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004.2  
32.973  
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( 3 30 2014 . )

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: , 2014. - 190 .

004.2  
32.973

© . . , 2014  
©

, 2014

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COMA – (Cache Only Memory Architecture);

DSM – (Distribute Shared Momory);

HDD – (Hard Disk Drive);

JBOD – (Just A Bunch Of Disks);

NORMA – (No Remote Memory Access);

NUMA – (Non-Uniform Memory Access);

SISD – (Single Instruction, Single Data);

SIMD – (Single Instruction, Multiple Data);

MISD - (Multiple Instruction, Single Data);

MIMD – (Multiple Instruction, Multiple Data);

SMP – (Symmetric Multi Processors);

MPP – (Massively Parallel Processing);

RISC – (Restricted (reduced) instruction set computer);

RAID – (Redundant Arrays of Inexpensive Disks)

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(CPU, Central Processor Unit - ),

( . chip).

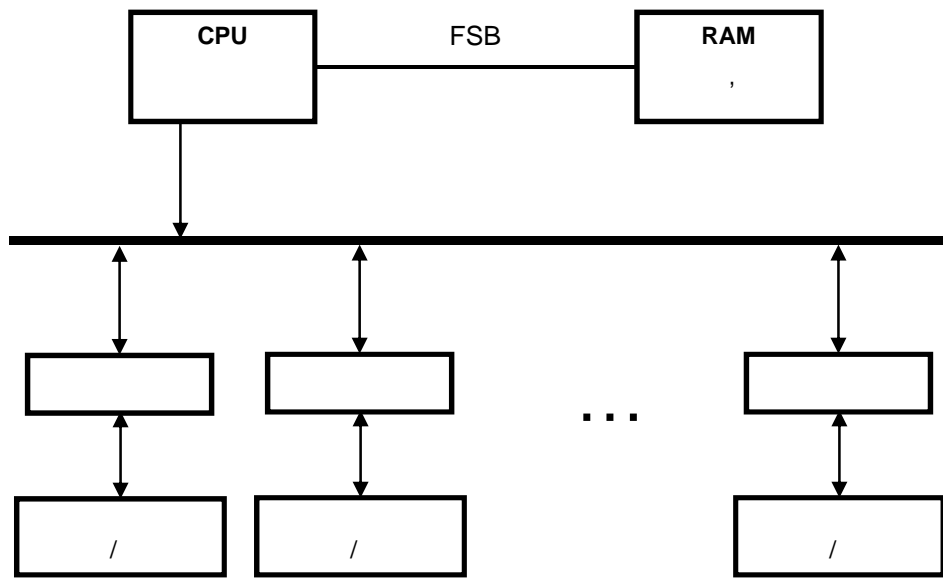
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2N, N -

( .1.1).



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2000-2005

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Power4

IBM 1999, 2001 . 2002

AMD Intel

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### 1.1.2

(Flynn),

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( 1.2):

**SISD** (Single Instruction, Single Data) –

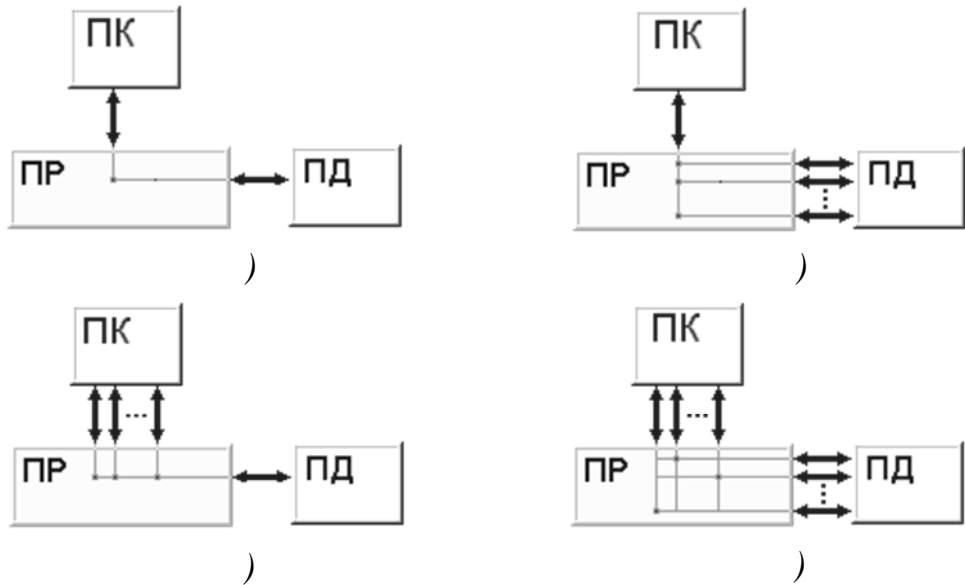
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. 1.2

) - SISD; ) - SIMD; ) - MISD; ) - MIMD.

**SIMD** (Single Instruction, Multiple Data) –

( ILLIAC IV, ICL DAP, Goodyear Aerospace MPP, Connection Machine),

**MISD** (Multiple Instruction, Single Data) –

**MIMD** (Multiple Instruction, Multiple Data) –

;

(  
MIMD.

1.3).

79

1024,

1024,

79.



.1.3

1.4).

79



. 1.4

Symmetric Multi Processors (SMP). SMP

SMP

: Symmetric Multi Processors Shared Memory Processors.

## 1.2. SIMD

### 1.2.1

SIMD-

SIMD

GFLOPS.

( ),

. SIMD-

:

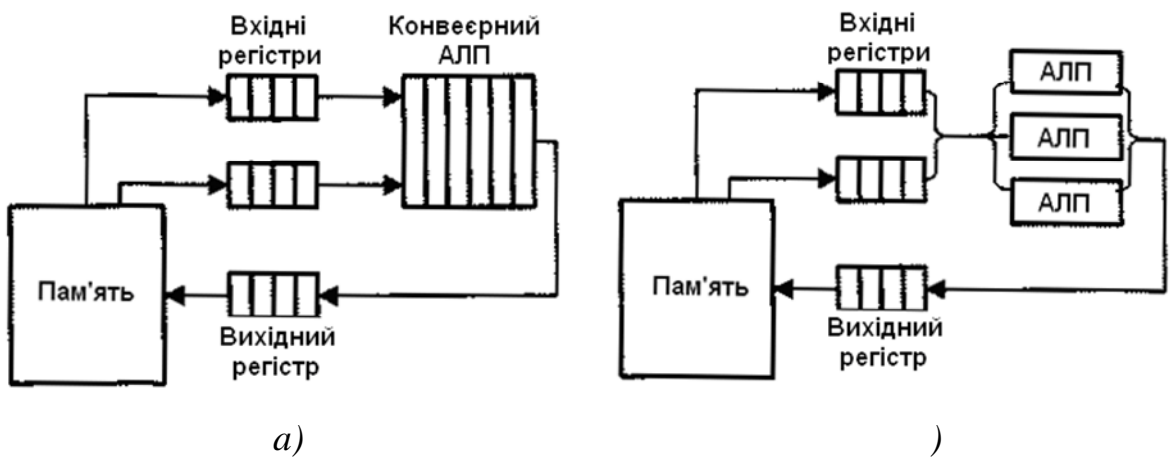
( )),

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. 1.4.



. 1.5

( . 1.5, )

( ) .

( . 1.6, ) .

( . 1.6, ) .

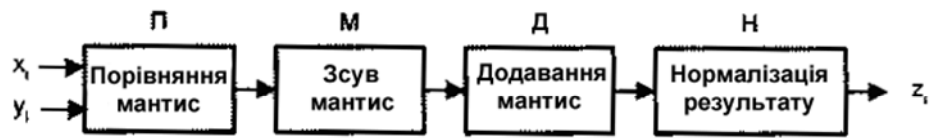
( . 1.6, ) .



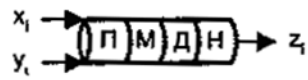
( . . 1.5, ).

. 1.6, .

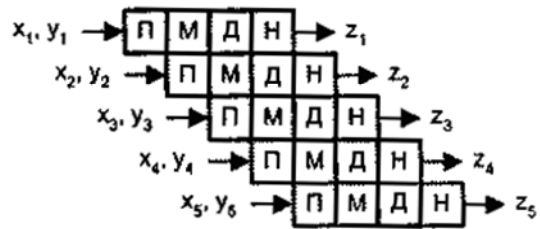
. 1.6, .



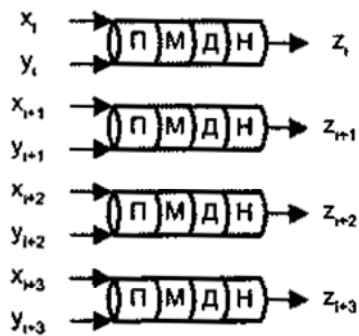
а)



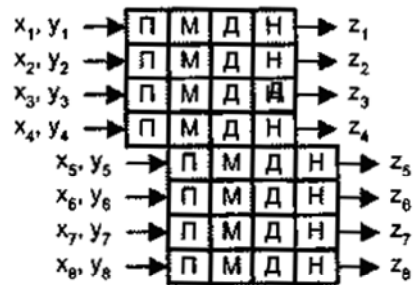
б)



в)



г)



д)

. 1.6

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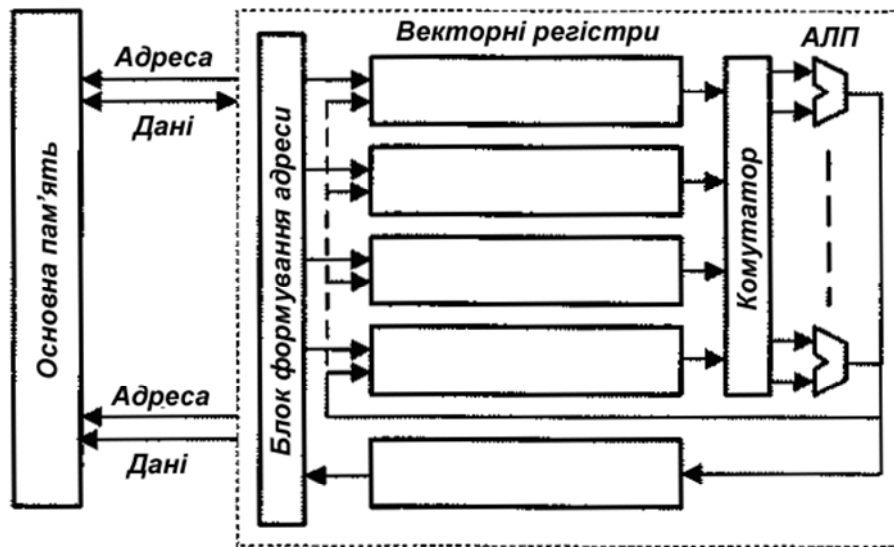
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. 1.7.



.1.7

( )

$X/Y$

$X(I/Y)$

FIFO,

50-100

$(V_a, V_b, V_c \dots)$

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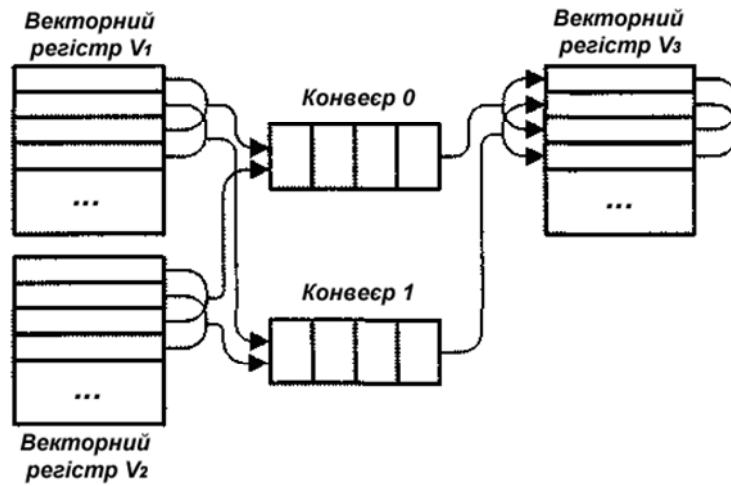
$$T = s + x N \quad (1.1)$$

$s$  — , — , ( 0.5, 1  
 2)  $N$ , — .

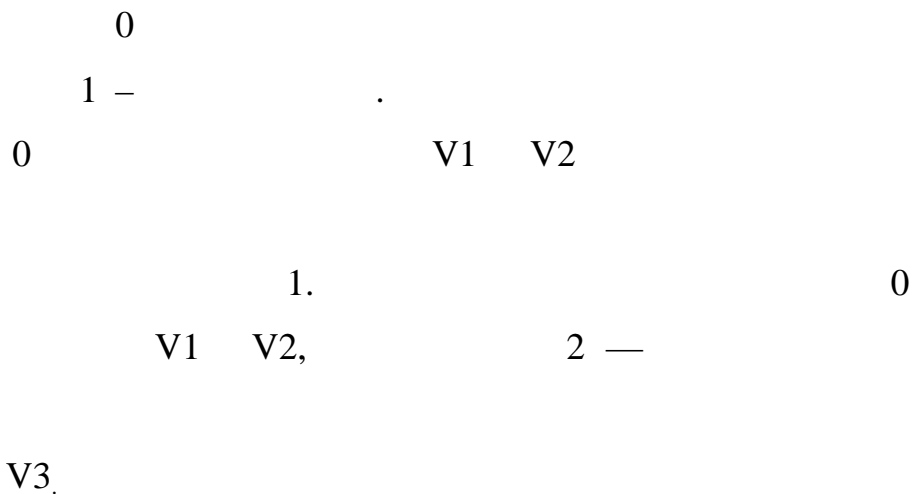
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Cray C90,

( .1.8).



. 1.8.



### 1.2.2

(array processor),

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( ), SIMD-

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, SIMD.

.1.9.

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. 1.9.

SIMD-

( )

(front-end computer).

SIMD-

SIMD-

( . 1.10).

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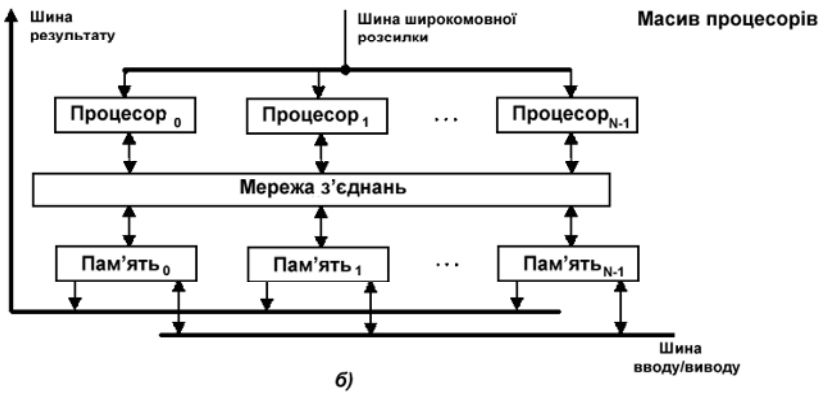
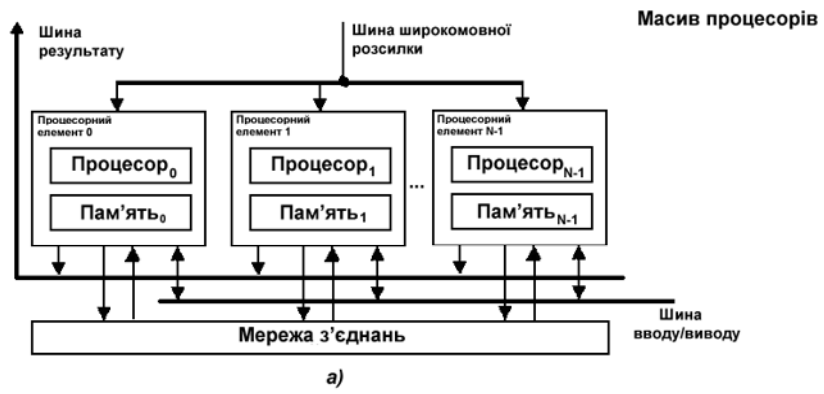
» (« - »), N

( )

( . 1.10, ).

—

MasPar MP-1, Connection Machine CM-2, GF11, DAP, , STARAN, ILLIAC.



. 1.10.

, — « — »;

— « — » —

.1.10, .

N

Burroughs

Scientific Processor (BSP), Texas Reconfigurable Array Computer TRAC.

