:

$$\frac{\varepsilon\varepsilon_0 E^2}{2} = \frac{\mu\mu_0 H^2}{2}. \qquad (24)$$

Instantaneous flux density of energy of EMW (Pointing's vector)

$$\vec{P}(x,t) = [\vec{E}\times\vec{H}]. \qquad (25)$$

The average value of Pointing's vector defines the wave intensity:

$$I = P_{AVE} = E_m \cdot H_m / 2. \qquad (26)$$

## EXAMPLE OF PROBLEM SOLUTION

**Example 4.** In the homogeneous isotropic non-magnetic medium with the dielectric permittivity of $\varepsilon = 9$ along the $x$-axis propagates plane EMW from wave source which is located at point $x_0 = 0$. The change of intensity of magnetic field is described by equation $H_z(x,t) = H_m \cdot \cos(\omega t - kx - \pi/2)$, when amplitude of magnetic field intensity in a wave 0,02 $A/m$. The oscillation period is 1 $\mu s$.

1) To rebuild the equations of change of electric field intensity and magnetic field intensity with numerical coefficients.

2) To draw the graph of wave at the moment of time of $t_1 = 1{,}5T$.

3) To define the Pointing's vector at the moment of time of $t_1 = 1{,}5T$ in the point with coordinate $x_1 = 1{,}25\lambda$ and plot it on the graph.

4) To define the wave intensity.

*Input data:*

$H_m = 0{,}02\ A/m$;

$T = 1\ \mu s = 10^{-6}\ s$;

$\varepsilon = 9;\quad \mu = 1$;

$H_z(x,t) = H_m \cdot \cos(\omega t - kx - \pi/2)$

*Find:* $H_z(x, t),\ E_y(x, t)$ – ?

Graph, $\vec{P}(x_1, t_1),\ I$ – ?

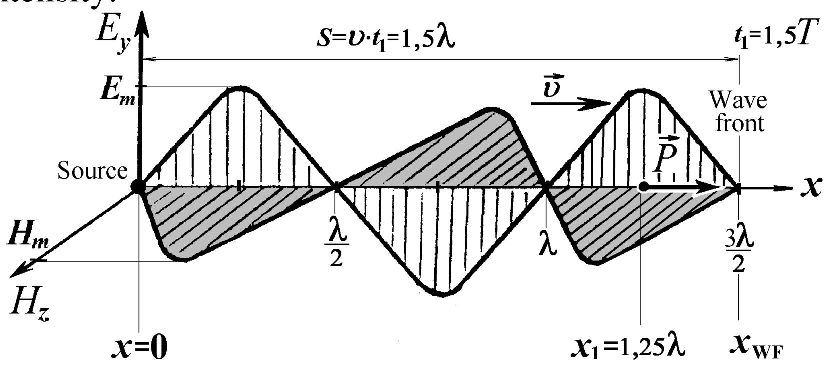



**Figure 3.6 –** Graph of wave at $t_1=1{,}5T$.

*Solution:*

1) Equations of given EMW have a general view:

$$E_y(x,t)=E_m \cdot \cos(\omega t - kx + \varphi_0); \qquad H_z(x,t)=H_m \cdot \cos(\omega t - kx + \varphi_0),$$

where $E_m$ and $H_m$ – amplitudes of intensities of electric and magnetic fields of the wave correspondently, $x$ – coordinate of a point of space; $t$ – time of propagation of a wave; $\varphi_0 = -\pi/2$ – initial phase of the wave source.

For the rebuilding this equations with numerical coefficients, it's necessary to define the cyclic frequency $\omega$ and wave number $k$, which are defined by equations:

$$\omega = \frac{2\pi}{T}; \tag{4.1}$$

$$k = \frac{2\pi}{\lambda}. \tag{4.2}$$

Period $T$ is given in the problem statement, the wave length $\lambda$ is a distance which is transited by a wave for a period:

$$\lambda = \upsilon \cdot T, \tag{4.3}$$

where $\upsilon$ – phase velocity of propagation of EMW. In non-magnetic medium with permeability $\mu=1$ and dielectric permittivity $\varepsilon$ the velocity $\upsilon$ of propagation of EMW is defined with the formula:

$$\upsilon = \frac{c}{\sqrt{\varepsilon \cdot \mu}} = \frac{c}{\sqrt{\varepsilon}}, \tag{4.4}$$

where $c = 3 \cdot 10^8\ m/s$ – the speed of light.