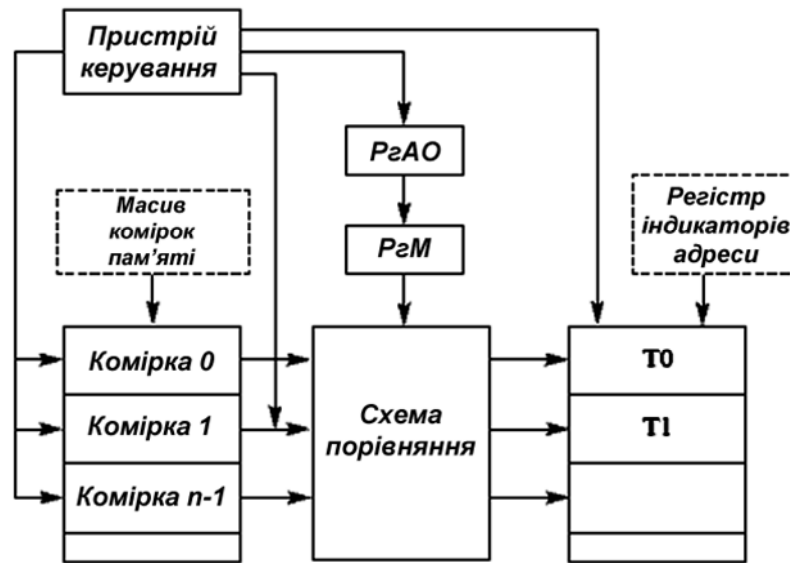


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SIMD- , -1,  
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### 1.2.3

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(SIMD = Single Instruction Multiple Data).



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( . 1.11)

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 SIMD (STARAN, PEPE) MIMD. ( )  
 ) (32 —  
 STARAN).

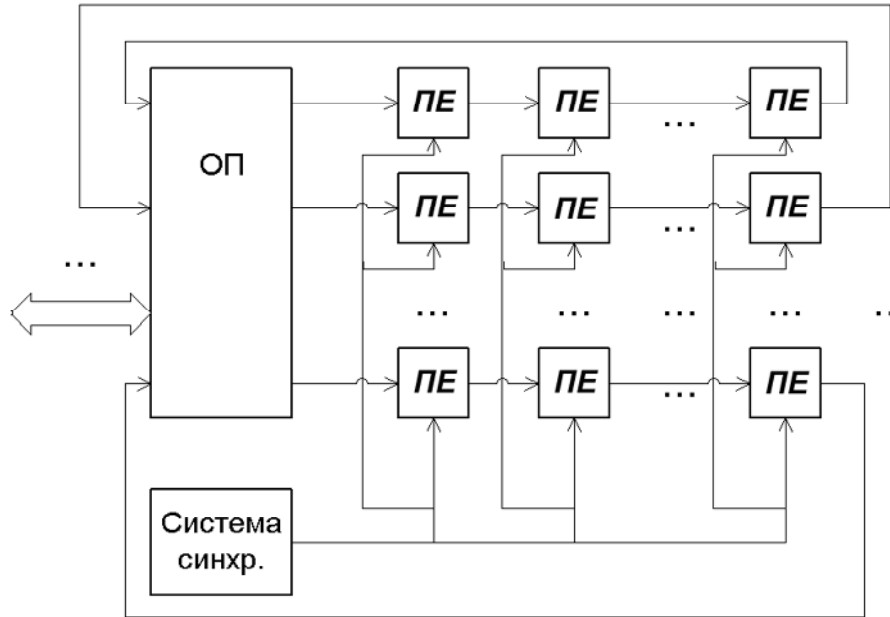
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 « » (systolic),  
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MISD,

MIMD-

MIMD

. 1.12.



. 1.12

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**(VLIW)**

(VLIW, Very Long Instruction Word)

80-

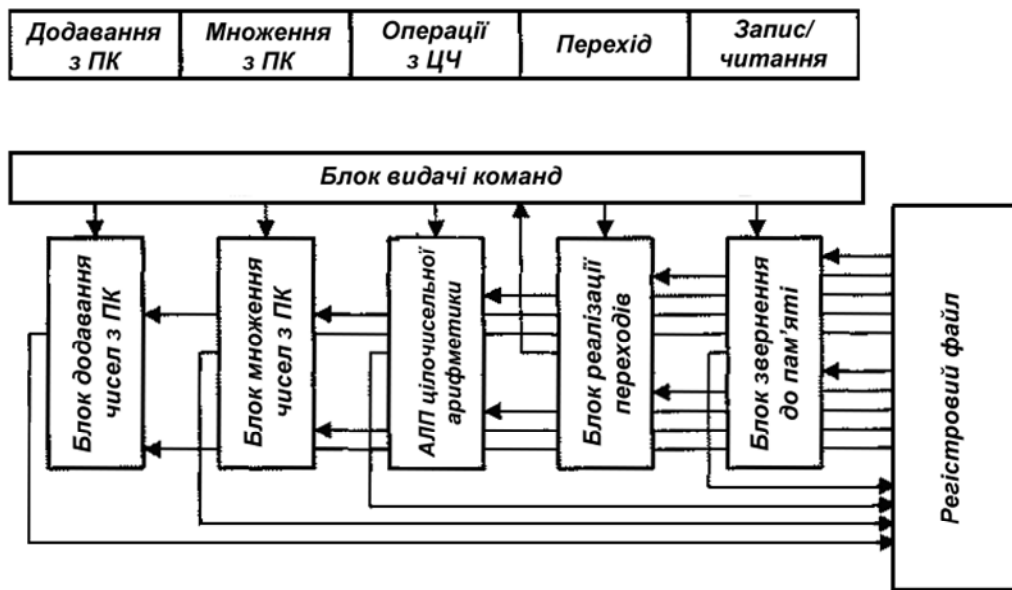
—

VLIW

. VLIW

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.1.13

256 1024

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. 1.13

VLIW-

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 VLIW- , ,  
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 RISC- , VLIW  
 RIS - .  
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 1 . / VLIW-  
 VLIW  
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 VLIW — ,  
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1994 ,  
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Leonard Adleman University of

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2001

"Nature",

99,8%.

2003

"Proceedings of the National Academy of Sciences"

Weizmann Institute of Science ( )

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330

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2001 .

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*Pseudomonas putida*

( - ),

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### 1.3.

#### 1.3.1

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1) , — ;

2) , ;

3) , . , , , , , , ( , , ) — . , — , , , .

### 1.3.2

$y_{m1}$

( )

( )

$= \{ , \dots, m \}$ ,

$Y = \{y_1, \dots,$

$X = \{x_1, \dots, x\}$ ,

$y = ( ), = 1, \dots, m.$

$F,$

$Y = F(X)$

$X,$

X.

.1.14.



. 1.14.

$X^*$   $Y^*$  ,  
 $= \{ n \}$  .  
 $X^*$   $Y^*$   
 $Y = F(X^*)$  ,  
 $Y^*$  .  
 $Y = F(X)$  ,  $X$

: 1)

( , ); 2)

, , ; 3)

, .

: , M

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. = ( b... q) -

A, ; S=(s<sub>1</sub>,...,s<sub>p</sub>) -

, ( ) ; = ( 1, ...,

r) ; Y=(y<sub>1</sub>,..., y<sub>m</sub>) —

, S C

Y=F( , S, C); S={S<sub>j</sub>} -

; = { j} —

z<sub>α</sub>,..., z<sub>ω</sub> ∈ (Y ∪ X)

z<sub>α</sub> ∈ z<sub>α</sub><sup>\*</sup>,..., z<sub>ω</sub> ∈ z<sub>ω</sub><sup>\*</sup> z<sub>α</sub><sup>\*</sup>,..., z<sub>ω</sub><sup>\*</sup> -

.  
= F(Y),

Y=F( , S, C).

: 5 C,

$$\max E = \max_{s \in S, c \in C} \Phi(Y) \tag{1.2}$$

:

$$z_{\alpha} \in z_{\alpha}^*, \dots, z_{\omega} \in z_{\omega}^* \tag{1.3}$$



X,

S

C,

(1.2),

(1.3).

$$Y = F( \dots, S, C ),$$

(1.2)

(1.3).

$$= F( \dots ),$$

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3. SIMD. - .
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(WLIV).
11. , ( ).
12. , .
13. , .
14. ?
15. ?
15. ?

2.1

2.1.1

IMD

SIMD

SIMD-

MIMD. MIMD-

MIMD

MIMD-

( )

MIMD-

(tightly coupled),

(SMP, Symmetric Multiprocessor)

(NUMA, Non-Uniform Memory Access).

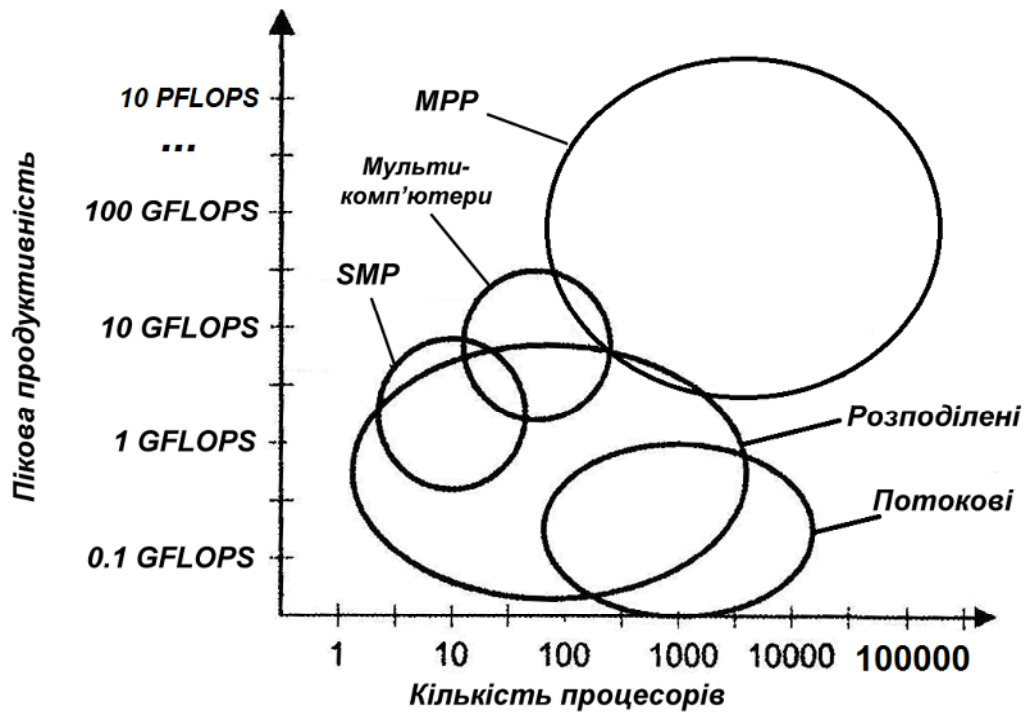
(loosely

coupled)

« »

( , Massively Parallel Processing)

MIMD-



. 2.1

MIMD-

( ),

MIMD

. 2.1.

2.1.2

SMP

Symmetric Multiprocessor).

. SMP

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- , ;
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- ;
- ( « »).

. SMP

SMP-

(master).

SMP-

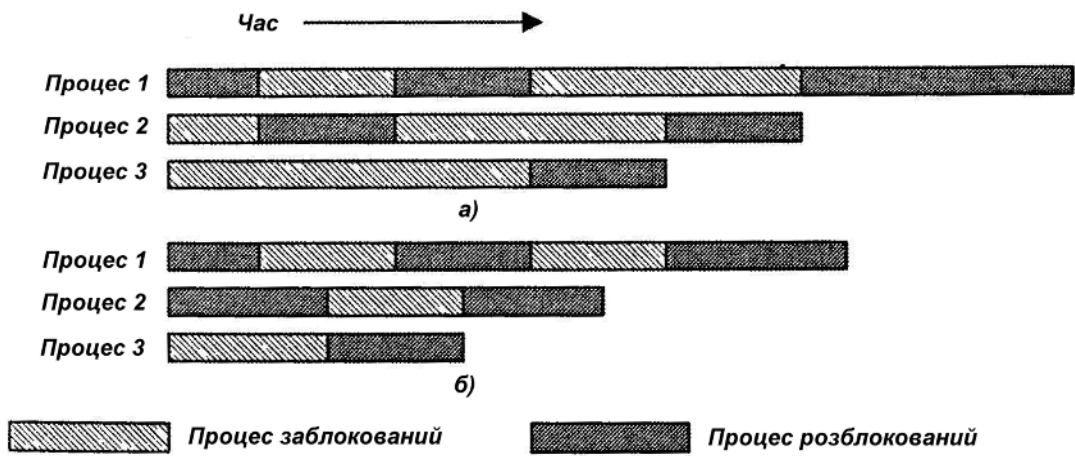
« »

(threads)

SMP-

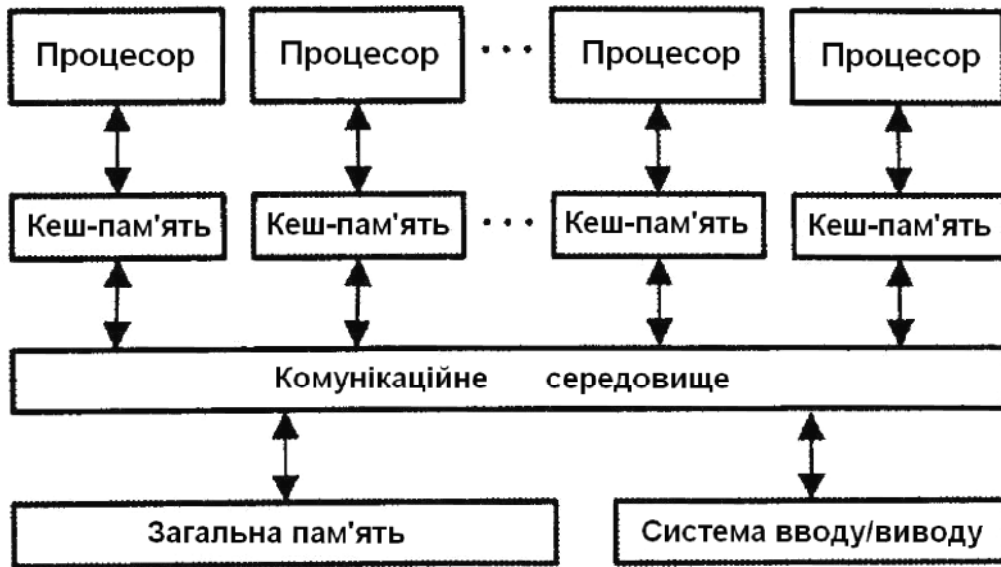
SMP-

( .2.2);



.2.2

2.3



2.3

SMP-

32

RISC-

DEC Alpha, Sun SPARC, MIPS

HP PA-RISC.

SMP-

CISC-

Intel AMD.

(L1)

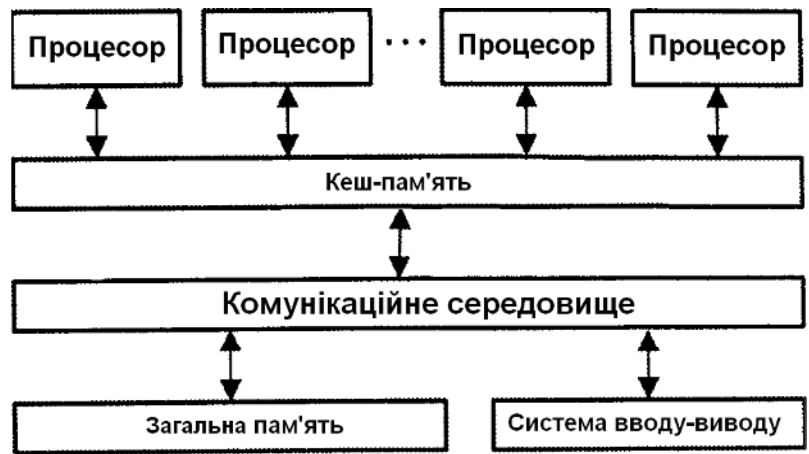
(L2)

SMP-

( 2.4).



, / .  
 , ( SMP-  
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 ( ).



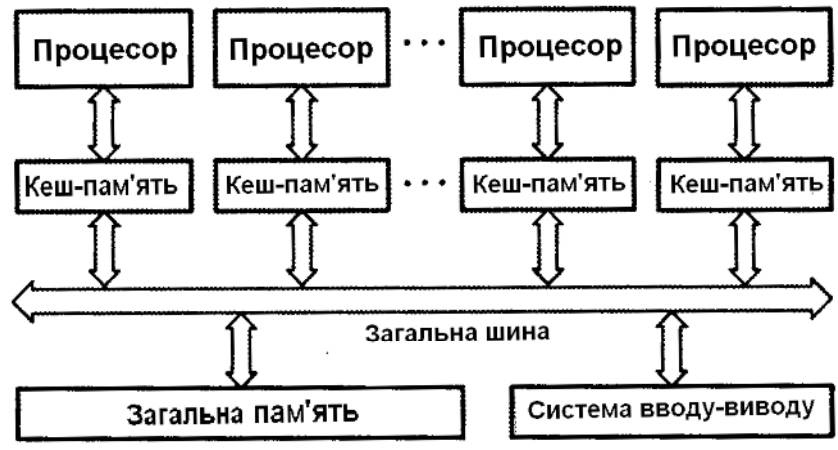
**.2.4. SMP-**

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SMP-

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( .2.5).



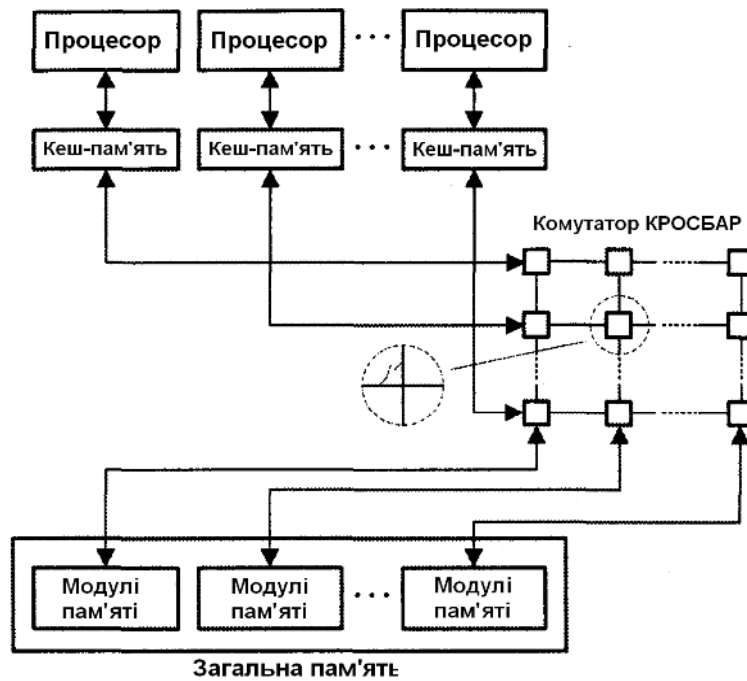
.2.5. SMP-

SMP-

SMP-

Compaq AlphaServer GS140 8400  
 Alpha 21264. SMP- HP  
 N9000 8 -8500,  
 SMP Thin Nodes RS/6000 IBM  
 POWER PC 604.  
 SMP-  
 86. : DELL Power  
 Edge, IBM Netfinity, HP NetServer.

« »  
 « » ( . 2.6)



. 2.6. SMP- « »

SMP-

SMP-

32 64. ,

Enterprise 10000,

64 , ' ,

GIGAPLANE-XB

Sun

Microsystems ( 16 16). IBM RS/6000 Enterprise Server Model S70

« »

12

RS64.

SMP-

ProLiant 8000

8500

Compaq

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,

Pentium III Xeon

( )

CC-NUMA

### 2.1.3

— . . . .

( ), , .

SMP .

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, « »,

: (workstation cluster),

(hypercom-puting), (network-based

concurrent computing), (ultracomputing).

:

DEC,

80-

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( ), ( ).

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Ethernet,

RS/6000 SP  
Memory Channel, ServerNet

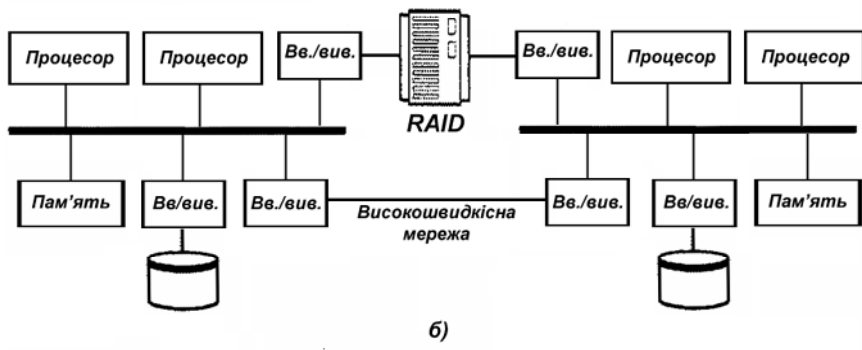
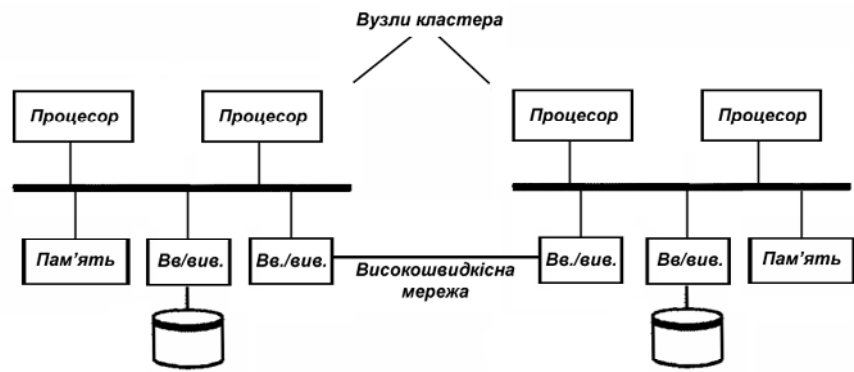
IBM,  
Compaq.

Digital

*heartbeat,*

( ),

. 2.7,



. 2.7

. 2.1.

**2.1.**

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( . . 2.7, ).

RAID( . 2.7, ).

( )

(Transmission Control Protocol), TCP  
- UDP (User Datagram Protocol)

Intel (Microsoft, Compaq .)

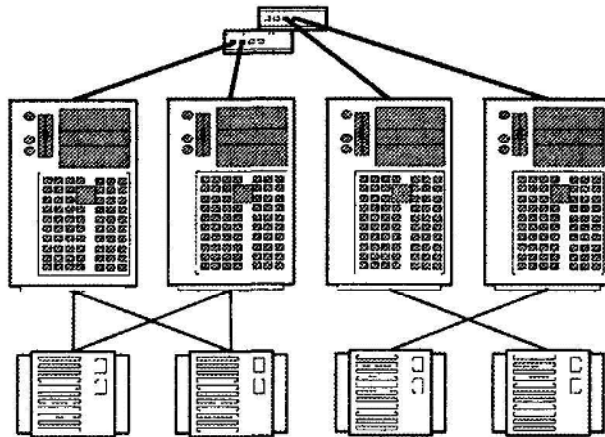
### Virtual Interface Architecture (VIA)

« » (keepalive).

( )

2-4

( .2.8).



.2.8

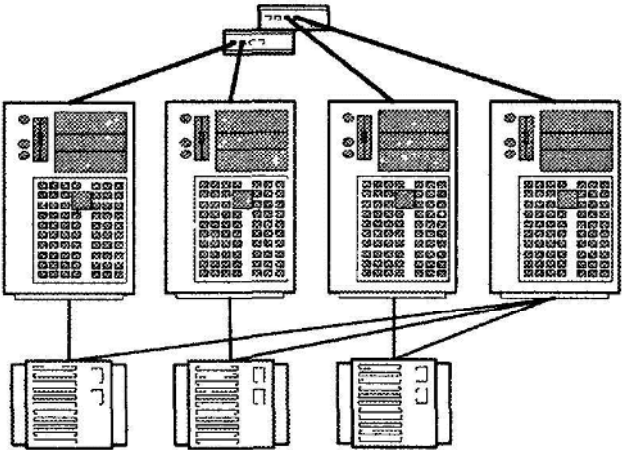
Informix XPS.

$N + 1$

$N+1$

2,3 4

( . 2.9).



. 2.9

$N + 1$

RAID 1 (

3).

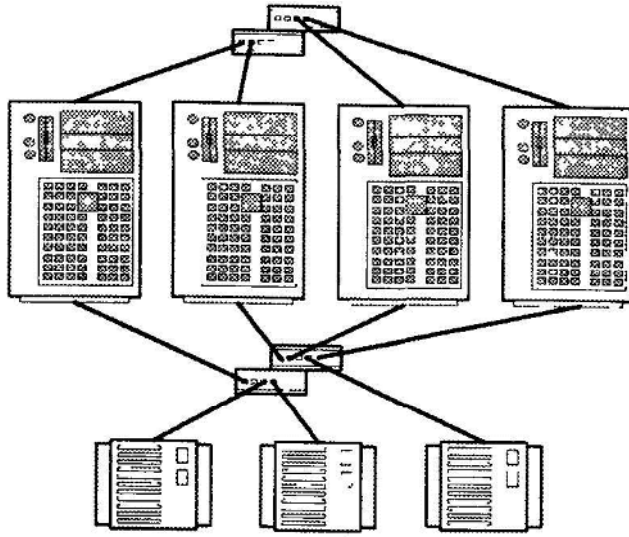
( )

$N N$

$N+1,$

$N N ( . 2.10)$

2, 3 4 ,



. 2.10 N N

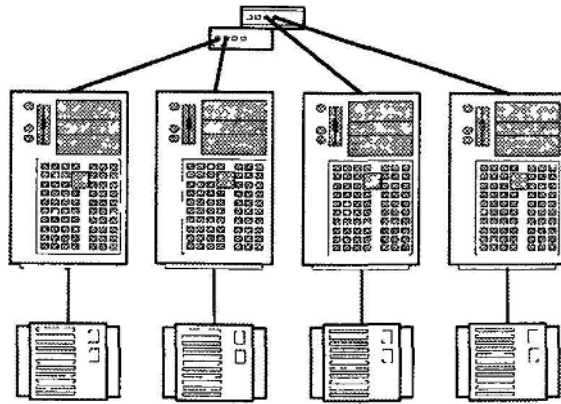
, , , RAID 1 ( ).

N N

Oracle Parallel Server,

( . 2.11)

Informix XPS.



. 2.11

## 2.1.4

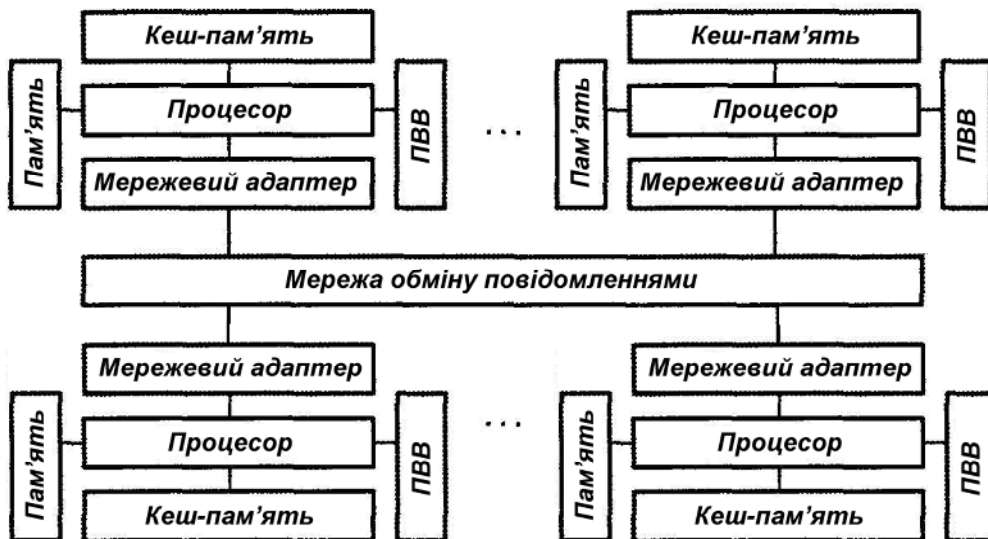
( )

Processing),

( , Massively Parallel

> 32

. 2.12.



. 2.12

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 MIMD  
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• , ;  
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 , ( ), —  
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 , , -  
 , ,  
 , , SPP1000/XA SPP1200/XA  
 , ,  
 , - ,  
 , .  
 , -  
 ; -  
 , Cray Jaguar, 224162 ,  
 6- Opteron, 1,75  
 ( $10^{15}$  ).  
 , IBM Roadrunner 12960  
 , PowerXCell 8i 3.2  
 , 1,04  
 .  
 , -  
 , , .  
 , ,



SNI/Pyramid RM1000)

( nCube).

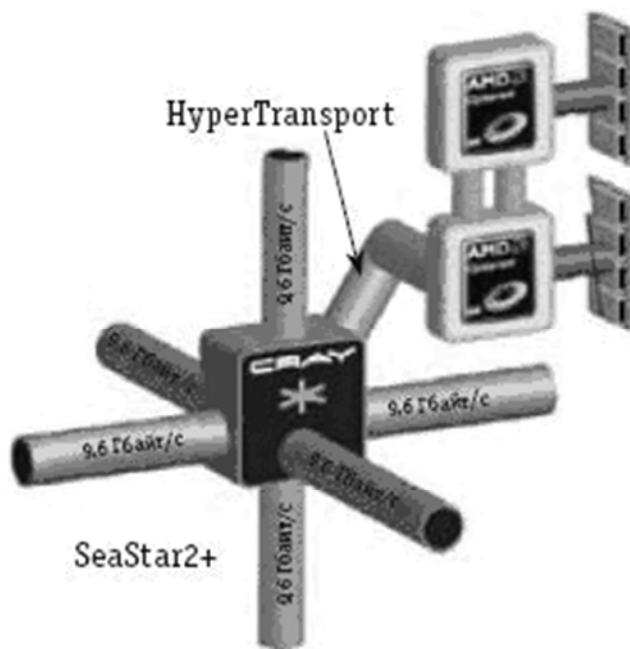
$D.$

$D \sim 2(m-1), \quad m -$

$D \sim n, \quad n -$

/ . , . , .  
 , Cray XT5 Cray XT6,  
 , Cray  
 Research,  
 , Roadrunner, , ,  
 Linpack 1 PFlop/s. 31-  
 ( 2008 ), TOP500 ,  
 , IBM  
 -  
 , Roadrunner 6120  
 AMD Opteron, 1.8 ,  
 12240 IBM Cell 8i, 3.2 ,  
 - TriBlades,  
 Infiniband. 560  
 , 226 .  
 - 2.35 , 437 MFlop/s/ .  
 IBM Roadrunner 133 .  
 , 1.376 PFlop/s, Linpack -  
 1.026 PFlop/s.  
 - TriBlade Opteron,  
 PowerXCell 8i , 16 , Opteron 16  
 , Cell. TriBlade LS21,  
 QS22. LS21 Opteron  
 16 , , 4 . QS22  
 PowerXCell 8i 8 , , 4 .

QS22 , PCI x8 LS21,  
 QS22. Infiniband 4x DDR.  
 - TriBlade , TriBlades  
 BladeCenter H.  
 49 , Cell 49  
 Opteron.  
 Roadrunner Red Hat Enterprise Linux  
 xCAT.  
 RoadRunner  
 .  
 ,  
 , Cray XT53-  
 XT6.  
 Cray XT3 Opteron ,  
 2005 , ,  
 . XT4 Opteron 2007 .  
 5 Barcelona/2,6 ( Shanghai/2,7  
 ) Istanbul/2,6 , 2008 .  
 , XT5 .  
 Cray Cray XT6 -  
 Opteron 6100.  
 - Cray XT , ,  
 , , « » SeaStar  
 . Cray XT5  
 6. 5 ASIC- ,  
 , SeaStar2+ , Opteron  
 DIMM ( .2.13). Cray XT4  
 , .



. 2.13

SeaStar2+

Opteron

HyperTransport 2.0 (

Opteron).

(

8

32

70 GFLOPS

124

GFLOPS.

XT5 – (Blaid),

Istanbul

500 GFLOPS;

1 2 2.

1 4 24 (24

)

12 TFLOPS

, 1,54

(

16

),

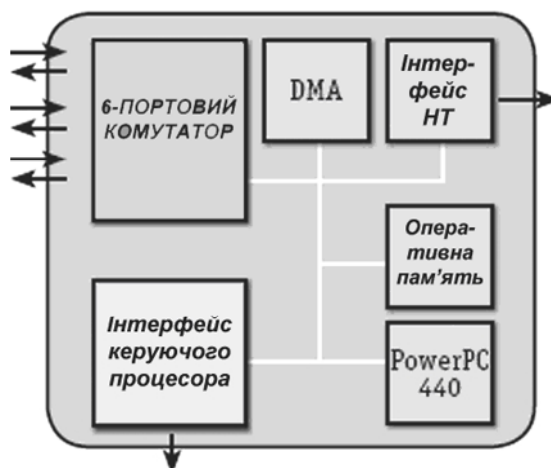
XT5

–

25 32 24.

2 PFLOPS

300  
5,  
2 , 700 0,6 1,4  
43 .



.2.14

SeaStar2+

SeaStar2+

( .2.14),

9,6 / .  
57,6 / ,  
2 .