Let's substitute in the formula (4.2) the expression of $\lambda$ from the formula (4.3) and $\upsilon$ from the formula (4.4):

$$k = \frac{2\pi}{\upsilon T} = \frac{2\pi\sqrt{\varepsilon}}{cT} \; . \tag{4.5}$$

From the equality of volume energy density of electric and magnetic field

$$\sqrt{\varepsilon_0 \varepsilon} \cdot E_m = \sqrt{\mu_0} \cdot H_m ,$$

we obtain the relation between the amplitudes of electric and magnetic intensities:

$$E_m = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon}} \cdot H_m . \tag{4.6}$$

The right part of the formula (4.1) gives the unit of measurement of cyclic frequency [*rad/s*]; let's check whether the right part of the formula (4.5) gives us the unit of wave number [*rad/m*], and right part of the formula (4.6) **–** the unit of intensity of electric field [*V/m*].

$$[k] = \frac{rad}{[c]\cdot[T]} = \frac{rad}{s \cdot m/s} = rad/m ;$$

$$[E_m] = \sqrt{\frac{[\mu_0]}{[\varepsilon_0]}}[H_m] = \sqrt{\frac{H/m}{F/m}} A/m = \sqrt{\frac{Wb \cdot V}{A \cdot C}} A/m = \sqrt{\frac{V \cdot s \cdot V}{A \cdot A \cdot s}} A/m = \frac{V}{A}\frac{A}{m} = V/m$$

Let's make the calculations and write down the equation *E* and *H* with numerical coefficients

$$\omega = \frac{2\pi}{10^{-3}} = 2\pi \cdot 10^{-6} \; s^{-1}; \;\; k = \frac{2\pi\sqrt{9}}{3 \cdot 10^{3} \cdot 10^{-6}} = 0{,}02\pi \; rad/m \; ;$$

$$E_m = \sqrt{\frac{4\pi \cdot 10^{-7}}{8{,}85 \cdot 10^{-12} \cdot 9}} \cdot 0{,}02 = 2{,}51 \; V/m ;$$

Then finaly equations of EMW: $\underline{E_y(x,t) = 2{,}5 \cdot \cos(2 \cdot 10^6 \pi \cdot t - 0{,}02\pi \cdot x - \pi/2) \; V/m};$
$\underline{H_z(x,t) = 0{,}02 \cdot \cos(2 \cdot 10^6 \pi \cdot t - 0{,}02\pi \cdot x - \pi/2) \; A/m.}$

2) Let's draw the graph of wave at the moment of time of $t_1 = 1{,}5T$.

At this time the source will have a phase, equal to

$$\Phi(x=0, t_1) = (2 \cdot 10^6 \pi \cdot 1{,}5 \cdot 10^{-6} - 0{,}02\pi \cdot 0 - \pi/2) = (3\pi - 0 - \pi/2) = \pi/2,$$

then intensities of electric and magnetic fields in a source will have a zero values, as a $\cos(\pi/2)=0$ (**Fig. 3.6**, point $x=0$).

Through distance, equal to wave length

$$\lambda = 2\pi / k \quad \text{or} \quad \lambda = 2\pi / 0{,}02\pi = 100m$$

this value will repeat, as a $\cos(\pi/2 - k\lambda) = 0$ (see **Fig. 3.6**, point $x=\lambda$).

During this time the wave will transit distance equal to position of a wave front:

$$x_{WF} = \upsilon \cdot t_1 = \frac{\lambda}{T} \cdot t_1 = \frac{\lambda}{T} \cdot 1{,}5T = 1{,}5\lambda ,$$

then in position of wave front intensities of electric and magnetic fields in a source will have a values same as source at $t=0$, that is zero, as a $\cos(\pi/2 - k \cdot 1{,}5\lambda) = 0$ (see **Fig. 3.6**, point $x = x_{WF}$).

3) Let's Calculate instantaneous value of modulus of the Pointing's vector (vector of energy fluxes density of EMW):

$P(x,t)= E_y(x,t)\cdot H_z(x,t)= E_m\cdot\cos(\omega t-kx+\varphi_0)\cdot H_m\cdot\cos(\omega t-kx+\varphi_0)= E_m\cdot H_m\cdot\cos^2(\omega t-kx+\varphi_0)$.

Let's check whether the obtained formula gives the unit of energy fluxes density $[W / m^2]$

$$[P]=[E_m]\cdot[H_m]=\frac{V}{m}\cdot\frac{A}{m}=\frac{J\cdot A}{C\cdot m^2}=\frac{J\cdot A}{A\cdot s\cdot m^2}=\frac{W}{m^2};$$

Let's substitute the numerical values:

$P(x,t)= 2{,}5\cdot 0{,}02\cdot\cos^2(2\cdot 10^6\pi\cdot t-0{,}02\pi\cdot x-\pi/2)$.

At the moment of time of $t_1=1{,}5T=10^{-6}s$ (given by the problem statement) and at the point with coordinate $x_1 =1{,}25\lambda= 1{,}25\cdot 2\pi / k$; then $x_1 =1{,}25\cdot 2\pi / 0{,}02\pi=125\ m$ we obtain:

$P(x_1,t_1) = 2{,}5\cdot 0{,}02\cdot\cos^2(2\cdot 10^6\pi\cdot 1{,}5\cdot 10^{-6} - 0{,}02\pi\cdot 125 - \pi/2) =$

$= 0{,}05\cdot\cos^2(3\pi - 2{,}5\pi - \pi/2) = 0{,}05\cdot\cos^2(0{,}5\pi - \pi/2)= 0{,}05\cdot(1)^2=50\ mW / m^2$.

Let's plot it on the graph obtained Pointing's vector (see **Fig. 3.6**, point $x=x_1 =1{,}25\lambda$).