HyperTransport, SeaStar2+

(Direct Memory Access, DMA),

SeaStar2+

Cray XT4

XT5, Cray XT5 – XT6 (

), 6 , ,

Cray Gemini,

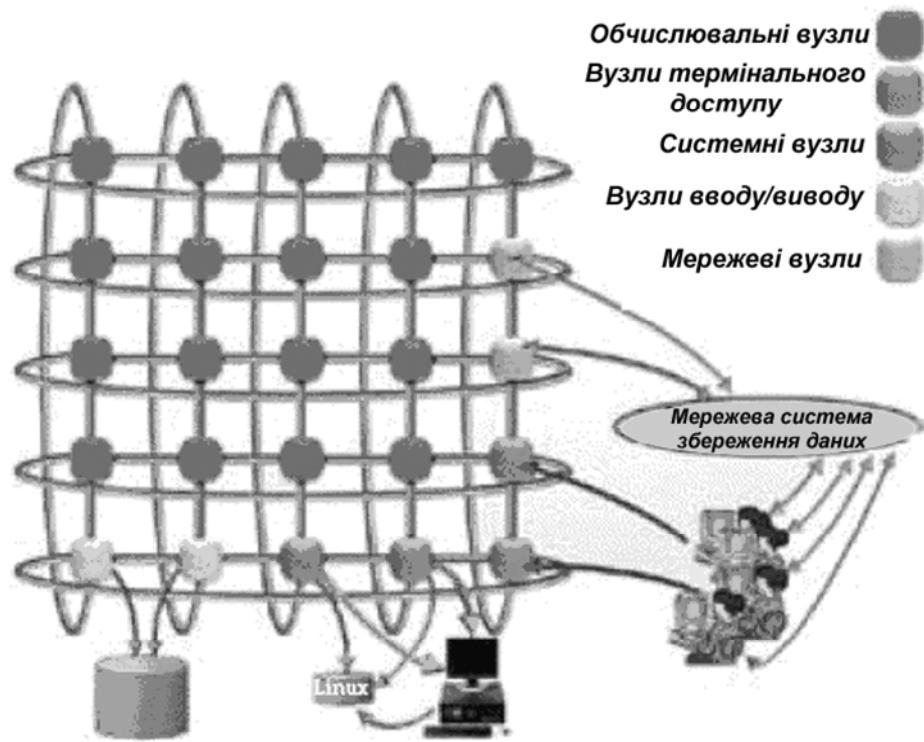
Baker.

AMD Bulldozer 12 16 ,

32-

( .2.15), « » PCI-Express. Gigabit Ethernet, 10 Gigabit Ethernet, Infiniband, Fibre Channel. Fibre Channel SATA.

, Cray XT. , Cray SIO- (Storage-I/O).



.2.15

Cray XT

. Cray Lustre -

; , 1 ray Sun Microsystems.



(L1)

(L2)

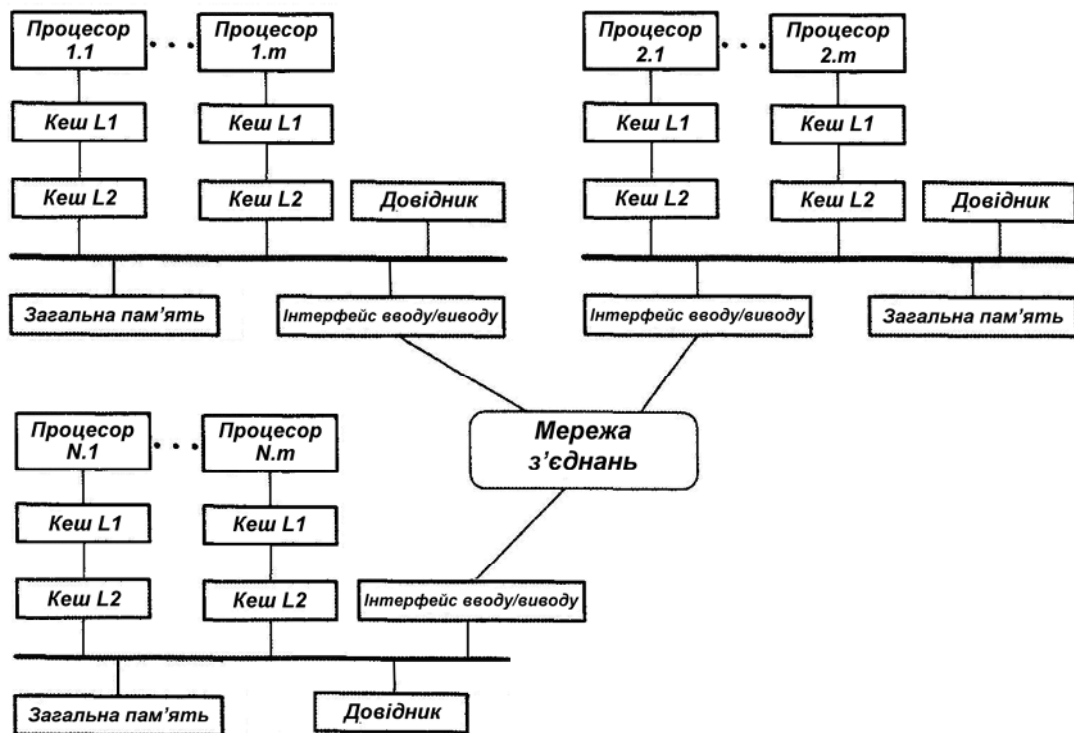
CC-NUMA

Silicon Graphics Origin

MIPS R10000,

Sequent NUMA-Q

Pentium II.



. 2.16.

CC-NUMA

CC-NUMA,



.  
 , , , ,  
 .  
 - ' .  
 — .  
 , ,  
 , , - ' .  
 ,  
 , , - ' .  
 . 3 2 ( 2.3)  
 798, 1. :  
 1. 2.3 2  
 798.  
 2. 2 ,  
 1.  
 3. 2 1,  
 1.  
 4. ' 1, 2.3,  
 798, .  
 5. ' 1 ,  
 1.  
 6. 1 .  
 7. ,  
 2.  
 8. 2 2,  
 , ,  
 .  
 9. - ' 2.3  
 2.3.

## 2.2

### 2.2.1

(point-to-point).  $N$ ,  $(x, y)$

(source node)  $(x, y)$   $(x_c, y_c)$   $(x_r, y_r)$

(recipient node)  $(x, y)$   $(x_c, y_c)$   $(x_r, y_r)$

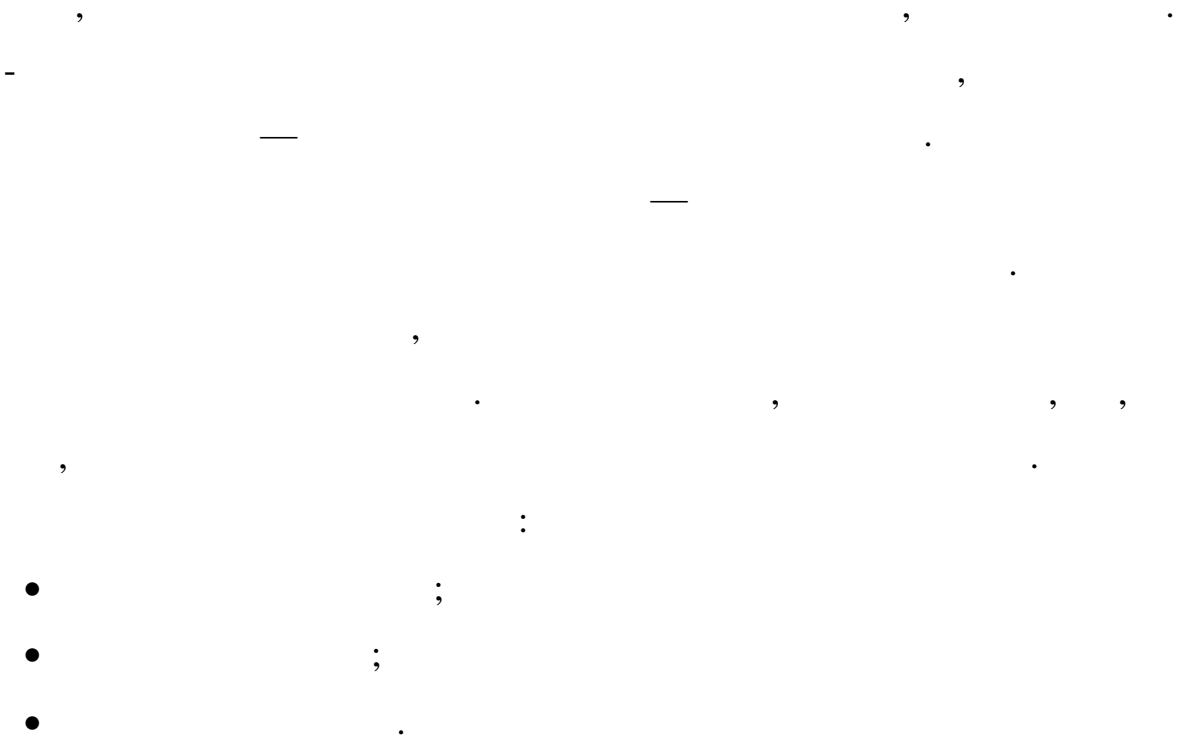
$(f_c, f_{xy})$   $(t_c, t_{xy})$   $(l)$   $(v): l = vt_c$

$b$   $b_c = wf_c$

(direct networks)

(indirect networks)

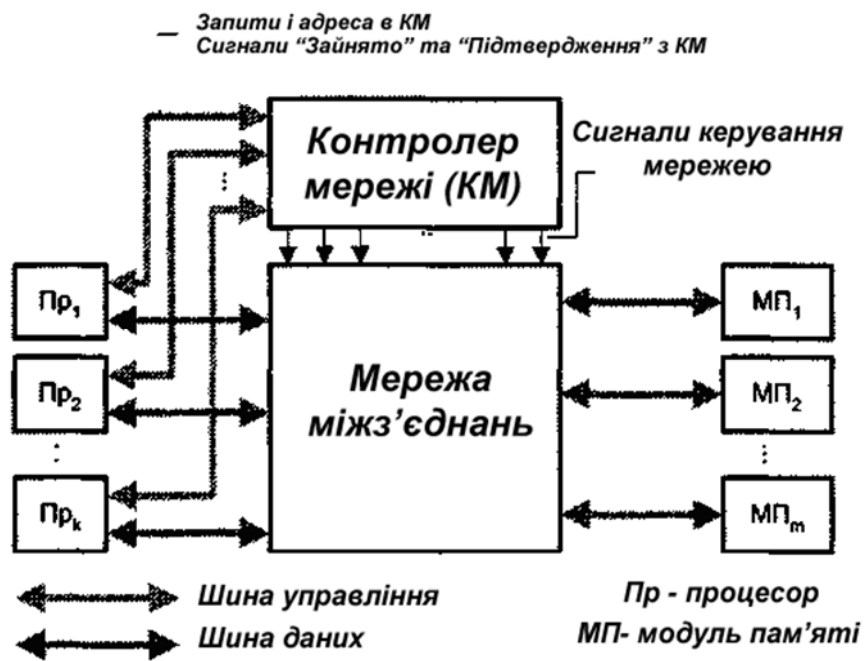
. ( << >> << >> >> << >>.)



—  
,  
( ),

» (tail drop),

( . 2.17).



.2.17

### 2.2.2

:

- (N);
- (I);
- (D);
- (d);
- (W);
- (T);
- (Q);
- (B);
- (b).

(network size)

(number of links) —

(network diameter),

$$= \{c_1, c_2, \dots, c_n\}$$

( , ),

D — ( , )

, « »

$$C_{lx} = l_x U_0, \quad l_x = \{ : r = \} —$$

,  $aC_{0x} = \{c:s_c = x\} —$

$$| C_{lx} / \quad | \quad 0x /$$

( , , ),

4096 (2<sup>12</sup> 64<sup>2</sup>),

126, — 12.

(network bandwidth)

(network latency) — ,

.

,

.

(network connectivity)

,

.

,

.

(bisection width).

(cut of network).  $C(N_1, N_2)$  — ,

N

$N_1$

$N_2$ .

$(N_1, N_2)$  — ,

$N_1$ ,

$N_2$ .

—

,

,  $|N_2|$   $|N_1|$   $|N_2| + 1$ .

,

:

$$B = \min_{bisection} |C(N_1, N_2)|. \tag{2.1}$$

,

,

,

.

(bisection bandwidth) —

.

,

,

,

,

.

$b$

$$b = \min B(N_1, N_2).$$

$b$

$$: b = b_c \times .$$

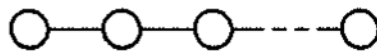


### 2.2.3

- ( ) ;
- ( , , , , ) ;
- ( ' , , ) ;

( . 2.18).

:  $D=N-1$ ;  $d=2$ ;  $I=N-1$ ;  $=1$ .



. 2.18.

1,

2.

( 2.19, ).

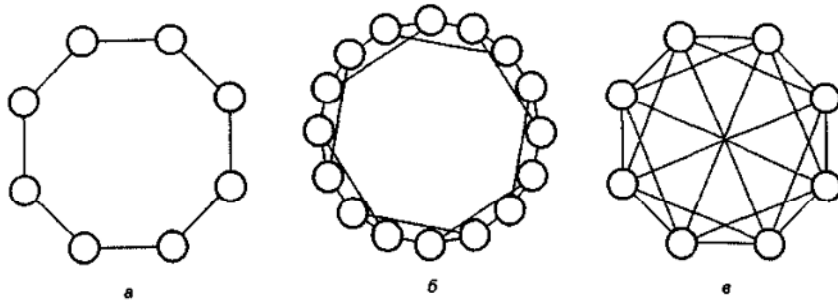
( )

$$D = \min \left[ \frac{N}{2} \right]; d = 2; I = N; B = 2.$$

1 (N/2)-1,

2.19,

3.



.2.19.

: — ; — ;

Ring,

IBM,

Token  
KSR 1 S I.

. 2.3,

$N$

$i \ j,$

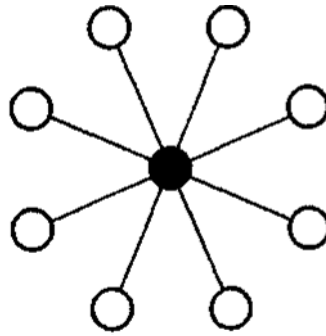
$|i - j|$

2.

2.20).

1;  $B=1$ .

— ( .  
:  $D = 2$ ;  $d = N - 1$ ;  $I = N -$

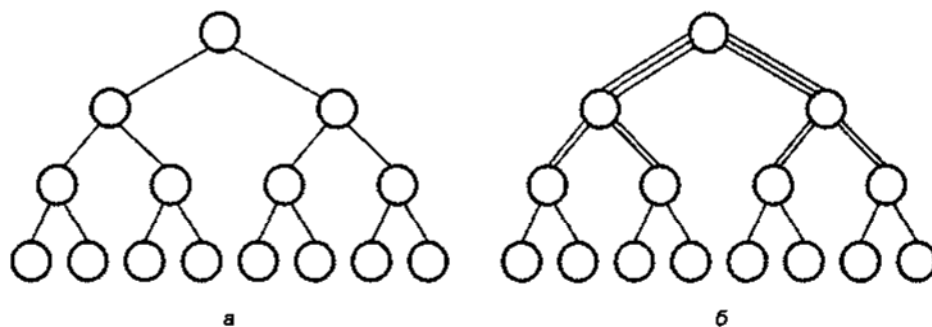


. 2.20

2.21, ).

$h$  — ( ),  
 $\max[\log_2 N]$ ,  
 $: D = 2(h-1); d = 3; I = N-1; B=1.$  ,  
 262 144 ( )  
 512, — 36.

DADO 1023 , .



.2.21.

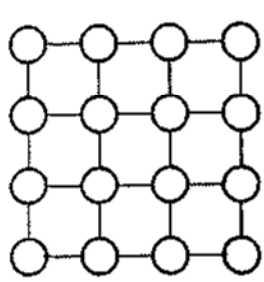
— ; — « » :

« » ( .2.21, ).

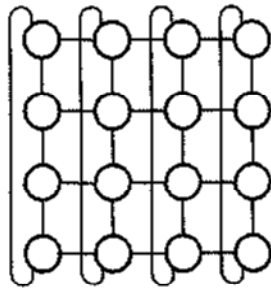
« »

-5.

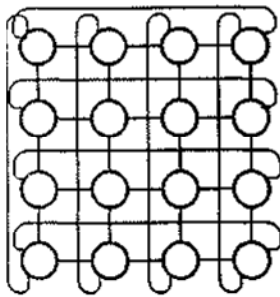
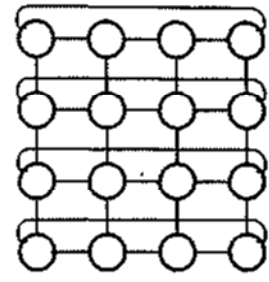
(mesh),



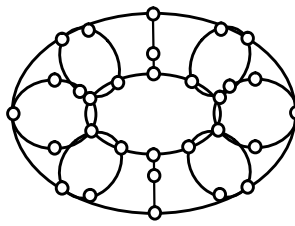
a



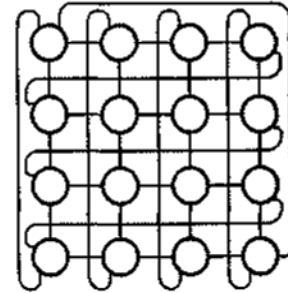
b



d



e



f

. 2.22.

- — ; — ;  
: — ; —

( . 2.22, ).

$$: D = 2( - 1); d = 4; I = 2N-2m; B = .$$

(wraparound)

( . 2.22, ).

( . 2.22, ).

$$D = 2 \min \left[ \frac{m}{2} \right]; d = 4; I = 2N; B = 2m. \tag{2.2}$$

4x8

. 2.22, .

(twisting).

ILLIAC. . 2.22,

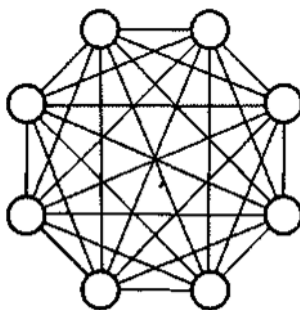
$$: D = m - 1; d = 4; I = 2N; = 2 .$$

Cray T3D,

: ILLIAC IV, , DAP, CM-2, Paragon .

,  
 , ( . 2.23),  
 « »  
 . , N ,  
 :

$$D=1; d=N-1; I=\frac{N(N-1)}{2}; B=\frac{N^2}{4}. \quad (2.3)$$



.2.23 ,

, ,  
 . , ,  
 , ,  
 N - 1 .  
 , ,

, .2.24. , ( .  
 2.24, ), . ,  
 ( .2.24, ), — , 8 ( .2.24, )

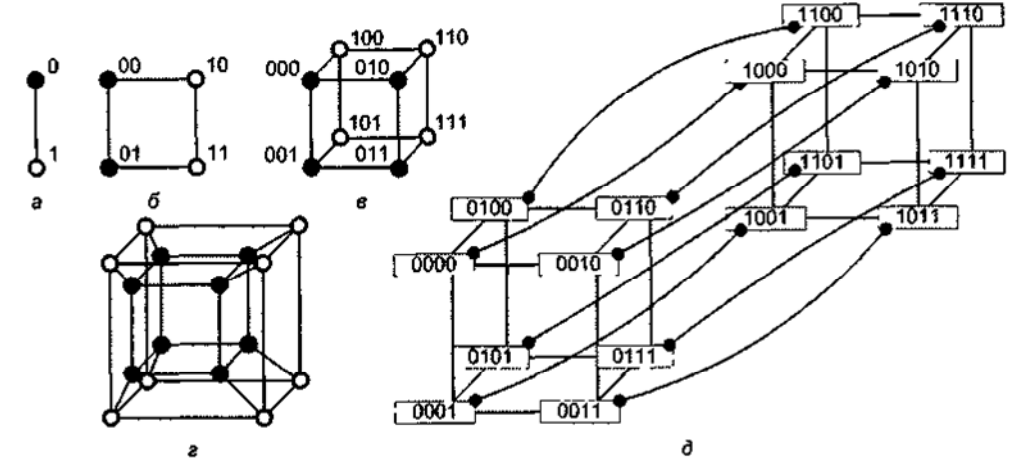
—  
 m- : ( -I) - ,  
 ,

(N= 2^m)

D = m; d = m; I = mN/2; B = m. (2.4)

1

0010  
0110 — , 0110 0101 — .  
m- ( - 1) - . 2.8, .  
( - 1) - ( -  
1) -



. 2.24.

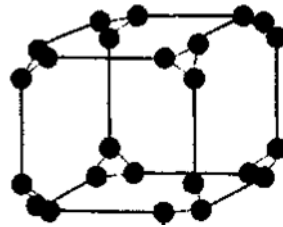
— ; — ; — ; — ; — ; — ; —

m-



$i$ - $0$  .  $i$ - $0$  ,  $(i-1)$ -  
 $i$ -

, ( 2.25).



2.25

$k$ -  $n$ -

$k$  ( $N = k^n$ ).

$n$ -  $k$ ,

$n$ -  $k$   $k$ - (

$-1)$ -

$k$ -  $n$ - :

$k$ - 1- — ;

$k$ - 2- — ;

$k$ - 3- — ;

4- 2- — 4x4;  
 2- n- — .

,

,

$k$

. 2.2.

### 2.2.

|         |               |     |                    |                   |  |                          |
|---------|---------------|-----|--------------------|-------------------|--|--------------------------|
|         |               |     |                    |                   |  | -                        |
|         | 1             | N-1 | $\frac{N(N-1)}{2}$ | $\frac{N^2}{4}$   |  | N                        |
|         | 2             | 1   | N-1                | 1                 |  | N                        |
|         | 2(h-1)        | 3   | N-1                | 1                 |  | h = log <sub>2</sub> N   |
|         | N-1           | 2   | N - 1              | 1                 |  | N                        |
|         | $\frac{N}{2}$ | 2   | N                  | 2                 |  | N                        |
|         | 2(m-1)        | 4   | 2N-2m              | $\sqrt{N}$        |  | m =                      |
|         | 2[m/2]        | 4   | 2N                 | 2                 |  | m = $\sqrt{N}$           |
|         |               |     | $\frac{nN}{2}$     | $\frac{N}{2}$     |  | N = log <sub>2</sub> N ; |
| -<br>n- | n[k/2]        | 2n  | nN                 | 2k <sup>n-1</sup> |  | N = k <sup>n</sup>       |

### 2.2.4

,

,

.

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,

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,

( ),

,

(two-sided).

(one-sided).

.

-

-

( — permutations).

(multicast),

( )

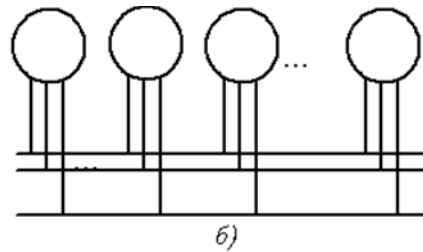
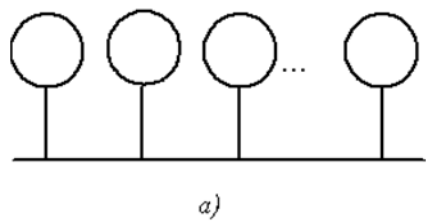
*n*

:

(strictly non-blocking)  
blocking).

(wide sense non-





. 2.26

: -

; -

(« »)

(crossbar

switch system)

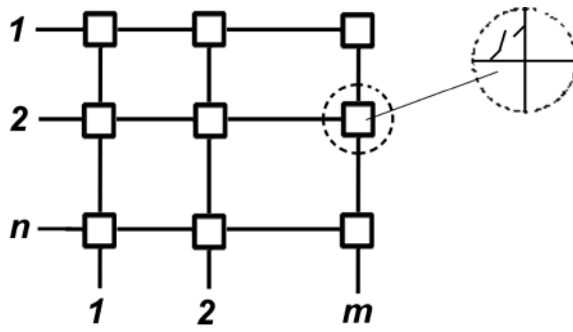
( )

« »,

( )

( . 2.27) -

min( , ).



.2.27

$n \ m$

«  $n^2$  »  
 1, —  $n/2$ .  
 ( )

256

VPP500 224 224.

Fujitsu

« »  $2 \times 2$ .

( . 2.28).

( ) -

:

.

,

.

.

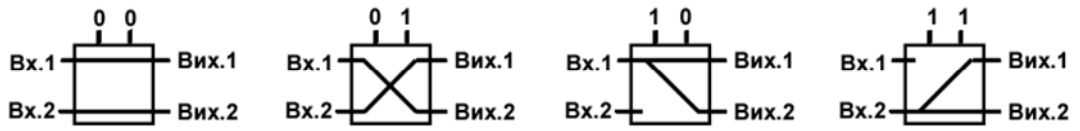
.

-

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. 2.28.

-

. 2.29.

-

( )

-

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,

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-

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.

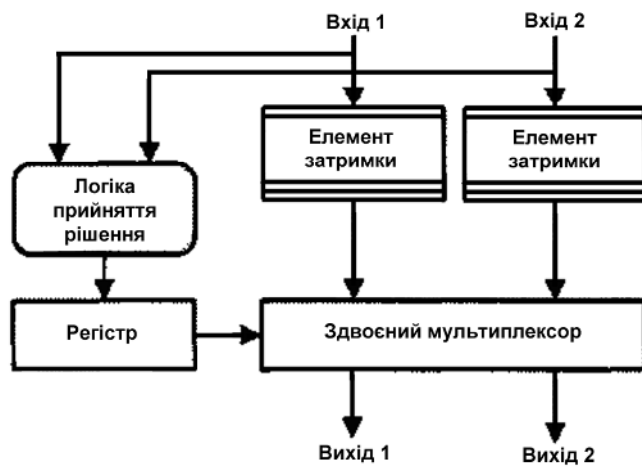
,

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-

,

.



. 2.29

### 2.2.5



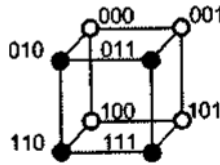
$N$ ,  $\dots$ ,  $= \log_2 N$ .  
 $b_i$ .

(exchange)

:

$$E_k(b_m, \dots, b_k, \dots, b_1) = (b_m, \dots, \bar{b}_k, \dots, b_1), \quad 1 \leq k \leq m. \quad (2.5)$$

( . 2.30).



. 2.30

(shuffle)

:

(perfect shuffle),

(unshuffle),

$i$ -

( $i^{\text{th}}$  subshuffle)

$i$ -

( $i^{\text{th}}$  supershuffle).

, . 2.31 —

I

:

$$S(b_m, b_{m-1}, \dots, b_1) = (b_{m-1}, b_{m-2}, \dots, b_1, b_m). \quad (2.6)$$

$i$

$i \ j$

$j$

—

:

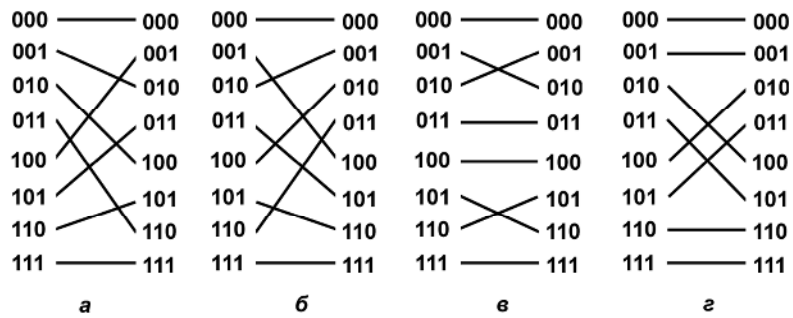
$$U(b_m, b_{m-1}, \dots, b_1) = (b_1, b_m, \dots, b_2). \quad (2.7)$$

$i$ - :

$$S_i(b_m, b_{m-1}, \dots, b_i, \dots, b_1) = (b_m, \dots, b_{i+1}, b_{i-1}, \dots, b_1, b_i). \quad (2.8)$$

$i$ - :

$$S^i(b_m, b_{m-1}, \dots, b_i, \dots, b_1) = (b_i, b_{m-1}, \dots, b_{i+1}, b_m, b_{i-1}, \dots, b_1). \quad (2.9)$$



. 2.31.

$m = 3$ :

— ; —  
— ; —

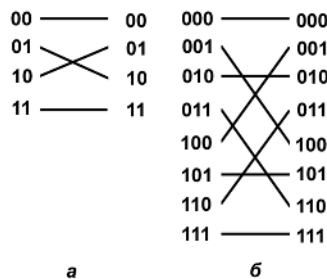
« » (butterfly) — « »

60-

( . 2.32).

:

$$B(b_m, b_{m-1}, \dots, b_1) = (b_1, b_{m-1}, \dots, b_2, b_m). \quad (2.9)$$



. 2.32

« » :

-  $m = 2$ ; -  $m = 3$ .

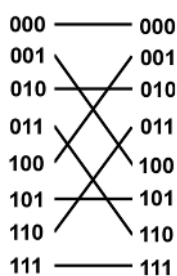
« »  
 ,  
 « » « »- .  
 ,  
 :

$$R(b_m, b_{m-1}, \dots, b_1) = (b_1, b_2, \dots, b_m). \quad (2.9)$$

= 3 . 2.33.

= 3

« », .



. 2.33

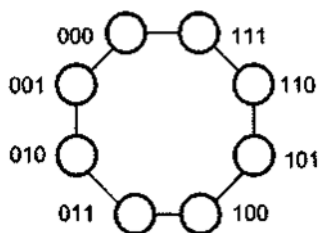
(m = 3)

:

$$SX(x) = (x+1) \bmod N \quad (N = 2^m) \quad (2.9)$$

= 3

( . 2.34).



. 2.34

(m = 3)

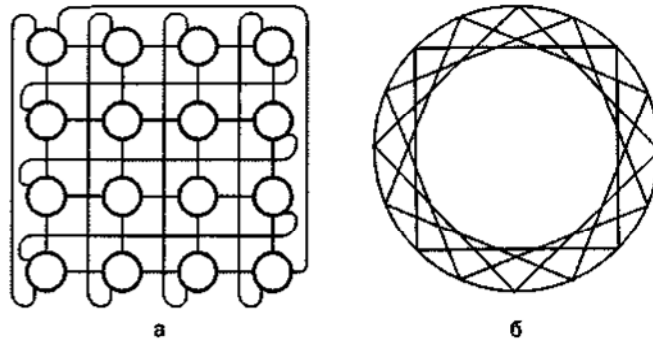
# ILLIAC IV

« »

ILLIAC IV,

ILLIAC IV:

$$\begin{aligned}
 R_{+1} &= (i+1) \bmod N; \\
 R_{-1} &= (i-1) \bmod N; \\
 R_{+r} &= (i+r) \bmod N \quad (0 \leq i \leq N-1); \\
 R_{-r} &= (i-r) \bmod N \quad (r = \sqrt{N}).
 \end{aligned}
 \tag{2.10}$$



. 2.35

ILLIAC IV:

$$N=4$$

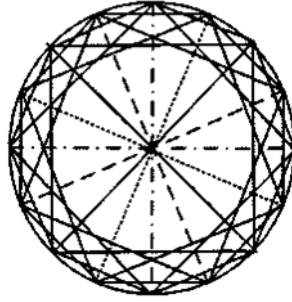
. 2.35.

4.

(barrel shift)

$$\begin{aligned}
 B_{+1}(j) &= (j+2^i) \bmod N, \\
 B_{-1}(j) &= (j-2^i) \bmod N, \quad 0 \leq j \leq N-1, \quad 0 \leq i \leq \log_2 N-1.
 \end{aligned}
 \tag{2.11}$$

. 2.36.



. 2.36

$\pm 2i$



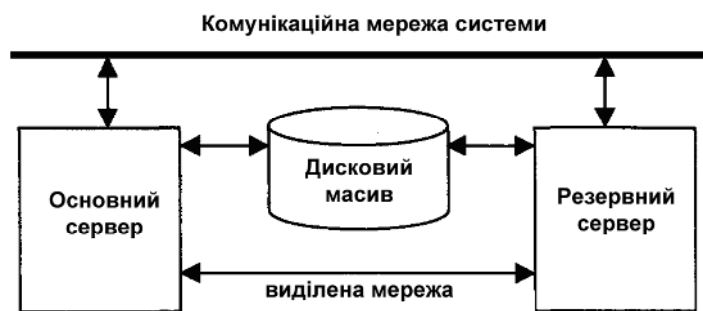
Serial Attached SCSI (SAS),

3 / ,

10

/ .

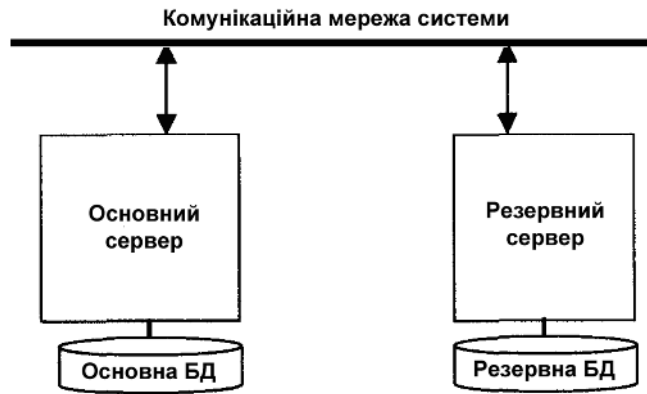
. 2.37.



. 2.37

( )

. 2.38.