4) The intensity of electromagnetic wave is the average energy in time, going through the unit plane, which is perpendicular to the direction of wave propagation;

$$I = P_{AVE} = \frac{1}{2}E_m H_m,$$

where $P$ – average value of vector modulus of energy fluxes density of EMW (modulus of Pointing's vector).

Let's make the calculations: $I = 0{,}5\cdot 2{,}51\cdot 0{,}02 = 2{,}51\cdot 10^{-2}\ W / m^2 = 25\ mW / m^2$.

***Results***: $E_y(x,t)=2{,}5\cdot\cos(2\cdot 10^6\pi\cdot t-0{,}02\pi\cdot x-\pi/2)\ V/m$;

$H_z(x,t)=0{,}02\cdot\cos(2\cdot 10^6\pi\cdot t-0{,}02\pi\cdot x-\pi/2)\ A/m$.

$P(x_1,t_1) =50\ mW / m^2$; $I = 25\ mW / m^2$.

## INDIVIDUAL TASKS FOR PROBLEM 3.6. ELECTROMAGNETIC WAVES

*In accordance with your variant to solve the following problem listed below (all necessary input data are reduced in the table 3.6).*

Plane electromagnetic wave propagates in homogeneous isotropic non-magnetic medium with dielectric permittivity ε. The intensity of electric field of wave changes by law $E = E_m\cdot\cos(\omega t- kx+\pi/2)$.

1) By input data, given in the table 3.6, find necessary parameters and equations of electric intensity $E$ and magnetic intensity $H$ with numerical coefficients.

2) Draw the graph of wave at the moment of time of $t_1$ .

3) To define the Pointing's vector at the moment of time of $t_1$ in the point with coordinate $x_1 = \lambda / 8$ and plot it on the graph.

4) Calculate the intensity of wave.

## TABLE OF TASK VARIANTS

**Table 3.6**

| Variant | *f*, *MHz* | *T*, μ*s* | ω, *rad/s* | λ, *m* | υ, *Mm/s* | *k*, *rad/m* | ε | $E_m$, *V/m* | $H_m$, *A/m* | $t_1$, μ*s* |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | – | – | – | 100 | – | – | 1,44 | 40 | – | 0,7 |
| **2** | – | – | – | – | 40 | π/10 | – | – | 0,5 | 0,75 |
| **3** | – | – | $2{,}5\cdot10^6\pi$ | 240 | – | – | – | 50 | – | 1 |
| **4** | 1 | – | – | – | 70 | – | – | – | 0,3 | 2 |
| **5** | – | 0,5 | – | – | – | – | 2,25 | 20 | – | 0,75 |
| **6** | 1,25 | – | – | – | – | π/20 | – | – | 0,4 | 1,4 |
| **7** | – | 2 | – | – | 200 | – | – | 30 | – | 3 |
| **8** | – | – | – | – | – | π/50 | 5,76 | – | 0,2 | 1 |
| **9** | 2 | – | – | 60 | – | – | – | 10 | – | 0,5 |
| **10** | – | – | $10^6\pi$ | – | – | – | 9 | – | 0,3 | 3,5 |
| **11** | – | 4 | – | 560 | – | – | – | 20 | – | 5 |
| **12** | – | – | $2\cdot10^6\pi$ | – | 80 | – | – | – | 0,4 | 1,5 |
| **13** | – | – | – | – | – | π/15 | 16 | 10 | – | 0,5 |
| **14** | – | – | – | 60 | 240 | – | – | – | 0,1 | 0,25 |
| **15** | 0,5 | – | – | – | – | – | 4 | 40 | – | 3 |
| **16** | – | – | $5\cdot10^6\pi$ | 40 | – | – | – | – | 0,2 | 0,5 |
| **17** | 1 | – | – | – | 180 | – | – | 50 | – | 1,5 |
| **18** | – | 0,8 | – | – | – | π/30 | – | – | 0,4 | 1 |
| **19** | – | – | – | 30 | – | – | 25 | 20 | – | 0,5 |
| **20** | – | – | – | – | 90 | π/36 | – | – | 0,3 | 1,4 |
| **21** | – | 8 | – | 320 | – | – | – | 10 | – | 10 |
| **22** | 2,5 | – | – | 120 | – | – | – | – | 0,1 | 0,6 |
| **23** | 5 | – | – | – | – | π/8 | – | 30 | – | 0,35 |
| **24** | – | – | $2\cdot10^6\pi$ | – | – | – | 36 | – | 0,5 | 1 |
| **25** | – | – | – | 260 | 130 | – | – | 20 | – | 3 |
| **26** | – | 2 | – | – | – | π/34 | – | – | 0,3 | 3,5 |
| **27** | – | 0,8 | – | – | – | – | 6,25 | 40 | – | 1 |
| **28** | – | 0,4 | – | – | 60 | – | – | – | 0,4 | 0,7 |
| **29** | 0,5 | – | – | – | – | – | 21,3 | 10 | – | 2 |
| **30** | – | – | $5\cdot10^5\pi$ | – | 160 | – | – | – | 0,2 | 7 |

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