. 2.38

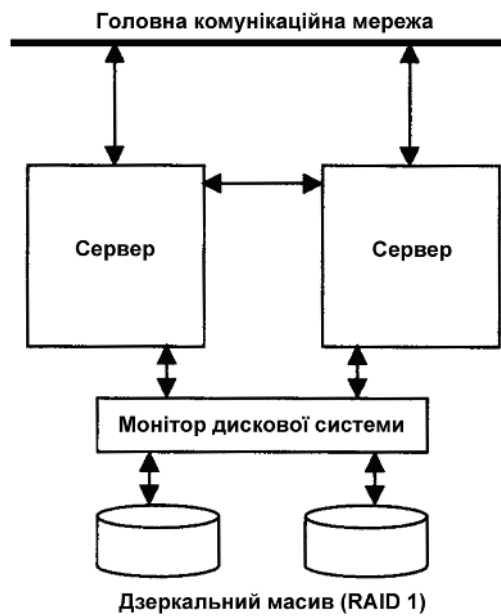
. 2.39.

Teradata, Oracle Informix,

nothing.

sharing





.2.39

### 2.3.2

(HDD).

RAID.

«RAID» (Redundant Arrays of Inexpensive

Disks)

«

».

RAID

1987

RAID –

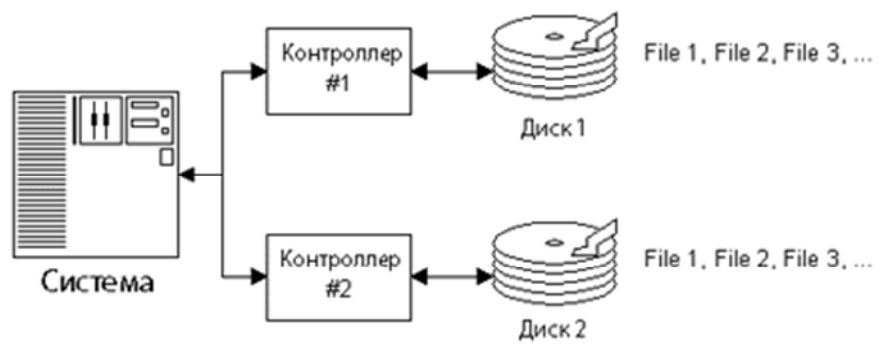
RAID-

RAID  
IDE/ATA SATA

RAID (Array),  
(Mirroring), (Duplexing), (Striping)  
(Parity).

RAID

RAID ( .2.40).



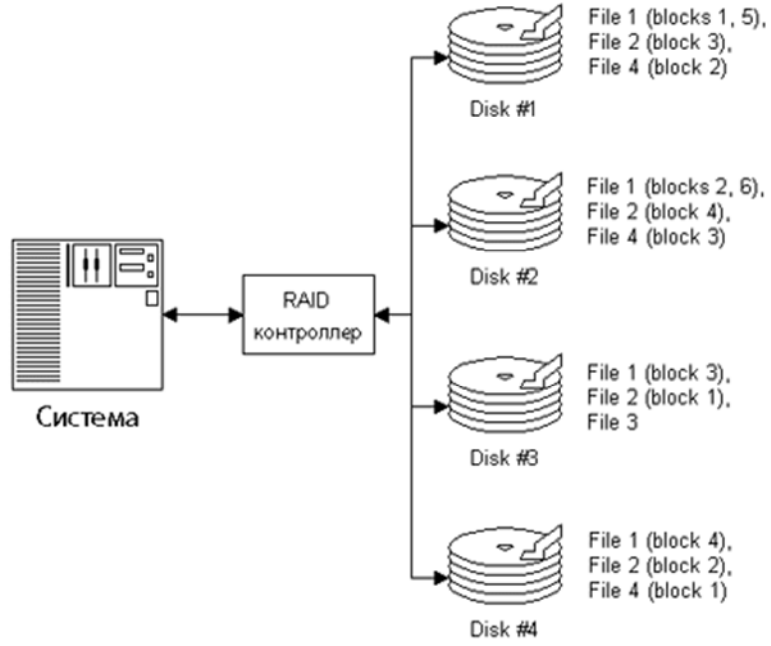
.2.40

RAID-

( .2.41) .

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« » – 1 ,  
– 512 ( ).

( ) ( ) .



. 2.41

$N$

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$N+1$

RAID-

$N+1$

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$N$

XOR.

XOR

$$(A \text{ XOR } B) \text{ XOR } B = A.$$

5

20%

RAID-

, RAID

(single)

(multiple)

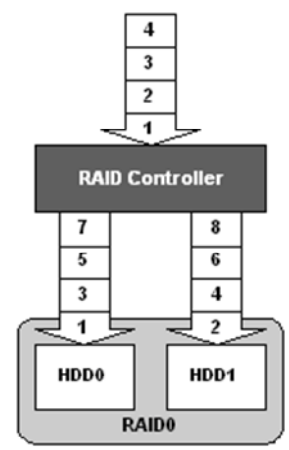
RAID.

8-

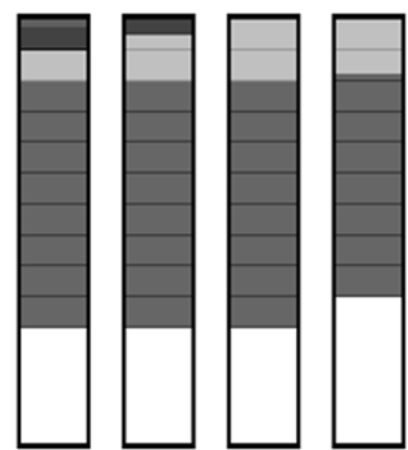
### RAID 0

( .242) .

( , 16 )



a)



b)

. 2.42 RAID 0. a) -

, ) -

RAID 0 4  
 - 16  
 - 20  
 - 100  
 - 500

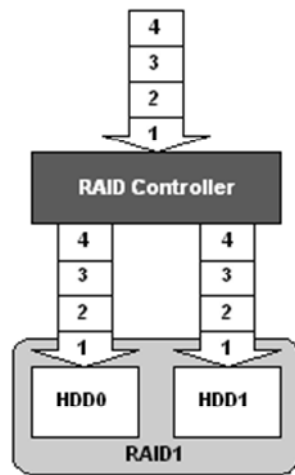
RAID 0

RAID-

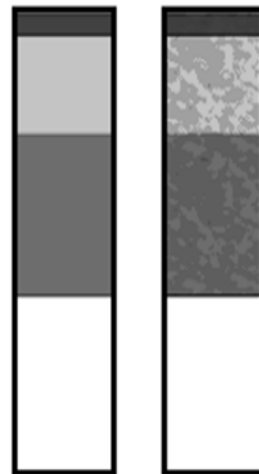
### RAID 1

( .2.43) .

RAID.



a)



b)

. 2.43 RAID 1: a) -

, ) -

### RAID 1

RAID 1

## RAID 2

RAID

(.2.44).

14

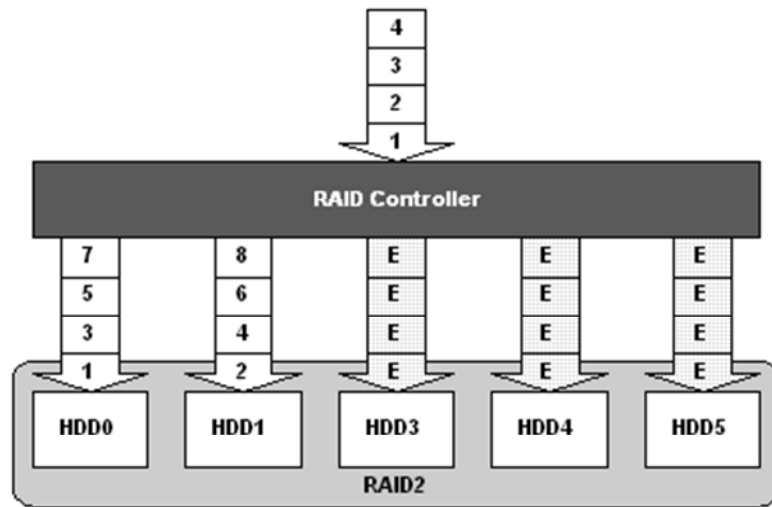
39

(!).

(10

32)

## RAID 2



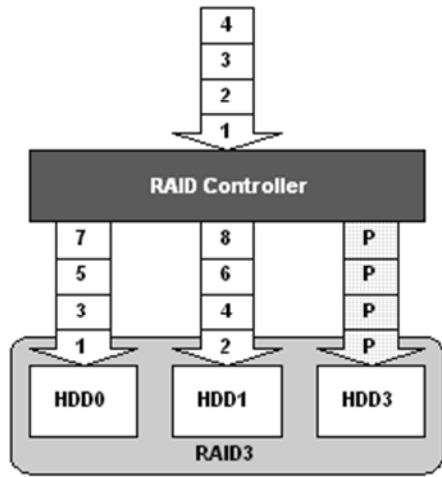
.2.44

RAID 2

## RAID 3

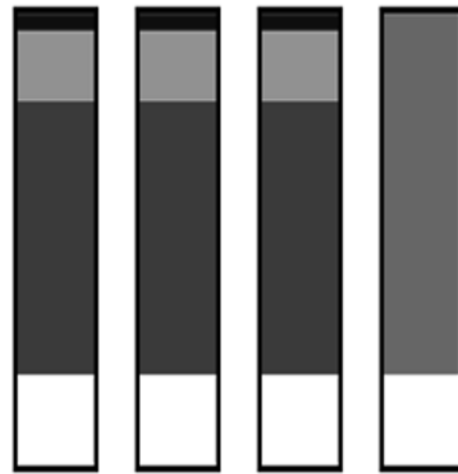
(.3.16).

1024



a)

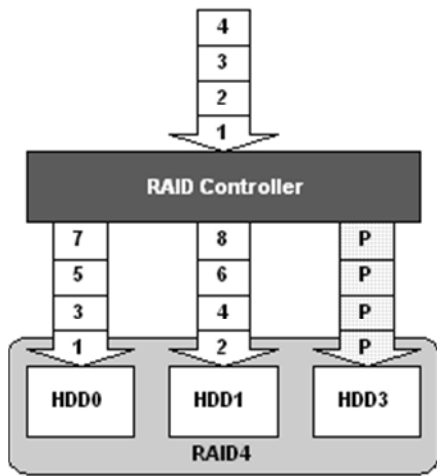
. 2.45 RAID 3: a) -



б)

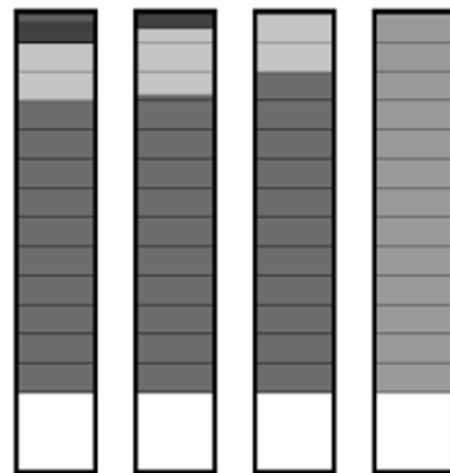
, ) -

### RAID 4



a)

. 2.46 RAID 4: a) -



б)

, ) -

### RAID 3

( .2.46) .

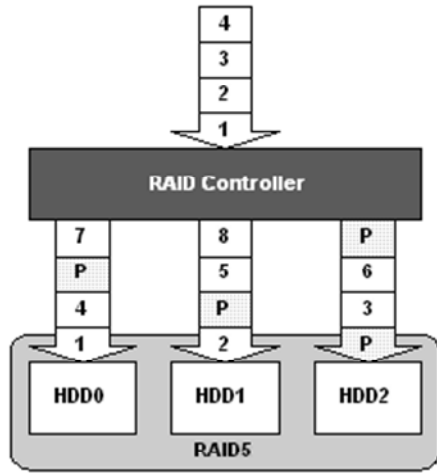
,



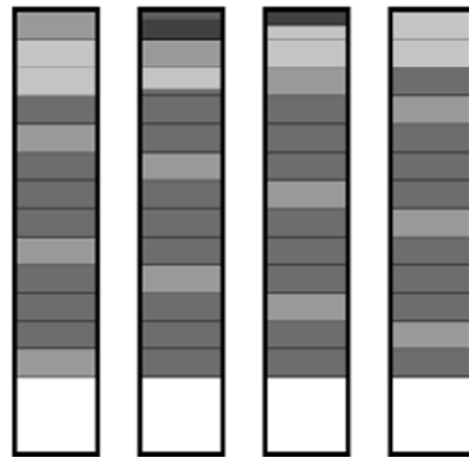
« »

RAID 3 RAID 5,

### RAID 5



a)



b)

.247 RAID 5: a) -

, ) -

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( .247) .

RAID 3,

RAID

**RAID 6**

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**RAID 5**

**RAID 6**

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**RAID 7**

, RAID 7 ,

**RAID 3**

Storage Computer Corporation.

, RAID 3,

, RAID 7

**RAID**

RAID

RAID,

RAID -

RAID 0

RAID 1, 3 5.

RAID

(set).

X+Y

RAID X,

RAID X

RAID Y.

RAID 0+1 (01) 1+0 (10)

RAID 0+1

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RAID 1+0 –

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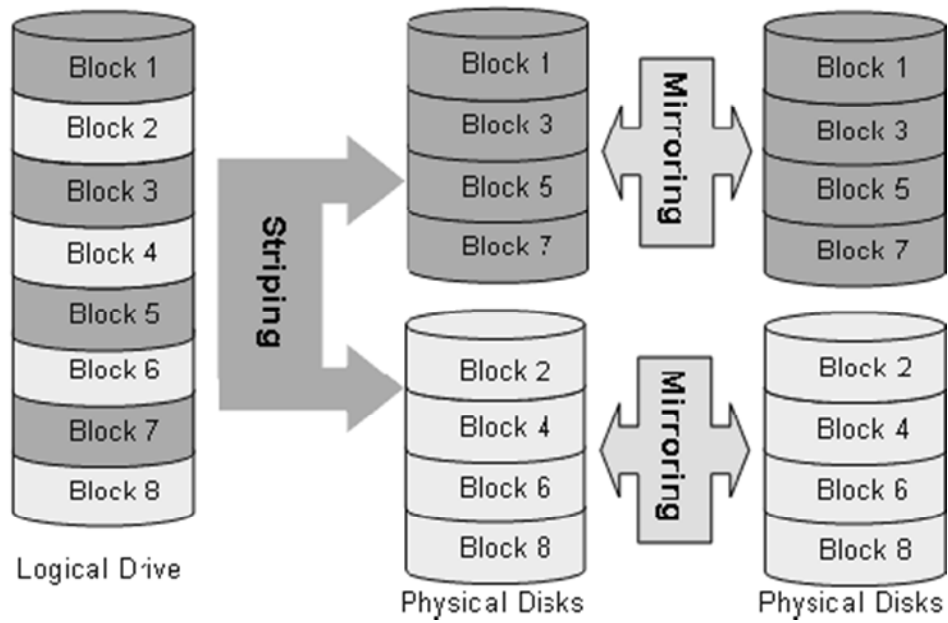
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( .248),



. 2.48

RAID 0+1

RAID 0+1

RAID

RAID 1+0.

10

(5 2)

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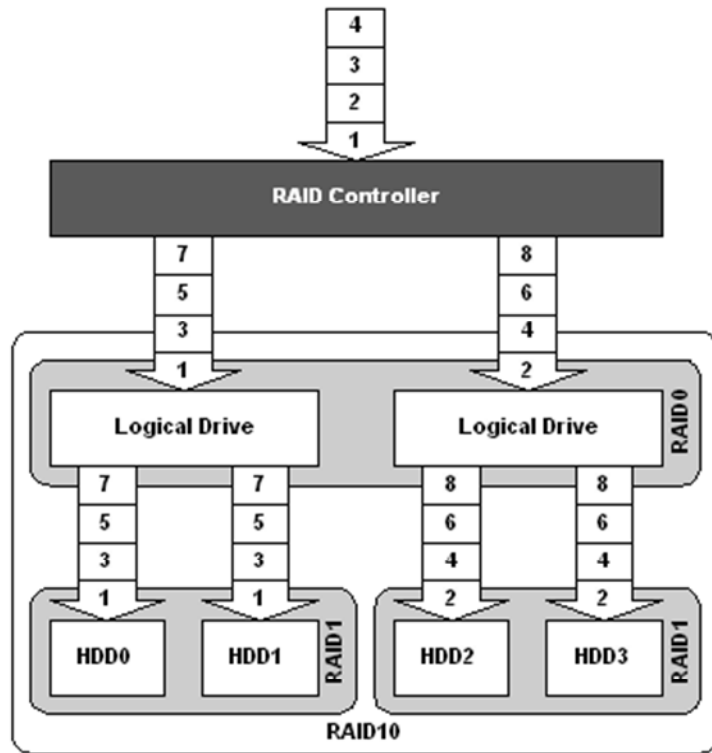
!

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50%.

RAID 01

( .2.49) .



. 2.49

RAID 1+0

RAID 0+3 (03) 3+0

0+3 3+0

5+3 (53).

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RAID 3

3+0.

RAID 3

- 6. RAID 3+0

0+3.

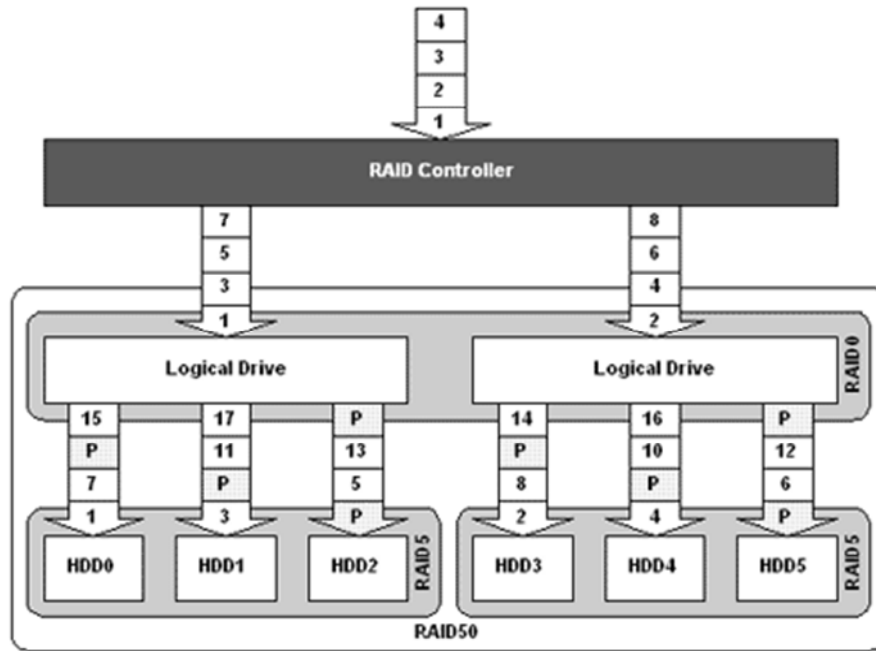
RAID 0+5 (05) 5+0 (50).

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. RAID 0+5

RAID 5.

RAID 5+0 ( .2.50).



. 2.50

RAID 5+0

RAID 5,

WEB-

RAID 1+5 (15) 5+1 (51)

RAID 15 51 -

1+5

, 5+1 -

RAID 5+1

RAID 5,

## RAID

2.3

RAID-

2.4

RAID-

2.3 -

RAID-

|                               | RAID 0      | RAID 1        | RAID 3                | RAID 5                | RAID 6                           |
|-------------------------------|-------------|---------------|-----------------------|-----------------------|----------------------------------|
| Технологія                    | Чергування  | Дзеркалювання | Чергування, парність  | Чергування, парність  | Чергування, парність             |
| Контроллер                    | Програмний  | Програмний    | Апаратний             | Апаратний Ні-End      | Спеціалізований                  |
| К-ть жорстких дисків          | 2, 4        | 2             | 3 і більше            | 3 і більше            | 3 і більше                       |
| Доступний робочий простір %   | 100         | 50            | 66 для 3,<br>75 для 4 | 66 для 3,<br>75 для 4 | 33 для 3<br>50 для 4<br>60 для 5 |
| Стійкість при відмові диска   | Немає       | Висока        | Висока                | Висока                | Дуже висока                      |
| Відновлення даних             | Немає       | Дуже швидке   | Швидке                | Швидке                | Дуже швидке                      |
| Швидкість випадкового читання | Дуже хороша | Хороша        | Хороша                | Дуже хороша           | Дуже хороша                      |
| Швидкість випадкового запису  | Дуже хороша | Хороша        | Погана                | Нормальна             | Погана                           |
| Швидкість лінійного читання   | Дуже хороша | Хороша        | Дуже хороша           | Дуже хороша           | Хороша                           |
| Швидкість лінійного запису    | Дуже хороша | Хороша        | Хороша                | Хороша                | Середня                          |
| Ціна                          | Найнижча    | Низька        | Середня               | Середня               | Висока                           |

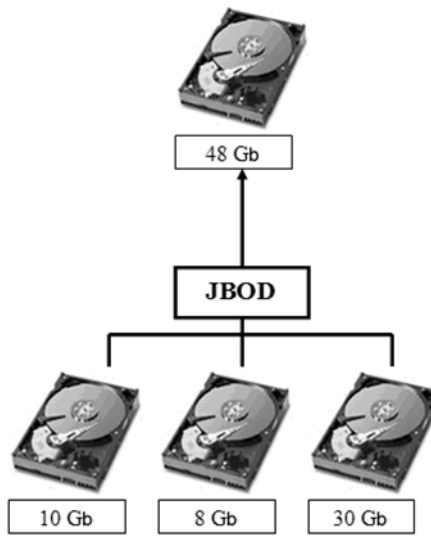
## JBOD

?

?

RAID, JBOD – Just A Bunch Of Disks.

( 2.51).



**. 2.51 JBOD**

**JBOD**

?

**2.4 -**

**RAID-**

|                               | <b>RAID 0+1</b>           | <b>RAID 1+0</b>           | <b>RAID 5+0</b>              | <b>RAID 5+1</b>                     |
|-------------------------------|---------------------------|---------------------------|------------------------------|-------------------------------------|
| Технологія                    | Чергування, дзеркалювання | Чергування, дзеркалювання | Чергування, парність         | Чергування, парність, дзеркалювання |
| Контроллер                    | Програмний                | Майже все                 | Спеціалізований              | Спеціалізований                     |
| К-ть жорстких дисків          | 4 min                     | 4 min                     | 6 min                        | 6 min                               |
| Доступний робочий простір %   | 50                        | 50                        | 66 для 2 страйпів по 3 диски | 33-40                               |
| Стійкість при відмові диска   | Дуже хороша               | Відмінна                  | Хороша                       | Відмінна                            |
| Відновлення даних             | Швидке                    | Дуже швидке               | Середнє                      | Швидке                              |
| Швидкість випадкового читання | Дуже хороша               | Дуже хороша               | Дуже хороша                  | Дуже хороша                         |
| Швидкість випадкового запису  | Хороша                    | Хороша                    | Хороша                       | Хороша                              |
| Швидкість лінійного читання   | Дуже хороша               | Дуже хороша               | Дуже хороша                  | Дуже хороша                         |
| Швидкість лінійного запису    | Хороша                    | Хороша                    | Хороша                       | Хороша                              |
| Ціна                          | Відносно висока           | Відносно висока           | Висока                       | Дуже висока                         |

1. ' IMD. SMP- ,
2. SMP- .
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7. , .
8. ' .
9. . , , .
10. ILLIAC. , ,
11. : , , .
12. : , ' .
13. : , .
14. , .
15. , .
16. RAID0. RAID1, RAID2, RAID3 .
17. RAID4, RAID5. .
18. RAID6, RAID7. .
19. RAID0+1 RAID3+0.
20. RAID0+1 RAID3+0.
21. RAID5+0 RAID5+1. JBOD.





3. ,  
( , ).

4. .

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6. .

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1. .

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5. , (HDD)

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IBM PC,

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MS Windows 3.x Windows 95,

RT-11,

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VMWare

i 86 VM

System/370

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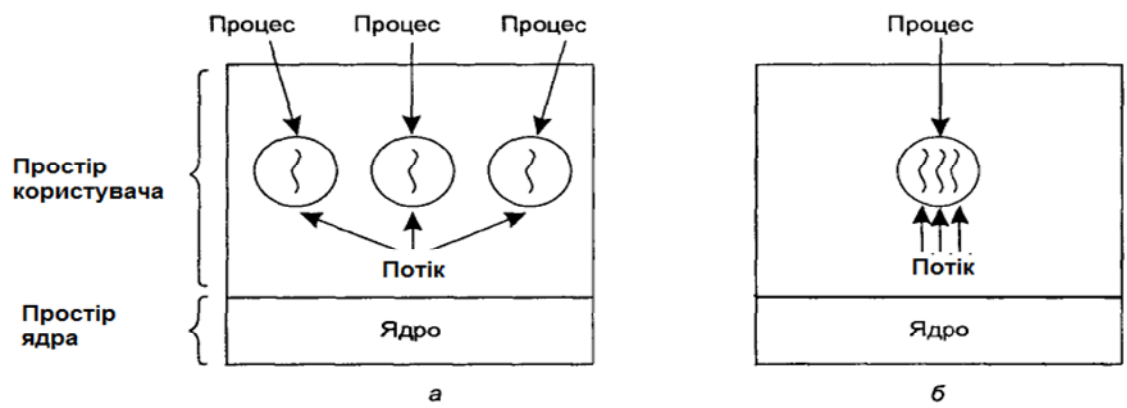
7. .

1-3

, 4-7 - .

## 3.2.

### 3.2.1



.3.1. )

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 ,
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 .
 : , ,
 , .
 , (shared memory)
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 , , , ,
 ,
 (message passing)
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 ,
 (messages).
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,  
, (mapped memory).  
,  
,  
,  
, (memory-mapped  
files).

### 3.2.2



### 3.2.3

(communication channel).

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, ( , ).

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: **send** (

) **receive** ( ).

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send receive

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send, ?

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receive, ?

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send

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send

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### 3.2.4

(named pipes)

(message queues).





(datagram sockets) —  
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, ( ) ,  
, - ( ,  
socket ( ).

1. , bind(  
) ). UNIX ,

2. listen( ) ,

3. listen( )

accept( ).

accept( )

connect( ).

bind( ).

send(),

— recv().

(Remote Procedure Call, RPC)

1. send

2.

RPC

3. receive

4.

5.

### 3.3.

#### 3.3.1

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1. , , , ,  
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,  
1 3, 10 5 10<sup>8</sup> .  
, 200  
, 10<sup>11</sup>  
10 10- ,  
100 Mflops (10<sup>8</sup> )

10<sup>7</sup> 100 . ,  
 10 , 1.7 Tflops (1.7X10<sup>12</sup>)  
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N<sup>4</sup> N<sup>5</sup>, N .  
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4. , , , ,  
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- ( : , , , . .);
- : (PVM, MPI); (Dynamite, Globus ).

### 3.3.2

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#### 3.1.

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|--|---|---|-------|
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|  |   |   | MIMD- |
|  |   |   | SIMD- |
|  | - |   |       |

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(Scheduler)

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pi

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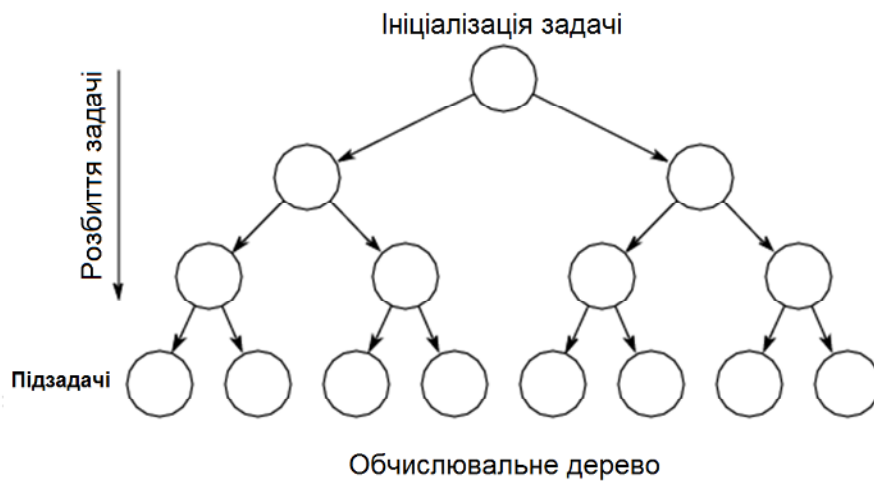
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cix

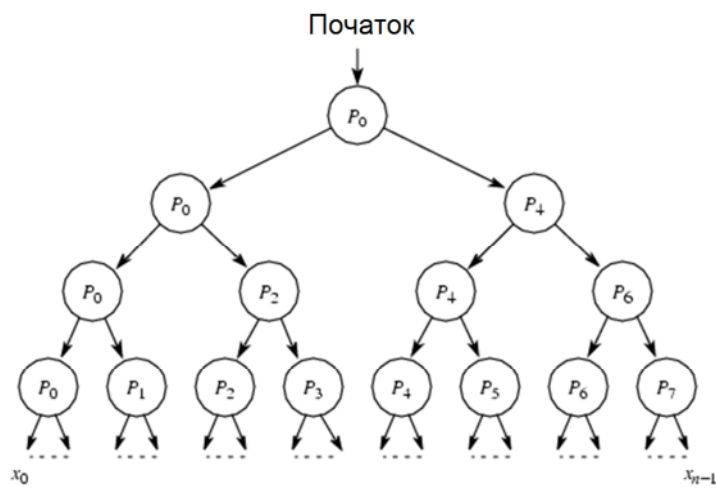
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( 3.2-3.4).

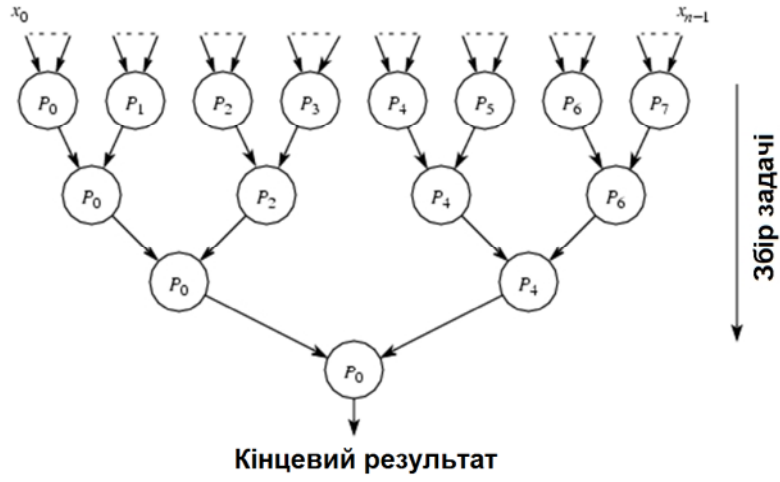


. 3.2



. 3.3





. 3.4

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### 4.1.2

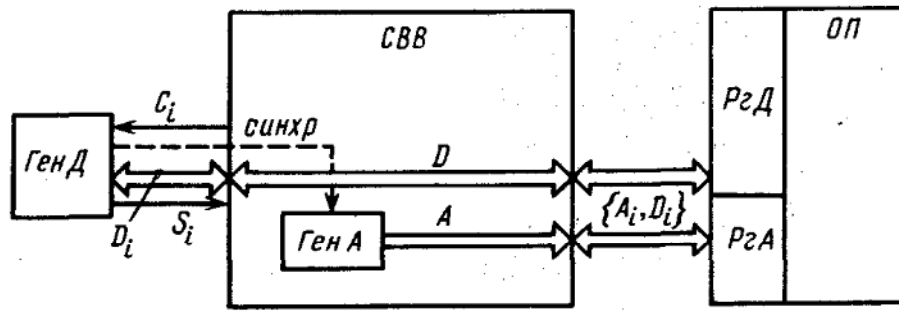
( )

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$S_{ij}$

$D_{ij}$

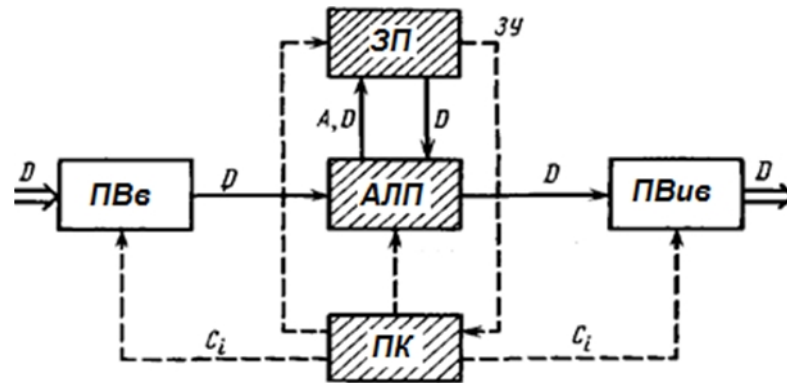
4.1.



4.1

( ),

. 4.2.



4.2

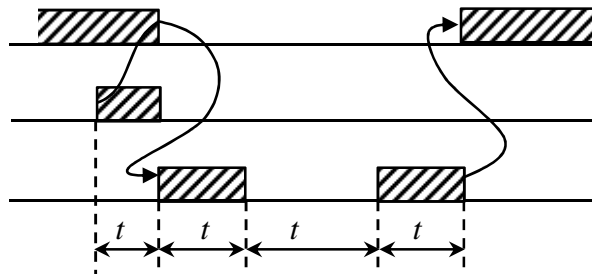
( )







### 4.1.3



### .4.4

( ).

## 4.2.1

### 4.2.

( ) (shared memory)

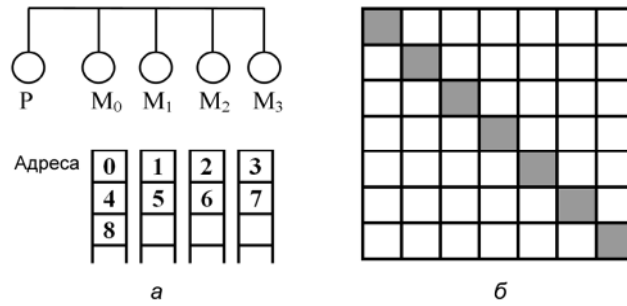
(distributed memory).

(closely coupled systems).  
 SIMD, MIMD.  
 SM-SIMD (Shared Memory SIMD) SM-MIMD  
 (Shared Memory MIMD).  
 (loosely  
 coupled systems).  
 SIMD, MIMD,  
 DM-SIMD (Distributed Memory SIMD) DM-MIMD  
 (Distributed Memory MIMD).  
 load R0, «  
 R0 ».  
 R0, — load  
 R0



4, ,  $v_1, v_4, v_8$ .  
 « » (stride).

( . 4.5, ).



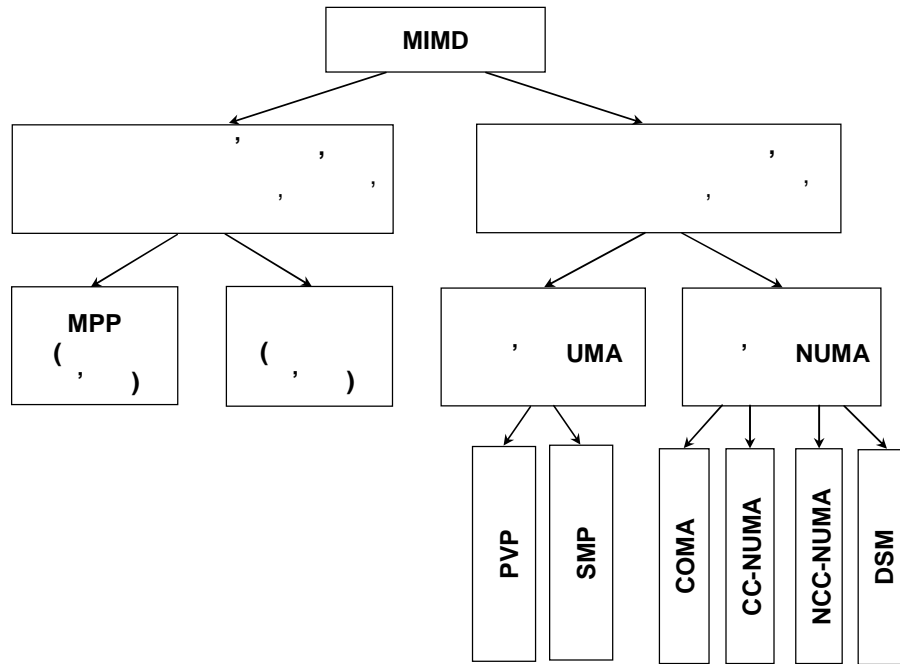
. 4.5 , : — 9 8 x 8.

## 4.2.2

. 4.6

MIMD ( SIMD).

UMA (Uniform Memory Access).



. 4.6.

UMA-

— (  $i$  ) ( )

. 4.7, .

$j$  (  $i$   $j$  )

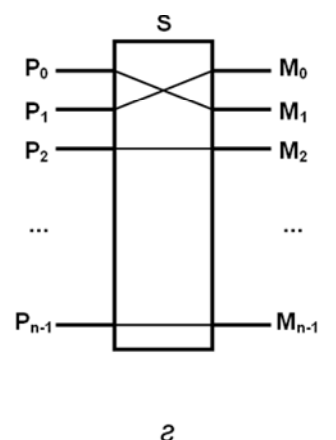
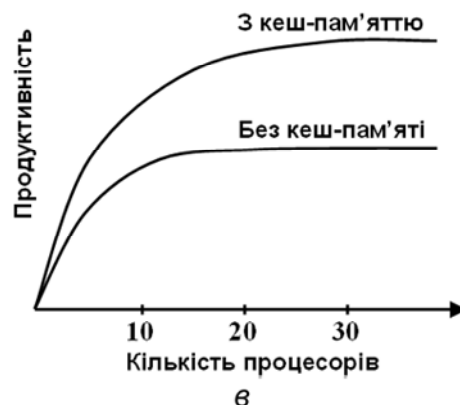
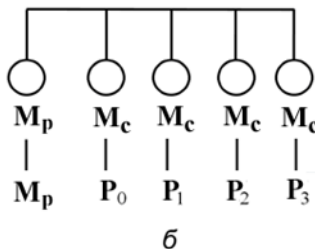
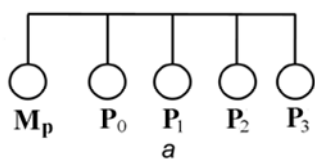
:

( . 4.7, )

11-

UMA

. 4.7, .



. 4.7.

UMA

4-8

32-64



NUMA (Non-Uniform Memory Access).

( . 4.3, )

( > 0,9)

( . 4.3, ),

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30

NUMA

, CC-NUMA NCC-NUMA.

( , Cache Only Memory

Architecture)

« »

NUMA.  
Cache Coherent Non-Uniform Memory Architecture)

CC-NUMA

(NCC-NUMA, Non-Cache Coherent Non-Uniform Memory Architecture)

CC-NUMA

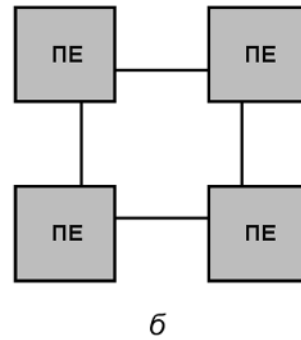
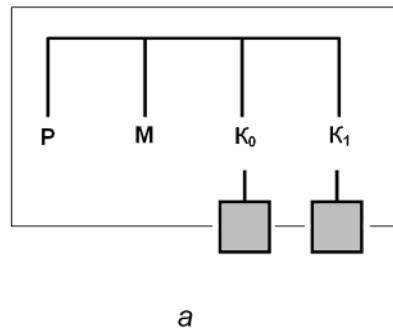
memory architectures). NUMA, (virtual shared CC-NUMA, DSM.

MIMD,

### 4.2.3

(NORMA, No Remote Memory Access).

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 ( 4.8, ) ( ).  
 ( ), ( )  
 / ( 0 K<sub>1</sub>). ( 4.8, )  
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.4.8.

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’ (DSM, Distribute Shared

Momory),

,

(SC-NUMA,

Software-Coherent Non-Uniform Memory Architecture).

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#### 4.2.4

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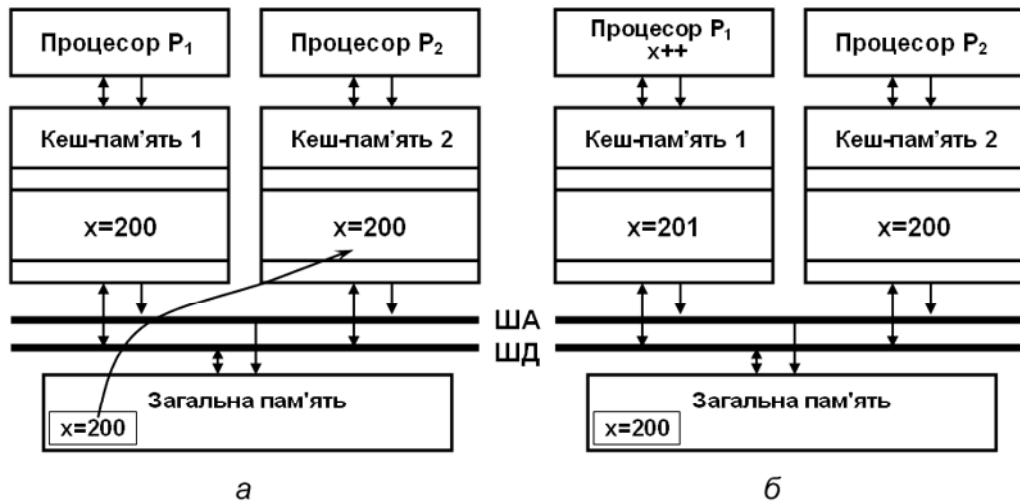
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(write through)

(write back).

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(write invalidate)

(write update).

(write broadcast).

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4

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6. ,
7. , : UMA, NUMA.
8. , : COMA, CC-NUMA, NCC-  
NUMA. .
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**5.1.2**

**C**

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 $b$   $( b \in )$ ,  
 $; b$   
 $( \in$   
 $)$ ,



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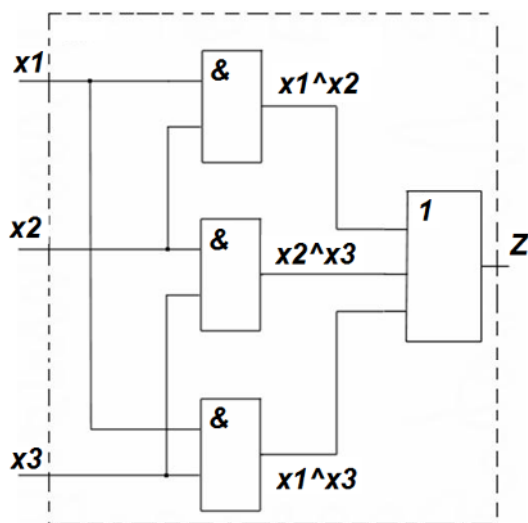
:  

$$(\ ) = 1 - [1 - (\ )]^{+1}, \quad (5.2)$$

- , .  
 , , 0,7  
 0,91, - 0,973.

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3».



.5.1

«2 3»

5», «4 7» . . .  
3»

«3

«2

. 5.1

$$: Z = x_1 x_2 + x_2 x_3 + x_1 x_3.$$

:

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n-







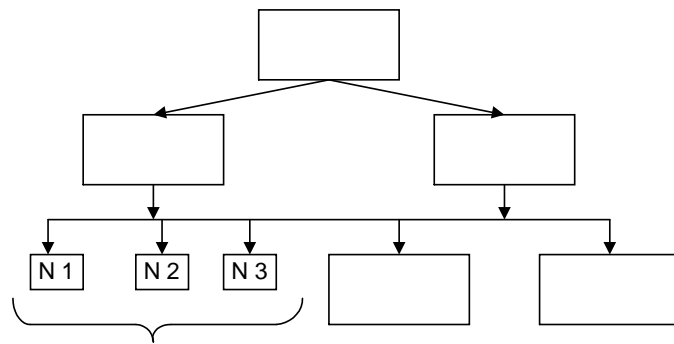
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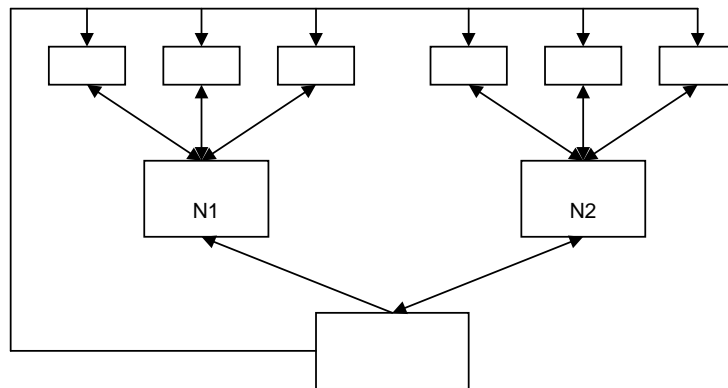
### 5.2.3

. 5.2.



. 5.2

. 5.3.



. 5.3.



## 5.2.4

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## 5.3.

### 5.3.1

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- 2) ;
- 3)

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### 5.3.2

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(heartbeat),

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2)

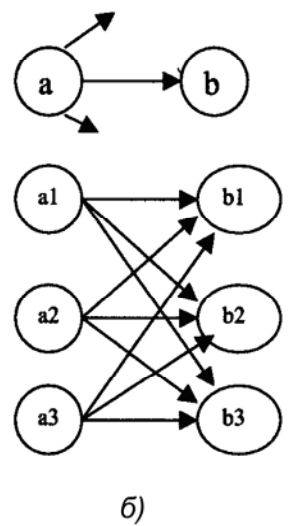
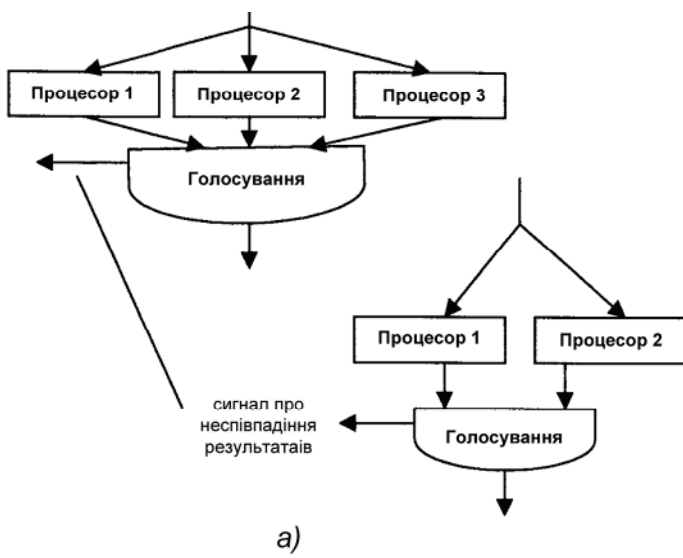
. 5.4,

, Tandem

. 5.4,

b,

$b_i$ ,  $b, i=1, 2, 3.$



. 5.4



$n$

$n^2$

$B_i, i=1, 2, 3,$

$a_i$

. 5.5.

. 5.5. .

5.5. .

2

1

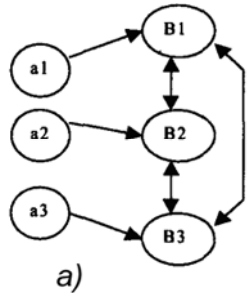
. 5.5. .

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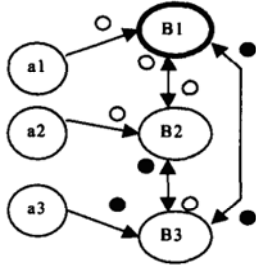
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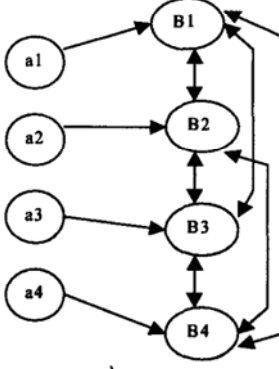
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a)



б)



в)

.5.5

### 5.3.3

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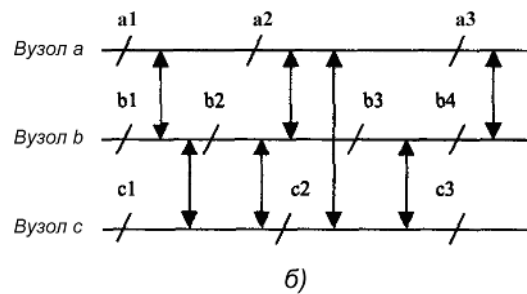
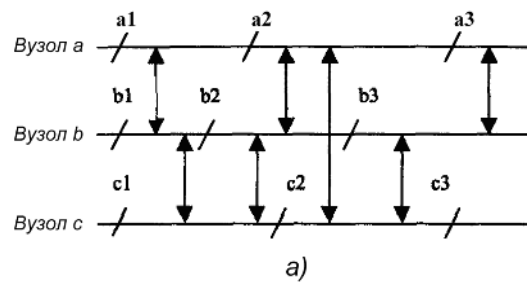
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3.3.  
 5.6, —  $a_1, 2, 3,$  —  $b_1, b_2, b_3,$   
 —  $c_1, 2, 3.$

3  
 —  $b$   
 3,  
 —  $b.$   
 —  $b$   $b_3,$

« ».



**.5.6**

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11. « »?

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, 2004. – 608 .
2. . . . .  
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3. . . . . – . . . . . , 1980 – 520 .
4. . . . . , . . . . . C++.  
– : . . . . . – 2002.
4. . . . . – . . . . . « . . . . » ,  
1999., - 320 .
5. . . . . , . . . . . :  
“ . . . . . ” , 2003
6. . . . . ’ . . . . . – . . . . . :  
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7. . . . . , . . . . . : , - . . . . . , 2002
8. . . . . : . . . . . :  
. . . . . , 2002.
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12. . . . . . . . . . .  
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14. . . . . . . . . . . / . . . . . , . . . . .

